(3) Distance travelled when the body is again at rest.

$v = 0$

Put $v = 0$ in (1)

$$0 = 30s - 2s^2$$

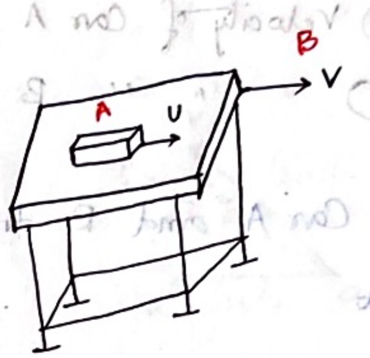
$$30\cancel{s} = 2s^{\cancel{2}}\,s$$

$$\boxed{s = 15m}$$

Distance travelled by body, when again at rest is 15m

Relative Motion :

- A particle is said to be in motion if it changes its position with resp to the surroundings taken as fixed, this type of motion known as individual motion of body.



$V_A \rightarrow$ Rate of change of distance between O to A.

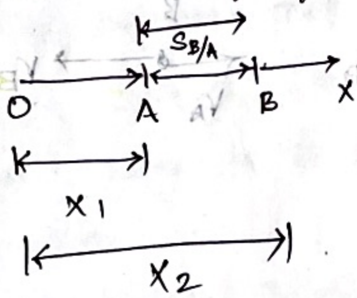
$= \frac{dx}{dt}$ $\quad a = \frac{d^2x}{dt}$

→ If the block is moving at velocity 'u' and at the same time is also pushed in the same direction at velocity 'v'. Both are moving in diff velocities, Now the velocity of block can be determined related to the velocity of table and this velocity is known as "relative velocity" and this motion is said to be relative motion".

relative velocity of A with respect to B is $(u-v)$ m/s.

$V_{A/B} = V_A - V_B$

1) Relative velocity of two particles moving in straight line.



relative velocity of B w.r.t. A

$\boxed{V_{B/A} = V_B - V_A}$

- here both A and B are moving in same direction. If the particles A moves in the opposite direction, then the -ve sign has to be used for V_A. Hence for our convenience, right hand side velocity is taken +ve and left hand side is taken as -ve.

Car A travels at a speed of 30m/s and Car B travels at a speed of 20m/s in the same direction. Determine

1. Velocity of Car A relative to Car B.
2. " " " B " " " A.

A ⟶ $V_A = 30m/s$

B ⟶ $V_B = 20m/s$

Let Car A and B travels in same direction say towards right

Velocity of car A = 30 m/s

" " " B = 20 m/s.

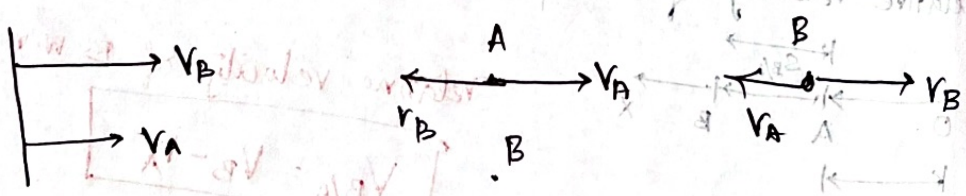
Velocity of Car A relative to Car B

$$V_{A/B} = V_A - V_B$$
$$= 10 m/s \ (\rightarrow +ve)$$

Velocity of Car B relative to Car A.

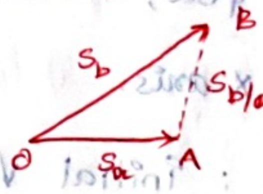
$$V_{B/A} = V_B - V_A$$
$$= -10 m/s \rightarrow 10 m/s \ (\leftarrow -ve).$$

Relative velocity of a particle from relative velocity diagram.



