DEPARTMENT OF AGRICULTURAL ENGINEERING

COURSE CODE & NAME: 19MEB201 & FLUID MECHANICS AND MACHINERY

II YEAR / III SEMESTER

UNIT - 2

TOPIC : BOUNDARY LAYER

INTRODUCTION

A real fluid flows past a stationary solid boundary a layer of fluid which comes in contact with the boundary surface. *Ludwig Prandtl* in 1904 observed that in the case of real fluids velocity gradient existed only in a thin layer near the surface. This layer was named as boundary layer.

Laminar and Turbulent boundary layer:

- If the length of the plate is more than the distance. The thickness of boundary layer will go on increasing in the down stream direction. Then the laminar boundary layer becomes unstable and motion of fluid within it, is disturbed and irregular which leads to a transition from laminar to turbulent layer.
- This short length over which the boundary layer flow changes from laminar to turbulent is called transition zone. The boundary layer is turbulent and continues to grow in thickness. The layer of boundary is called turbulent boundary layer.

Boundary layer thickness:

It is defined as the distance from the boundary of the solid body measured in the y-direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity of the fluid.

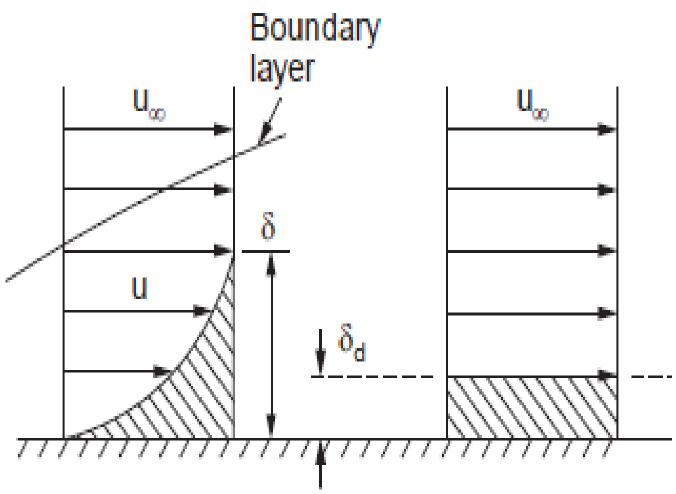
Displacement Thickness

It is the distance, measured perpendicular to the boundary by which the free stream is displaced an account of formation of boundary layer and denoted by δ^*

The reduction in volume flow is given by (for unit width)

$$= \int_0^{\delta} \rho\,(u_\infty - u)dy$$

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{u_\infty}\right) dy$$



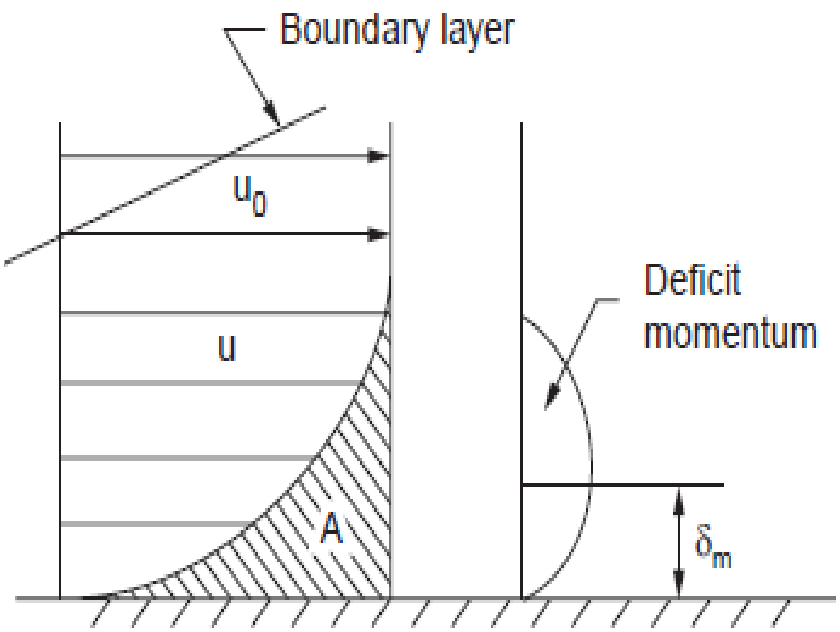
Momentum Thickness (θ)

