Unit - 4

Dynamics of Particle

Kinematics:

↳ study of motion of a moving body without considering the forces which cause the motion, and kinetics is the study of motion of a moving body with also considering the external forces which causes the motion.

Types of Plane motion:

Translation motion of a particle can be classified into 2 types

1. Rectilinear motion
2. Curvilinear motion.

→ The motion of a particle along a straight line is known as rectilinear motion (or straight line motion).

→ The motion of a particle along a curved path is known as curvilinear motion.

Displacement:-

↳ The displacement of a moving particle is the change in its position, during which the particle remains in motion. It is a vector quantity, has both magnitude and sense of direction.

Speed:-

↳ It is the distance travelled by the particle or body along its path per unit time. It is a scalar quantity has magnitude only. m/s, km/hr

$$Speed = \frac{\text{Distance travelled}}{\text{time taken}}$$

Velocity:-

$$Velocity = \frac{\text{Distance travelled in particular direction}}{\text{time taken}}$$

. When a particle moves in one direction and it covers ual distance in equal intervals of time, then the velocity of the rticle is known as uniform Velocity.

- When there is a change in direction, or change in magnitude change in magnitude and direction, then the velocity is known as ariable velocity.

Acceleration:

→ Rate of change of velocity.

$$\text{Acceleration, } a = \frac{\text{change of velocity}}{\text{Time taken}} = \frac{\Delta x}{\Delta t} \cdot \frac{dv}{dt}.$$

Average velocity:

↳ $\text{Average velocity} = \frac{\text{Change in Position}}{\text{Change in time}} = \frac{\Delta x}{\Delta t}$

Average Speed:-

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

If a particle starts from a point and then if it returns to the same point, average velocity is zero, But average speed is not zero.

Instantaneous Velocity:

↳ the Instantaneous velocity at any instant of time is the limit of average velocity as the increment of time approaches zero.

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$V = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2}.$$

Rectilinear motion with uniform acceleration :

A particle which is moving with uniform acceleration is analyzed by the equation of motion.

let u = Initial velocity s = Distance travelled by the particle

v = Final Velocity

t = time taken by the particle to change from u to v (sec)

a = acceleration of the particle

$$a = \frac{v-u}{t}$$

$$\boxed{v = u + at} \rightarrow ①$$

average velocity $= \frac{u+v}{2}$

$$s = \left(\frac{u+v}{2}\right) t$$

$$s = ut + \frac{1}{2}at^2 \rightarrow ②$$

$$t = \frac{v-u}{a}$$

$$v^2 = u^2 + 2as \rightarrow ③.$$

* If a body starts from rest, the initial velocity is zero ($u=0$)

* If a body comes to rest, the final velocity is zero ($v=0$).

(P) A car moving with a velocity of 20 m/s. The car bought to rest by applying brakes in 6 sec.

Find ① retardation ② Distance travelled by the car after applying the brakes.

Initial velocity $u = 20m/s$

Final velocity $v = 0$ (∵ car is bought to rest)

$t = 6$ sec.

(1) Retardation (−ve acceleration)

$$v = u + at$$

$$0 = 20 + (a \times 6)$$

$$a = -3.33 m/s^2$$

$$\text{retardation} = 3.33 m/s^2.$$

(2) Distance travelled.

S = Distance travelled by the car after applying the brakes

$$S = ut + \frac{1}{2}(at)^2$$

$$S = (20 \times 6) + \frac{1}{2}(-3.33 \times 6^2) = 60m.$$

(P) A Train starts from rest and attains a velocity of 45kmph in 2mins with uniform acceleration. Calculate

(1) acceleration (2) Distance travelled in this time, 2min

(3) Time required to reach a velocity of 36kmph

$u = 0$ $v = 45kmph$

$$v = \frac{45 \times 1000}{3600} m/s = 12.5 m/s$$

$$t = 2min = 2 \times 60 = 120 sec$$

(1) Acceleration (a)

$$v = u + at$$

$$12.5 = 0 + (a \times 120)$$

$$a = 0.104 m/s^2$$

② Distance travelled (s) in 2mins.

$$s = ut + \frac{1}{2}at^2 = (0 \times 120) + \left(\frac{1}{2} \times 0.104 \times 120^2\right)$$

$$= 748.8m.$$

③ Time required to attain velocity of 36 kmph

$u = 0$

$v = 36 \text{ kmph} = \frac{36 \times 1000}{3600} = 10 m/s$

$v = u + at$

$10 = 0 + (0.104 \times t)$

$t = 96.15 sec$

(P) A Burglar's car had a start with an acceleration of $2m/s^2$. A police vigilant party came after 5 sec and continued to chase the burglar's car with a uniform velocity of 20m/s. Find the time taken in which the police van will overtake the burglar's car?

Soln:

Initial velocity of Burglar's car = 0

acceleration of Burglar's car = $2 m/s^2$

Police van came after 5 seconds of the start of Burglar's car

uniform velocity of police van = 20m/s.

To find.

→ Time taken ~~in which~~ by the police van to overtake the Burglar's car

→ let t ⇒ time taken by police to overtake the Burglar's car.