~~Rotatbioe~~

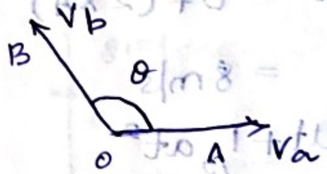
Relative velocity of two particles moving in a plane

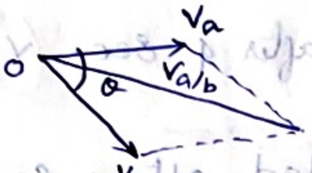


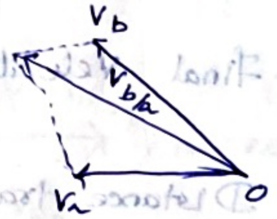
$$S_{B/A} = S_B - S_A$$

$$V_{B/A} = V_B - V_A \qquad a_{B/A} = a_B - a_A$$

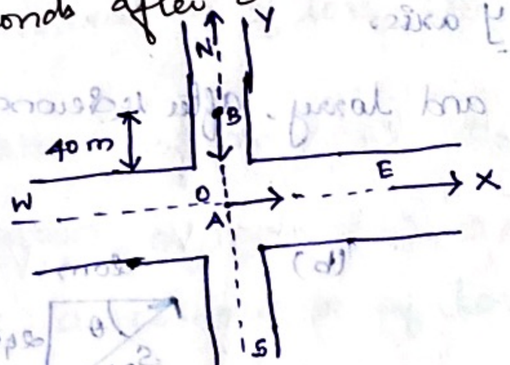
Relative velocity diagram







(P) A motor A is travelling from west to east at a constant speed of 18kmph. When the motor A cross north-south road as shown in diagram, a lorry B starts from rest, 40m north of the intersection and moves with a constant acceleration of $2m/s^2$. Determine the position, velocity and acceleration of lorry relative to motor A, 4 seconds after observation.



Solution:

The reference axes OX (towards East) and OY (towards North) are shown in the fig. First of all, let us analyse the motion of motor and lorry at 4 seconds time, after the observations are made.

Motor A: initial velocity $u = 18\ Km/hr = \frac{18 \times 1000}{3600} = 5m/s$

acceleration $a = 0$ ($\because$ travelling at constant speed)

$\therefore$ Distance travelled after 4 seconds = Constant speed × time

$= 5 \times 4$

$= 20m$

(ie) after 4 sec, motor A is at 20m from the Origin 'O' along x axis.

Lorry B : Initial Velocity $u = 0$ ($\because$ starts from rest)

acceleration $a = 2\ m/s^2$; time $t = 4$ sec

$\therefore$ Final Velocity after 4 sec, $v = u + at = 0 + (2 \times 4)$
$= 8\ m/s$

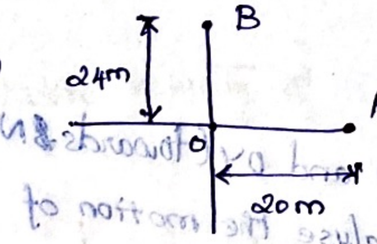
$\therefore$ Distance travelled after 4 sec, $s = ut + \frac{1}{2}at^2$

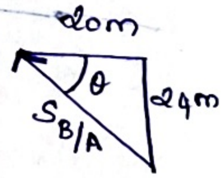
$= 0 + \left[\frac{1}{2} \times 2 \times 4^2\right]$

$= 16m.$

Position of the lorry after 4 second is $(40 - 16) = 24m$ from the origin along y axis.

The position of motor and lorry after 4 seconds are shown in fig (a)

(a) 

(b) 

Relative position of lorry (B) w.r.t. motor (A) after 4 sec.

$S_{B/A}$ = resultant of 24m and 20m in relative displacement in diagram in fig (b)

$= \sqrt{24^2 + 20^2} = 31.24\ m$ and $\theta = \tan^{-1}\left[\frac{24}{20}\right]$

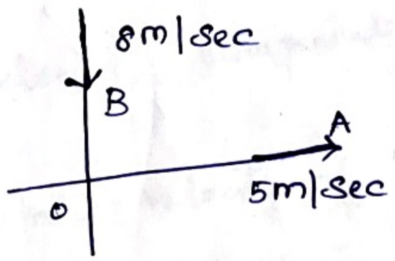
$= 50.19°$

Relative velocity of lorry (B) w.r.t motor (A) after 4 sec.