As the distance through which total loss of momentum per second should be equal to if it were passing a stationary plate and denoted by θ.

$$\theta = \int_0^{\delta} \rho u(u_\infty - u)dy$$

$$\theta = \int_0^{\delta} \left[\frac{u}{u_\infty} - \left(\frac{u}{u_\infty}\right)^2\right] dy$$

$$= \int_0^{\delta} \frac{u}{u_\infty}\left[1 - \frac{u}{u_\infty}\right] dy$$



Energy Thickness:

It is defined as the distance measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in kinetic energy of the flowing fluid on account of boundary layer formation.

$$\delta_e = \frac{1}{u_\infty^3}\int_0^\delta u(u_\infty^2 - u^2)dy = \int_0^\delta \frac{u}{u_\infty}\left[1 - \frac{u^2}{u_\infty^2}\right]dy$$

Problem: 1

The velocity distribution in the boundary layer is given by $\frac{u}{U} = \frac{y}{\delta}$*, where* u *is the velocity at a distance* y *from the plate and* $u = U$ *at* $y = \delta, \delta$ *being boundary layer thickness. Find:*

(i) The displacement thickness (ii) The momentum thickness (iii) The energy thickness (iv) The value of $\frac{\delta^*}{\theta}$.

Solution:

The displacement thickness δ^*

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^{\delta} \left(1 - \frac{y}{\delta}\right) dy = \left[y - \frac{y^2}{2\delta}\right]_0^{\delta}$$

$$\delta^* = \left(\delta - \frac{\delta^2}{2\delta}\right) = \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

The momentum thickness, θ

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$\theta = \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{\delta}{6}$$

The energy thickness, δ_e

$$\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y^2}{\delta^2}\right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^3}{\delta^3}\right) dy$$

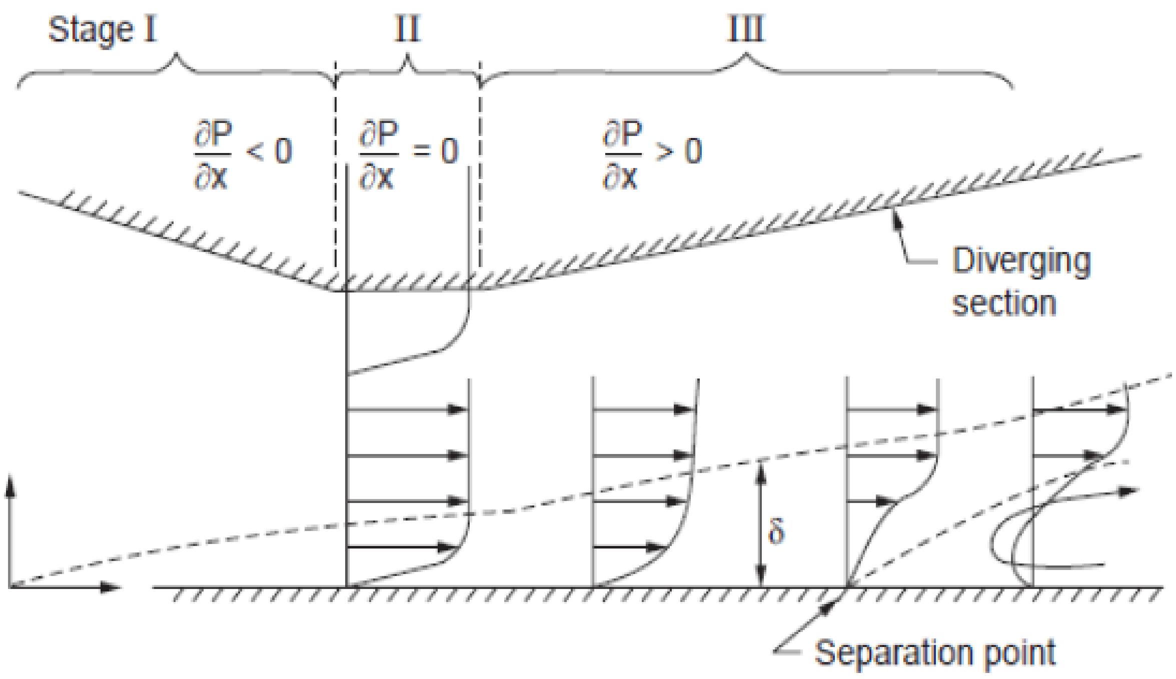
$$= \left[\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3}\right]_0^\delta = \frac{\delta}{4}$$

