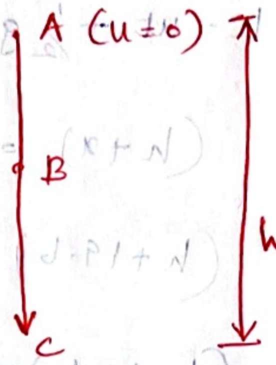
(P) A body falling from rest falls $1/5^{th}$ of total distance of its fall in the last second. When was the body dropped and from what height it was dropped? $g = 9.81\ m/s^2$



In AC

$t = t$ sec height $= h$

In AB

$t = (t-1)$

$BC = \frac{1}{5}h$ $AB = \frac{4}{5}h$

$s = ut + \frac{1}{2}gt^2$

AB $\Rightarrow s = ut + \frac{1}{2}gt^2$

$s = 0 \times (t-1) + \frac{1}{2} \times 9.81 \times (t-1)^2$

$s = 4.9(t-1)^2$

AC $\Rightarrow s = ut + \frac{1}{2}gt^2$

$= (0 \times t) + \frac{1}{2} \times 9.81 \times t^2$

$h = 4.9t^2$

$AB = \frac{4}{5}h$

$4.9(t-1)^2 = \frac{4}{5} \times 4.9t^2$

$(t-1)^2 = \frac{4}{5}t^2$

$5(t-1)^2 = 4t^2$

$5(t^2 + 1 - 2t) = 4t^2$

$t^2 - 10t + 5 = 0$

$t = 9.472$ sec. (because t is > 1).

then height $h = 4.9 \times (9.47)^2 = 439.62\,m$

$\boxed{h = 439.62\,m}$

(P) A stone is dropped into a well and the sound of splash is heard in 6 seconds. If the Velocity of sound is 160 m/s. find the depth upto water level in well.

$g = 9.81\, m/s^2$.

Soln:

↳ The stone is dropped into well and its splash sound is heard in 6 seconds includes, the time taken by the stone to reach the water level in the well and the time taken by its sound.

$t_1 \rightarrow$ time taken by stone to reach the water level

$h \rightarrow$ depth of water level in the well

$$h = ut_1 + \frac{1}{2}at_1^2$$

$$= (0 \times t_1) + \frac{1}{2} \times 9.81 \times t_1^2$$

$$\boxed{h = 4.9\, t_1^2} \rightarrow (1)$$

for the sound to travel this distance h, if the time taken is t_2

Distance = uniform velocity × time

$$h = 160 \times t_2$$

$$h = 160t_2 \rightarrow (2)$$

But $t_1 + t_2 = 6$

then, $t_2 = 6 - t_1$

put $t_2 = 6 - t_1$ in (2)

$$h = 160(6-t_1) \rightarrow ③$$

equaling ③ & ①.

$$160(6-t_1) = 4.9t_1^2$$

$$960 - 160t_1 = 4.9t_1^2$$

$$4.9t_1^2 + 160t_1 - 960 = 0$$

$$t_1 = 5.178 \text{ sec}$$

$$h = \cancel{160} \; 4.9t_1^2 = 4.9(5.17)^2 = 131.37 \text{ m}$$

Depth of water level = $\underline{\underline{131.37 \, m}}$

Rectilinear motion with variable acceleration:

→ particle is moving in a st. line with variable acceleration which leads to variable Parameter like velocity, displacement, acceleration, etc. etc.

case ①: Displacement is a function of time.

$$s = f(t) \rightarrow ①$$

$$v = \frac{ds}{dt}$$

$$v = \frac{d(f(t))}{dt} \rightarrow ②$$

$$a = \frac{dv}{dt} = \frac{d^2}{dt^2}(f(t)) \rightarrow ③$$

(P) A particle moves along a st. line with variable acceleration. If the displacement is measured in m and given by relation interms of time taken t, as follows

$$S = 3t^3 + 2t^2 + 7t + 3$$

Det

(1) Velocity of particle at start and after 3 sec.

(2) The acceleration of particle at start and after 3 sec.

Given

$$S = 3t^3 + 2t^2 + 7t + 3$$

$$V = \frac{ds}{dt}$$

$$V = \frac{d}{dt}(3t^3 + 2t^2 + 7t + 3)$$

$$V = 9t^2 + 4t + 7 \rightarrow (1)$$

(1) Velocity at start

To find the velocity of particle at start, substitute

$t = 0$ in eqn (1)

$$V_0 = (9 \times 0) + (4 \times 0) + 7$$

$$V_0 = 7 \, m/s.$$

Velocity after 3 sec

sub $t = 3$ in (1)

$$V_3 = 9(3)^2 + 4(3) + 7$$

$$V_3 = 100 \, m/s.$$

(2) Acceleration

$$a = 18t + 4 \rightarrow (2)$$

acceleration at start, sub $t = 0$

$$a = 4 \, m/s^2$$

Acceleration after 3 sec

sub $t = 3$ in eqn (2)

$$a_3 = (18 \times 3) + 4 = 58 \, m/s^2$$

(P) The eqn of motion of a particle moving in a st. line with variable acceleration is given by

$$S = 15t + 3t^2 - t^3$$

Calculate

(1) the velocity and acceleration at start

(2) the time, at which the particle attains its max velocity

(3) the max. velocity of particle.

$$S = 15t + 3t^2 - t^3$$

$$v = \frac{ds}{dt} = \frac{d}{dt}(15t + 3t^2 - t^3)$$

$$= 15 + 6t - 3t^2 \rightarrow (1)$$

$$a = \frac{dv}{dt} = 6 - 6t \rightarrow (2)$$

(1) Velocity at start

sub $t = 0$ in eqn (1)

$$v_0 = 15 + (6 \times 0) - 3(0)^2 = 15 \, m/s$$

(2) Acceleration at start

$t = 0$ in eqn (2)

$$a_0 = 6 - 6(0) = 6 \, m/s^2$$

(3) Velocity of particle will be max, when the differentiation of velocity with time is equal to zero.