Motion of Burglar's car

$u = 0$ $a = 2 m/s^2$ $t = (t+5)$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times (t+5)^2$$

$$s = (t+5)^2$$

Motion of police ~~van~~ vigilant Party

uniform velocity = 20 m/s

⇒ Distance travelled by police van from the starting point of Burglar's car and to overtake it

s = uniform velocity × time taken

$= 20t$

Police van overtakes the Burglar's car, hence the distance travelled by Burglar's car and police van should be equal

$$(t+5)^2 = 20t$$

$$t^2 + 25 + 10t - 20t = 0$$

$$t^2 - 10t + 25 = 0$$

$$t = \frac{10 \pm \sqrt{10^2 - (4 \times 1 \times 25)}}{2 \times 1} = \frac{10}{2} = 5 \text{ sec}$$

(P) A motor starts from rest and uniformly accelerated to a speed of 20kmph over a distance of 200m. Calculate the acceleration and time taken. If further acceleration raises the speed to 50kmph in 8 seconds. find the acceleration and further distance moved?

Soln: Case (1).

Given $u = 0$ $s = 200m$

$v = 20kmph = \frac{20 \times 1000}{3600} = 5.55 \, m/s.$

To find acceleration:

$v^2 = u^2 + 2as$

$(5.55)^2 = 0^2 + 2(a \times 200)$

$a = 0.077 \, m/s^2$

$v = u + at$

$5.55 = 0 + (0.077 \times t)$

$t = 72.14 \, sec$

Case (2) $u = 5.55 \, m/s$

u = Final Velocity in case (i)

$V = 50 \, kmph = \frac{50 \times 1000}{3600} = 13.89 \, m/s$

$t = 8 \, sec$

To find acceleration $v = u + at$

$13.89 = 5.55 + (a \times 8)$

$a = 1.0418 \, m/s^2$

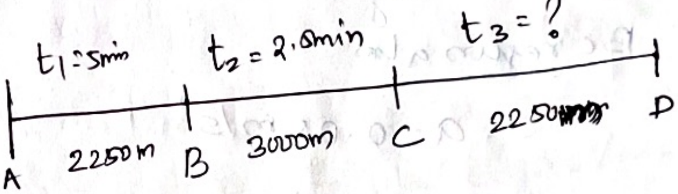
To find further distance moved.

$s = ut + \frac{1}{2}at^2$

$= (5.55 \times 8) + (\frac{1}{2} \times 1.0418 \times 8^2)$

$= 77.78 \, m$

(4) A train is travelling from A and D along the track as shown in diagram. Its initial velocity at A is zero. The train takes 5min to cover the distance AB, 2250m length and 2.5 min to cover BC; 3000m in length on reaching the station C, the brakes applied and the train stops 2250m beyond, at D (i) Find the retardation on CD (ii) The time it takes the train to get from A to D (iii) Its average speed for the whole distance.



Soln:

Given AB = 2250m CD = 2250m

BC = 3000 m

$t_1 = 5min$ $t_2 = 2.5min$ $t_3 = ?$

Brakes are applied at C

Train starts from A and stops at D.

(1) Retardation of train on CD.

$u = 0$ (At A)

$s = 2250\,m$

Final velocity at B = V

$t = 5\,min = 300\,sec.$

$$s = ut + \frac{1}{2}at^2$$

$$2250 = 0 + \left(\frac{1}{2} \times a \times 300^2\right)$$

$$a = 0.05\,m/s^2$$

$$v = u + at$$
$$= 0 + (0.05 \times 300)$$

$v = 15 m/s$

Next Consider the motion of train on BC

Initial velocity at B $u = 15 m/s$

Final velocity at C : v

$s = 3000 m$

$t = 2.5 min = 150 sec$

The train is moving on constant acceleration from A to C so on BC region also

$a = 0.05 m/s^2$

$v = u + at$

$= 15 + (0.05 \times 150)$

$v = 22.5 m/s$

We will consider the motion of the train on CD

Initial velocity (at C) $u = 22.5 m/s$

Final Velocity (at D) $= v = 0$

$s = 2250 m$ $t = ?$

$v^2 = u^2 + 2as$

$0 = (22.5)^2 + (2 \times a \times 2250)$

$a = -0.1125 m/s^2$ (retardation).

$v = u + at$

$0 = 22.5 - (0.1125 \times t)$

$t = 200 sec$ (or) $3.33 min$.

(2) Time taken from A to D

$T = t_1 + t_2 + t_3$

$= 5 + 2.5 + 3.333$

$= 10.833 min$

Average Speed

for whole distance = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{2250 + 3000 + 2250}{10.833 \times 60}$

$= 11.538 \, m/s$

$= \frac{11.538}{1000} \times 3600 = 41.53 \, kmph$

(P) A particle moves along st. line according to the eqn $x = t^3 - 12t^2 + 10$ where x is in meters and t is in seconds. Find the velocity of the particle at which acceleration is zero.

Given $x = t^3 - 12t^2 + 10$

$v = \frac{dx}{dt} = 3t^2 - 24t$

$a = \frac{d^2x}{dt^2} = 6t - 24$

Time at acceleration is zero

$0 = 6t - 24$

$t = 4 \, sec.$

Velocity at 4 sec (ie) when $a = 0$

$V_4 = 3 \times 4^2 - 24(4)$

$= -48 \, m/sec.$

(P) A Body moves along a straight line so that its displacement from a fixed point on the line is given by $s = 4t^3 - 6t^2 + 20$. Find the displacement, velocity and acceleration at end of 3 sec.