$S = 4t^3 - 6t^2 + 20$

$V = \frac{ds}{dt} = 12t^2 - 12t$

$a = \frac{dv}{dt} = 24t - 12$

(1) displacement at 3 sec

$S_3 = (4\times3)^3 - (6\times3)^2 + 20 = 74\,m$

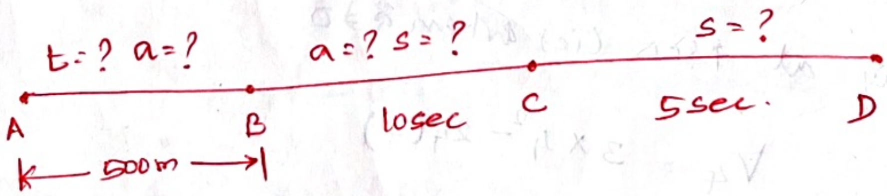
(2) Velocity at 3 sec

$V_3 = 12\times3^2 - 12(3) = 72\,m/sec$

(3) Acceleration at 3sec

$a_3 = (24\times3) - 12 = 60\,m/s^2.$

(8) A car starts from rest and accelerates uniformly to a speed of 80kmph over a distance of 500m. Find time and acceleration. Further acceleration raises the speed to 96kmph in 10 seconds. Find the acceleration and distance. Brakes are applied to bring the car to rest under uniform ~~acceleration~~ retardation in 5 sec. Find the distance covered during braking.



Consider the motion of car from A to B

$u = 0$ ($\because$ starts from rest)

$S = 500\,m$ $\quad v = 80\,kmph$ (velocity at B

$V = \frac{80\times1000}{3600} = 22.22\,m/s$

To find a:

$$v^2 = u^2 + 2as$$

$$(22.22)^2 = 0 + (2 \times a \times 500)$$

$$a = 0.494 \text{ m/s}^2$$

To find t

$$v = u + at$$

$$22.22 = 0 + (0.494)(t)$$

$$t = 44.98 \text{ sec.}$$

Consider the motion of car from B to C

u = Velocity at B

$= 22.22 \text{ m/s}$. $t = 10 \text{ sec}$

v = Velocity at C = 96 kmph = 26.67 m/s.

To find a

$$v = u + at$$

$$26.67 = 22.22 + a \times 10$$

$$a = 0.445 \text{ m/sec}^2$$

To find s

$$s = ut + \tfrac{1}{2} at^2$$

$$s = (22.22 \times 10) + (\tfrac{1}{2} \times 0.445 \times 10^2)$$

$$s = 244.45 \text{ m}$$

Consider the motion of car from C to D

u = Velocity at C

$= 26.67 \text{ m/s}$

$v = 0$ ($\because$ Bought to rest)

$t = 5 \text{ sec}$

$s = ?$

To find a

$v = u + at$

$0 = 26.67 + (a \times 5)$

$a = -5.334 m/s^2$ (retardation)

To find s

$s = ut + \frac{1}{2}at^2$

$s = (26.67 \times 5) + \frac{1}{2}(-5.334)(5^2)$

$= 66.675 m$ (

(P) Two trains A and B leave the same station on parallel lines. A starts with an uniform acceleration of $1/6 \, m/s^2$ and attains a speed of 24 kmph, when steam is reduced to keep the speed constant. B leaves 40 seconds after, with uniform acceleration of $1/3 \, m/s^2$ to attain a max speed of 48 kmph. When will it overtake A?

Consider to Motion of Train A

$u = 0 \quad a = 1/6 \, m/s^2 \quad V = 24 \text{ kmph} = 6.67 m/s^2$

The speed of the train is accelerated to attain a speed of 24 kmph and then train is moving with constant speed.

$T_{24 kmph} = \frac{v-u}{a} = \frac{6.67 - 0}{1/6} = 40 sec.$

Distance travelled by Train A in 40 sec

$= ut + \frac{1}{2}at^2$

$= 0 + (\frac{1}{2} \times \frac{1}{6} \times 40^2)$

$= 133.33 m.$

for $t > 40$ sec

$s = d/t$

$d = st$

Distance travelled by A in t sec $= 133.33 + 6.67(t-40)$

$$S = \frac{20t}{3} - 133.33 \rightarrow (1)$$

Consider the motion of Train B

$u = 0 \quad a = \frac{1}{3}\, m/s^2 \quad v = 48\,kmph = 13.34\, m/s$

Time taken by train B to attain a speed of 48 kmph

is $\left(\frac{v-u}{a}\right) + 40$

$$= \left(\frac{13.34 - 0}{1/3}\right) + 40 = 80\ sec.$$

($\because$ ~~Train~~ Train B starts 40 sec after T_A starts).