$\frac{dv}{dt}$ is given by eqn (2) hence equate it to zero

$$6-6t=0$$

$$6t=6$$

$$\boxed{t = 1\,sec}$$

The velocity will be max when $t = 1\,sec$.

Max Velocity

$V = 15 + 6t - 3t^2$ sub $t = 1$

$V_{max} = V_1 = 15 + 6(1) - (3\times1) = 18\,m/s$

Case (2) Acceleration is a function:

acceleration, $a = f(t) \rightarrow (1)$

integrating the eqn (1) w.r.t 't' we get

velocity, $v = \int f(t)\,dt \rightarrow (2)$

integrating the eqn (2) we get displacement

$s = \int v\,dt$

$= \iint f(t)\,dt \rightarrow (3)$

(P) The eqn of motion of a particle is given, acceleration (a) in terms of (t) is given below

$a = 3t^2 + 2t + 4$. It is observed that the velocity of particle is $12\,m/s^2$ after 4 seconds and the displacement of particle is 8 after 4 sec. Determine

(1) Velocity after 8 sec

(2) Displacement after 2 seconds.

Given $a = 3t^2 + 2t + 4$

Bounday Conditions

$t = 4$ sec $\quad V = 12\, m/s$

$t = 4$ sec $\quad S = 8m$.

Velocity $v = \int a\, dt$

$$v = \int (3t^2 + 2t + 4)\, dt$$

$$= \frac{3t^3}{3} + \frac{2t^2}{2} + 4t + c_1$$

$$V = t^3 + t^2 + 4t + c_1 \rightarrow (1)$$

To find the constant of integration C_1, apply the boundary condition of velocity

$V = 12\, m/s \quad t = 4$ sec

$$12 = 4^3 + 4^2 + (4 \times 4) + C_1$$

$$C_1 = -84$$

Now sub C_1 in eqn (1)

$$\therefore V = t^3 + t^2 + 4t - 84.$$

Displacement

$$S = \int v\, dt = \int (t^3 + t^2 + 4t - 84)\, dt$$

$$S = \frac{t^4}{4} + \frac{t^3}{3} + 4\frac{t^2}{2} - 84t + C_2 \rightarrow (2)$$

To find the constant of integration C_2, apply the boundary condition of diplacement $t = 4$ sec $S = 8m$.

$$\therefore 8 = \frac{4^4}{4} + \frac{4^3}{3} + 2(4)^2 - (84 \times 4) + C_2$$

$$C_2 = 226.66$$

sub C_2 in eqn (2)

$$S = \frac{t^4}{4} + \frac{t^3}{3} + 2t^2 - 84t + 226.66$$

(1) Velocity after 8 sec

sub $t = 8$ in eqn.

$$V = t^3 + t^2 + 4t - 84$$

$$= 8^3 + 8^2 + (4 \times 8) - 84 = 524 \text{ m/s}.$$

(2) Displacement after 2 sec

sub $t = 2$ in displacement eqn

$$S = \frac{t^4}{4} + \frac{t^3}{3} + 2t^2 - 84t + 226.66$$

$$= \frac{2^4}{4} + \frac{2^3}{3} + 2(2)^2 - (84 \times 2) + 226.66$$

$$= 73.32 \text{ m}$$

(3) Acceleration is a function of velocity.

(P) A particle starts from rest with an acceleration given by the relation $a = \frac{2}{(2+V)}$ m/s². Determine the distance covered by the particle. When it acquires a velocity of 72 kmph.

$V \rightarrow$ velocity of particle in m/s.

Given $a = \frac{2}{2+v}$

$a = v \cdot \frac{dv}{ds}$.

$v \cdot \frac{dv}{ds} = \frac{2}{2+v} \Rightarrow v(2+v)\,dv = 2ds \rightarrow (1)$

$(2v+v^2)\,dv$

On integrating.

$$\frac{2v^2}{2} + \frac{v^3}{3} = 2s + c$$

Given that velocity starts from rest

so $t = 0$ $v = 0$ $s = 0$ and $c = 0$

$$v^2 + \frac{v^3}{3} = 2s \rightarrow (2)$$

To find the distance covered by the particle when it attains speed of 72 kmph.

$$v = \frac{72 \times 1000}{3600} = 20\ m/s$$

sub $v = 20\ m/s$ in (2)

$$20^2 + \frac{20^3}{3} = 2s$$

$$\boxed{s = 1533.3\ m}$$

(4) Acceleration is a function of distance.

(P) The acceleration of a body starting from rest is given by the relationship

$a = 15 - 2s$.

where a is the acceleration in m/s^2 ; S is the distance travelled in m. Determine

① Velocity of body when it has travelled 4m.

② distance travelled when the body is again at rest.

Given

$$a = 15 - 2S.$$

$$a = v \cdot \frac{dv}{ds} = 15 - 2s$$

$$v\,dv = (15 - 2s)\,ds.$$

On integrating

$$\frac{v^2}{2} = 15s - \frac{2s^2}{2} + c$$

$$= 15s - s^2 + c$$

To find c

The body start from rest $t = 0$ $v = 0$ $s = 0$

$$\boxed{c = 0}$$

$$\frac{v^2}{2} = 15s - s^2$$

$$v^2 = 30s - 2s^2 \rightarrow ①$$

When $s = 4m$ in ①

$$v_4^2 = 30(4) - 2(4)^2$$

$$v_4 = \sqrt{30(4) - (2 \times 4^2)}$$

$$v_4 = 9.38 m/s$$