The value of $\frac{\delta^*}{\theta}$, $\qquad \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$

FLOW SEPARATION IN BOUNDARY LAYERS

In a flowing fluid when solid body is immersed, a thin layer of fluid called the boundary layer is formed adjacent to the solid body.

Pressure gradient in the direction of flow is negative $\frac{dp}{dx} < 0$ (when the pressure decreases the direction of flow, the flow is accelerated).



Pressure gradient in the direction of flow is positive $\frac{dp}{dx} > 0$ (the pressure force acts opposite to the direction of flow and increasing the viscous force, the thickness of boundary layer increases in the direction of flow this phenomenon is called separation).

The separation depends upon the following factors,

- The Curvature of the surface.
- Reynolds number of flow.
- The roughness of the surface.

Experimental results are used to predict such conditions.

- If $\left(\frac{\partial u}{\partial y}\right)_{y=0} = (-)\, is\ negative$ the flow has separated.
- If $\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$ the flow is on the verge of separation
- If $\left(\frac{\partial u}{\partial y}\right)_{y=0} = (+)\, is\ positive$ the flow will not separate or flow will remain attached with the surfaces.

Separation occurs in the following cases:

- Diffuser
- Open channel transitions
- Pumps
- Fans
- Aerofoils
- Turbine blades

Methods of preventing the separation of boundary layer:

- Suction of the slow moving fluid by a suction slot.
- Supplying additional energy from blower.
- Providing a bypass in the slotted wing.
- Rotating boundary in the direction of flow.
- Providing a small divergence in the diffuser.
- Providing guide blades in a bend.
- Providing a trip –wire ring in the laminar region for the flow over a sphere.

Problem: 2

For the following velocity profiles determine whether flow is attached or detached or on the verge of separation.

(i) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ *(ii)* $\frac{u}{U} = -2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^3 + 2\left(\frac{y}{\delta}\right)^4$

(iii) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3 - 2\left(\frac{y}{\delta}\right)^4$

Solution:

(i) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$ or $u = 2U\left(\frac{y}{\delta}\right) - U\left(\frac{y}{\delta}\right)^2$

Differentiating w.r.t. y

$$\frac{du}{dy} = 2U\left(\frac{1}{\delta}\right) - 2U\left(\frac{y}{\delta}\right) \times \frac{1}{\delta}$$

At $y = 0$, $\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{2U}{\delta}$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is positive, and then given *flow is attached.*

$(ii)\ \frac{u}{U} = -2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^3 + 2\left(\frac{y}{\delta}\right)^4$

$$u = -2U\left(\frac{y}{\delta}\right) + U\left(\frac{y}{\delta}\right)^3 + 2U\left(\frac{y}{\delta}\right)^4$$

$$\frac{\partial u}{\partial y} = -2U\left(\frac{1}{\delta}\right) + 3U\left(\frac{y}{\delta}\right)^2 \times \frac{1}{\delta} + 8U\left(\frac{y}{\delta}\right)^3 \times \frac{1}{\delta}$$

At $y = 0, \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{2U}{\delta}$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is negative, and then given *flow is detached.*