Distance covered by Train B in 40 sec $= ut + \frac{1}{2}at^2$

$$= 0 + \frac{1}{2}\left(\frac{1}{3} \times 40^2\right)$$

$$= 266.67\, m$$

For $t > 80$ sec

Distance travelled by B in t sec

0.522

$$= 266.67 + 13.34(t-80)$$

$s = D/t$

1066.6
1063.2

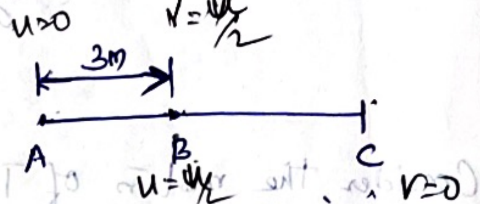
$$S = \frac{40}{3}t - 800 \rightarrow (2)$$

Distance covered by both trains are same

$$\frac{40}{3}t - 800 = \frac{20t}{3} - 133.33$$

$$\boxed{t = 100\ sec}$$

) A Bullet fired into a target loses ~~half~~ half its velocity after penetrating 3 m. How much further will it penetrate.



Solⁿ : $\boxed{S = 3}$

Let the Bullet be fired at A with velocity 'u'

$\because V_B = \frac{V_A}{2} = \frac{u}{2}$

Motion from A to B

$$V^2 = u^2 + 2as$$

$$\left(\frac{u}{2}\right)^2 = u^2 + 2as$$

$$\frac{u^2}{4} = u^2 + 2as \Rightarrow \frac{u^2}{4} = u^2 + 2a(3)$$

$\cancel{u^2 = 4u^2 + 24a}$ $\quad -3u^2 = 24a$

$\cancel{-3u^2 = 24a}$ $\quad \boxed{a = -0.125u^2}$

Let the Bullet penetrate further and rest at C.

Motion from B to C

Initial velocity $B = \frac{u}{2}$

$V_c = 0$

$S = ?$

$$V^2 = u^2 + 2as$$

$$0 = \left(\frac{u}{2}\right)^2 + 2(-0.125u^2)(s)$$

$$0.25u^2 s = \frac{u^2}{4}$$

$$\boxed{s = 1m}$$

Motion of Particle under Gravity:

→ Motion of a Particle under gravity is the special case of rectilinear motion under constant acceleration, known as acceleration due to gravity. denoted by g.

$$g = 9.81\ m/s^2$$

→ When a particle is dropped down from a height, the earth attracts it, hence the velocity of particle will go on increases as it comes nearer to earth and, hence it will be max when it strikes the ground, so g is +ve when moves downwards.

→ when moves upwards during projectile, the velocity will ↓ and velocity is zero when it reaches the max height. so g is -ve.

Rectilinear Motion

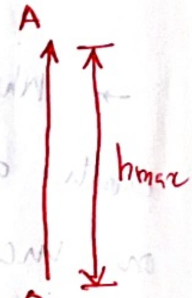
	Horizontal Motion	Vertical motion	Vertical upward motion
1)	$v = u + at$	$v = u + gt$	$v = u - gt$
2)	$s = ut + \frac{1}{2}at^2$	$h = ut + \frac{1}{2}gt^2$	$h = ut - \frac{1}{2}gt^2$
3)	$v^2 = u^2 + 2as$	$v^2 = u^2 + 2gh$	$v^2 = u^2 - 2gh$
4)	$S_n = u + \frac{a}{2}(2n-1)$	$h_n = u + \frac{g}{2}(2n-1)$	$h = u - \frac{g}{2}(2n-1)$

* When a body starts moving vertically downwards its initial velocity $u=0$

* When a body is projected ~~vertica~~ vertically upwards, at the highest point, its final velocity $v=0$.

* Upward motion

⇒ Maximum height attained by upward particle



$$\boxed{h_{max} = \frac{u^2}{2g}}$$

⇒ Time taken by the particle to reach maximum height

$$\boxed{t = \frac{u}{g}}$$

t ⇒ time takeem by particle, projected upwards.

Total time to return surface = 2X time up

$$T = 2t$$

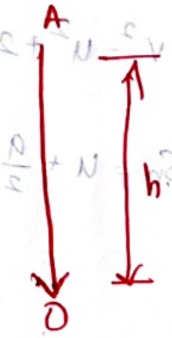
$$\boxed{T = \frac{2u}{g}}$$

$u = \sqrt{2g \times h_{max}}$ (If h_{max} is known)

$u = t \times g$. (If time is known)

Downward Motion :

Striking velocity of the particle moving downwards from the position of rest



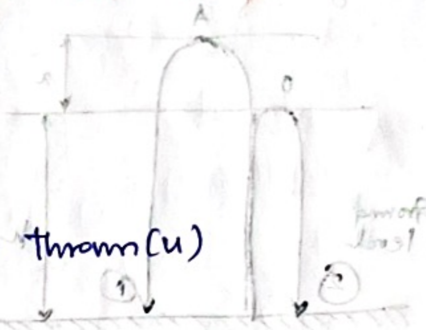
$$\boxed{v = \sqrt{2gh}}$$

(P) A stone is thrown vertically upwards. It reaches the max height 12m. Determine

(1) The velocity with which the stone was thrown.

