Solving the Difference Equations using Z-Transforms

(i) $Z[y(n)] = F(z)$

(ii) $Z[y(n+1)] = Z[F(z) - y(0)] = zF(z) - zy(0)$

(iii) $Z[y(n+2)] = z^2F(z) - z^2y(0) - zy(1)$.

(iv) $Z[y(n+3)] = z^3F(z) - z^3y(0) - z^2y(1) - zy(2)$

1) Solve $y(n+2) - 3y(n+1) + 2y(n) = 2^n$ given $y(0) = 0$; $y(1) = 0$

Taking Z. OBS

$$Z[y(n+2)] - 3Z[y(n+1)] + 2Z[y(n)] = Z[2^n]$$

$$[z^2F(z) - z^2y(0) - zy(1)] - 3z[zF(z) - zy(0)] + 2F(z) = \frac{z}{z-2}$$

$$z^2F(z) - z^2y(0) - zy(1) - 3zF(z) + 3zy(0) + 2F(z) = \frac{z}{z-2}$$

$$z^2F(z) - 3zF(z) + 2F(z) = \frac{z}{z-2}$$

$$F(z)[z^2 - 3z + 2] = \frac{z}{z-2}$$

$$F(z) = \frac{z}{(z-2)[z^2-3z+2]} = \frac{z}{(z-2)^2(z-1)}$$

$$\frac{z}{(z-2)^2(z-1)} = \frac{A}{(z-2)} + \frac{B}{(z-2)^2} + \frac{C}{(z-1)}$$

$$z = A(z-2)(z-1) + B(z-1) + C(z-2)^2$$

put $z=1 \Rightarrow C=1$

put $z=2 \Rightarrow B=2$

put $z=0 \Rightarrow A=-1$

$$\frac{z}{(z-2)^2(z-1)} = -\frac{1}{(z-2)} + \frac{2}{(z-2)^2} + \frac{1}{z-1}$$

$$z^{-1}\left[\frac{z}{(z-2)^2(z-1)}\right] = -1\, z^{-1}\left[\frac{1}{z-2}\right] + 2\, z^{-1}\left[\frac{1}{(z-2)^2}\right] + z^{-1}\left[\frac{1}{z-1}\right]$$

$$z^{-1}[F(z)] = -(2)^{n-1} + (n-1)\,2^{n-1} + (1)^{n-1}$$