Mass moments of inertia:

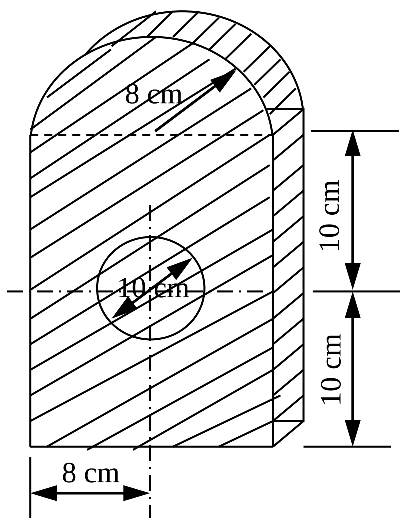
The moments of inertia of solid figures are refined as "mass moment of inertia". It is denoted by (I_{max}) or simply I_M.

Problem 21: A thin steel plate 4mm thick is cut and bent as shown in fig. If the density of the Steel is $7850\ kg/m^3$, determine the mass moment of inertia of the plate with respect to the centroidal axes xx and yy.

Solution:

Location of centre of gravity Divide the plate into three components

Due to symmetry, $\bar{x}$=8cm



Component 1: Rectangular plate,

$$\left(\frac{16}{100} \times \frac{20}{100} \times \frac{40}{100}\right) m$$

$$mass, m_1 = \rho tbd$$

$$= 7850 \times \frac{4}{1000} \times \frac{16}{100} \times \frac{20}{100} = 1.005\ kg$$

$$y_1 = \frac{10}{100} = 0.1\ m$$

Component 2: Semi-circular plate, $\frac{8}{100} m$ radius

$$mass, m_2 = \rho t \left(\frac{\pi r^2}{2}\right)$$

$$= 7850 \times \frac{4}{1000} \times \frac{\pi}{2} \times 0.08^2 = 0.3157\ kg$$

$$y_2 = 0.2 + \frac{4 \times 0.08}{3\pi} = 0.234\ m$$

Component3: Circular hole, 0.05m radius

$$mass, m_3 = \rho t \pi r^2$$

$$= 7850 \times \frac{4}{1000} \times \pi \times 0.05^2$$

$$= 0.2466\ kg$$

$$y_3 = 0.1m$$

$$\bar{y} = \frac{m_1 y_1 + m_2 y_2 - m_3 y_3}{m_1 + m_2 - m_3} = 0.14m$$

Mass moment of inertia about xx axis

$$(I_{XX})_{mass} = (I_1)_{mass} + (I_2)_{mass} - (I_3)_{mass}$$

From parallel axis theorem,

$$(I_1)_{mass} = (I_{G1})_{mass} + m_1 \bar{h}_1^2$$

$$= \frac{m_1 d_1^2}{12} + m_1(\bar{y} - y_1)^2$$

$$= \left(\frac{1.005 \times (0.2)^2}{12}\right) + (1.005 \times (0.14 - 0.1)^2)$$

$$= 4.958 \times 10^{-3}\ kg.m^2$$

$$(I_2)_{mass} = (I_{G2})_{mass} + m_2 \bar{h}_2^2$$

$$= 0.2176\ m_2 r^2 + m_2(\bar{y} - y_2)^2$$

$$= 3.229 \times 10^{-3}\ kg.m^2$$

$$(I_3)_{mass} = (I_{G3})_{mass} + m_3 \bar{h}_3^2$$

$$= \frac{m_3 r^2}{4} + m_3(\bar{y} - y_3)^2$$

$$= 5.486 \times 10^{-4}\ kg.m^2$$

$$\therefore\ (I_{XX})_{mass} = 7.6384 \times 10^{-3} kg.m^2$$

Mass moment of inertia about YY axis

$$(I_{YY})_{mass} = (I_1)_{mass} + (I_2)_{mass} - (I_3)_{mass}$$

$$= \frac{m_1 b^2}{12} + \frac{m_2 r^2}{4} - \frac{m_3 r^2}{4}$$

$$= 2.495 \times 10^{-3} kg.m^2$$