



DEPARTMENT OF MATHEMATICS

UNIT-V Z-TRANSFORM

PROPERTIES of Z-TRANSFORM

i) Linear property:

$$\mathcal{Z}[af(n) + bg(n)] = aF(z) + bG(z)$$

$$\mathcal{Z}[af(n) + bg(n)] = \sum_{n=0}^{\infty} (af(n) + bg(n)) z^{-n}$$

$$= \sum_{n=0}^{\infty} a f(n) z^{-n} + \sum_{n=0}^{\infty} b g(n) z^{-n}$$

$$= a F(z) + b G(z)$$

ii) First shifting Theorem:

$$\mathcal{Z}[f(t)] = F(z) \text{ then } \mathcal{Z}[e^{-at} f(t)] = F[ze^{aT}]$$

$$\mathcal{Z}[e^{-at} f(t)] = F[e^{-ant} f(nt)]$$

$$= \sum_{n=0}^{\infty} e^{-ant} f(nt) z^{-n}$$

$$= \sum_{n=0}^{\infty} f(nT) (ze^{aT})^{-n}$$

$$= F(ze^{aT}).$$



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Result:

$$(i) z [e^{-at} f(t)] = F(z) \Big|_{z \rightarrow ze^{at}}$$

$$(ii) z [e^{at} f(t)] = F(z) \Big|_{z \rightarrow ze^{-at}}$$

(iii) Second shifting Theorem:

$$(i) z [f(n+1)] = z F(z) - z f(0)$$

(iv) Scaling in z-transform (or) multiplication by α^n :

$$z [\alpha^n f(n)] = \{F(z)\} \Big|_{z \rightarrow z/\alpha}$$

(v) Differentiation in z-domain:

$$z [n f(n)] = -z \frac{d}{dz} [\{F(z)\}]$$



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1) Find $z[e^{-iat}]$

$$z[e^{-iat}] = z[1 \cdot e^{-iat}]$$

$$= [z(1)]_{z \rightarrow ze^{iat}}$$

$$= \left[\frac{z}{z-1} \right]_{z \rightarrow ze^{iat}}$$

$$= \frac{ze^{iat}}{ze^{iat} - 1}$$

2) Find Z-transform of $\cos at$ & $\sin at$

$\checkmark z[\cos at]$ & $z[\sin at]$

$$z[e^{iat}] = z[1 \cdot e^{iat}]$$

$$= [z(1)]_{z \rightarrow ze^{iat}}$$

$$= \left[\frac{z}{z-1} \right]_{z \rightarrow ze^{iat}}$$



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$$= \frac{ze^{-iat}}{ze^{-iat} - 1}$$

Divide by e^{-iat}

$$= \frac{z}{ze^{-iat} - e^{iat}}$$

$$= \frac{z}{z - [\cos at + i \sin at]}$$

$$= \frac{z}{(z - \cos at) - i \sin at}$$

$$z[\cos at + i \sin at] = z[z - \cos at + i \sin at] \quad \begin{array}{l} \text{[Taking} \\ \text{conjugate]} \end{array}$$
$$(z - \cos at)^2 + (\sin at)^2$$

$$\Rightarrow z[\cos at] = \frac{z[z - \cos at]}{(z - \cos at)^2 + (\sin at)^2} = \frac{z[z - \cos at]}{z^2 - 2z \cos at + 1}$$

$$z[\sin at] = \frac{z \sin at}{z^2 - 2z \cos at + 1}$$



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$$1) z[n^2+2n+3]$$

$$\begin{aligned}z[n^2+2n+3] &= z[n^2] + 2z[n] + z[3] \\&= z[n^2] + 2z[n] + 3z[1] \\&= \frac{z[z+1]}{(z-1)^3} + 2 \cdot \frac{z}{(z-1)^2} + 3 \cdot \frac{z}{z-1} \\&= \underbrace{z^2 + z + 2z[z-1] + 3z[z-1]^2}_{(z-1)^3} \\&= \underbrace{z^2 + z + 2z^2 - 2z + 3z^3 + 3z^2 - 6z^2}_{(z-1)^3} \\&= \frac{3z^3 - 3z^2 + 2z}{(z-1)^3} = \frac{z[3z^2 - 3z + 2]}{(z-1)^3}\end{aligned}$$