(iii) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3 - 2\left(\frac{y}{\delta}\right)^4$

$$u = 2U\left(\frac{y}{\delta}\right)^2 + U\left(\frac{y}{\delta}\right)^3 - 2U\left(\frac{y}{\delta}\right)^4$$

$$\frac{\partial u}{\partial y} = 4U \times \left(\frac{y}{\delta}\right) \times \frac{1}{\delta} + 3U\left(\frac{y}{\delta}\right)^2 \times \frac{1}{\delta} - 8U\left(\frac{y}{\delta}\right)^3 \times \frac{1}{\delta}$$

At $y = 0, \left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is the given flow is *on the verge of separation.*

Drag Force and Coefficient of Drag

Drag is the component of force acting parallel to the direction of motion. Using the method of dimensional analysis the drag force can be related to flow Reynolds number by

$$\frac{F_D}{\rho A V^2} = f(R_e)$$

Defining coefficient of drag as the ratio of drag to dynamic pressure, it is seen that

$$C_D = f(R_e) = \frac{F_D}{(1/2)\rho A V^2}$$

This applies to viscous drag only. In case wave drag is encountered, then

$$C_D = f(R_e, F_r)$$

If compressibility effect is to be considered

$$C_D = f(R_e, M)$$

Friction coefficient over flat plate in laminar flow, at a location was defined by

$$C_{fx} = \tau_w/(1/2)\rho AV^2 = 0.664/Re_x^{0.5}.$$

Over a given length the average value is obtained as twice this value. For a flat plate of length L, in *laminar flow*

$$C_D = 1.328/Re_L^{0.5}$$

In turbulent flow in the range $5 \times 10^5 > R_e < 10^7$

$$C_D = 0.074/Re_L^{0.2}$$

For Re_L up to 10^9 an empirical correlation due to Schlichting is

$$C_D = 0.455/(\log Re_L)^{2.58}$$

For *combined laminar and turbulent flow* in the range $5 \times 10^5 > R_e < 10^7$

$$C_D = 0.074/Re_L^{0.2} - 1740/Re_L$$

For the range $5 \times 10^5 > R_e < 10^9$

$$C_D = 0.455/(\log Re_L)^{2.58} - 1610/Re_L$$

The values of C_D for laminar flow is in the range 0.002 to 0.004.

Problem:3

A plate 450 $mm \times 150$ mm has been placed longitudinally in a stream of crude oil (Sp.gravity 0.925 and kinematic viscosity of 0.9 stoke) which flows with a velocity of $6\ m/sec$ *. Calculate*

i. *The friction drag on the plate*

ii. *Thickness of boundary layer at the trailing edge.*

iii. *Shear stress at the trailing edge.*

Solution:

Length of the plate, $L = 450\ mm = 0.45\ m$

Width of the plate, $B = 150\ mm = 0.15\ m$

Specific gravity of oil, $S = 0.925$

kinematic viscosity of oil, $\nu = 0.9\ stoke = 0.9 \times 10^{-4}\ m^2/sec$

The friction drag on the plate, F_D

Reynolds number at the end of plate,

$$Re_L = \frac{UL}{\nu}$$

Since $Re_L < 5 \times 10^5$, the flow over the plate is entirely laminar.

∴ Average coefficient of drag,

$$C_D = 1.328/Re_L^{0.5} = \frac{1.328}{\sqrt{30000}} = 0.007667$$

Drag on the one side of the plate,

$$C_D = \frac{F_D}{(1/2)\rho A V^2}$$

$$F_D = 8.63\ N$$

Thickness of boundary layer at the trailing edge

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

∴ thickness at the trailing edge $(x = 0.45\, m)$, $Re_x = 30000$

$$\delta = \frac{5x}{\sqrt{Re_x}} = 0.013\, m$$

Shear stress at the trailing edge.

Local coefficient of drag,

$$C_{fx} = \tau_w/(1/2)\rho A V^2 = 0.664/Re_x^{0.5}$$

$$\tau_w = 63.77\ \ N/m^2$$

THANK YOU