(2) the time taken to reach maximum height

(3) Total time ~~tak~~ taken by stone to return to ground surface, after projected upwards.



$h_{max} = 12m$

(1) Velocity with which the stone was thrown (u)

At max height $v = 0$.

$$v^2 = u^2 - 2gh$$
$$0 = u^2 - 2(9.81) \times 12$$
$$u^2 = 2 \times 12 \times 9.81$$
$$u = 15.34 \, m/s$$

(2) Time taken to reach maximum height

at max height $v = 0$

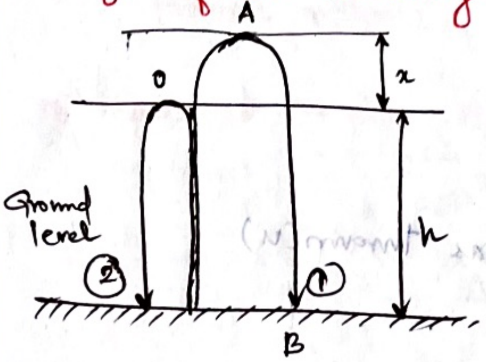
$$v = u - gt$$
$$0 = 15.34 - 9.81t$$
$$\boxed{t = 1.56 \text{ sec}}$$

(3) Total time taken T

Total time taken by stone T is equal to twice the time taken by stone to reach the max. height

$$T = 2t$$
$$T = 2(1.56)$$
$$\boxed{T = 3.12 \text{ sec}}$$

(P) A stone is projected upwards from the roof of a building with a velocity of 19.6 m/s and another stone is thrown downwards from same point 3 seconds ~~the~~ later. If both the stone reach the ground at the same time, determine the height of the building. Take $g = 9.8 m/s^2$



Motion of first stone (Thrown upwards)

$t \Rightarrow$ total time taken by stone (1) to strike the ground.

t_1 = time taken to reach the max height 'x'

t_2 = Time taken to strike the ground from max height

$$\boxed{t = t_1 + t_2}$$

Consider the upward motion from O to A

$v = u - gt$ ($u = 19.6 m/s$ time $= t_1$)

$0 = 19.6 - 9.81 \times t_1$

$t_1 = 2$ sec.

using the eqn $h = ut + \frac{1}{2} g t^2$

$x = ut - \frac{1}{2} g t^2$

$x = (19.6 \times 2) - \frac{1}{2}(9.81 \times t_1^2)$

$x = (19.6 \times 2) - (\frac{1}{2} \times 9.81 \times 2^2)$

$$\boxed{x = 19.6 m}$$

Consider the motion from A to B

Total distance $= (h + x)$ $u = 0$ time $= t_2$

Total distance = h + 19.6

$t_2 = t - t_1 = (t-2)$ sec

$h = ut + \frac{1}{2}gt^2$ (or) $(h+x) = ut_2 + \frac{1}{2}gt_2^2$

$(h+x) = \frac{1}{2} \times 9.8 \times (t-2)^2$

$(h+19.6) = \frac{1}{2} \times 9.8 \times (t-2)^2$

$(h+19.6) = 4.9(t-2)^2 \rightarrow$ (1)

Motion of second stone (Thrown downwards)

$u = 0$ time taken $= t - 3$ (3 seconds later)

$h = ut + \frac{1}{2}gt^2$

$h = 0 + \frac{1}{2} \times 9.8 \times (t-3)^2$

$h = 4.9(t-3)^2 \rightarrow$ (2)

sub (2) − (1)

$(\cancel{h} + 19.6) - \cancel{h}) = (4.9(t-2)^2) - (4.9(t-3)^2)$

$19.6 = 4.9[(t-2)^2 - (t-3)^2]$

$19.6 = 4.9((\cancel{t^2} + 4 - 4t) - (\cancel{t^2} + 9 - 6t))$

$= (-5 + 2t)\, 4.9$

$2t - 5 = 4$ $\boxed{t = 4.5 \text{ sec}}$

sub $t = 4.5$ in (2)

$h = 4.9(t-3)^2$

$= 4.9(1.5)$

$h = 11.025$ m $\Rightarrow$ Height of the building.