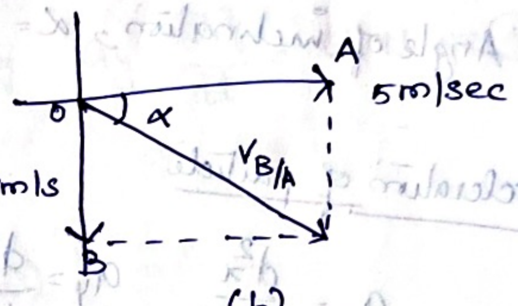
Velocity of lorry (B), $V_B = 8 m/s$

Velocity of motor (A), $V_A = 5 m/s$

These velocities are shown in (a) and the relative diagram is shown in (b)



(a)



(b)

$V_{B/A} = \sqrt{8^2 + 5^2} = 9.43 m/s$ and $\alpha = \tan^{-1}(8/5)$

$= 57.99°$

Relative acceleration of lorry (B) w.r.t motor (A) after 4 sec.

acceleration of lorry (B), $a_B = 2 m/s^2$

acceleration of motor (A), $a_A = 0$

∴ relative acceleration of lorry B w.r.t motor A,

$a_{B/A} = \sqrt{2^2 + 0^2} = 2 m/s^2$

Curvilinear Motion :

- When the path described by a moving particle is a curved line, then the motion of the particle is known as curvilinear motion of translation.

- If the continous path described by a moving particle confined to a plane then it is ~~motion~~ known as 'plane motion', Cartesian system

$$V_x = \frac{dx}{dt} \qquad V_y = \frac{dy}{dt}$$

$$V = \sqrt{V_x^2 + V_y^2}$$

Angle of inclination, $\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

acceleration of particle

$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2}$$

Resultant acceleration, $a = \sqrt{a_x^2 + a_y^2}$

angle of inclination, $\tan\phi = \frac{a_y}{a_x}$

P) The motion of a particle along a curved path is given by eqn.

$x = t^2 + 8t + 4$ and $y = t^3 + 3t^2 + 8t + 4$

Determine.

(1) Initial velocity of particle

(2) Velocity of the particle at $t = 2$ sec.

(3) Acceleration of particle at $t = 0$

(4) " " at $t = 2$ sec

Soln:

Given $x = t^2 + 8t + 4$ $\quad y = t^3 + 3t^2 + 8t + 4$

Velocity components of particle

orizontal component of velocity $V_x = \frac{dx}{dt} = 2t+8 \rightarrow (1)$

velocity component of velocity $V_y = \frac{dy}{dt} = 3t^2+6t+8 \rightarrow (2)$

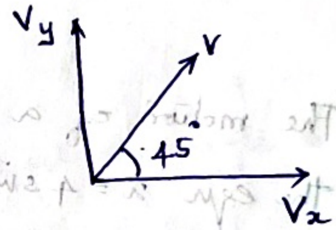
Acceleration component of particle

Horizontal component of acceleration $a_x = \frac{d^2x}{dt^2} = 2 \rightarrow (3)$

Velocity component of velocity $a_y = \frac{d^2y}{dt^2} = 6t+6 \rightarrow (4)$

1) Initial velocity of Particle

put $t=0$ in eqn (1) and (2)



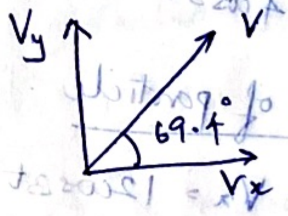
$V_x = 8 m/s$

$V_y = 8 m/s$

$V = \sqrt{V_x^2+V_y^2} = \sqrt{8^2+8^2} = 11.31 \ m/s$

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \left(\frac{8}{8}\right) = 1 = 45^\circ$

2) Velocity at 2sec.



put $t = 2sec$ in (1) and (2)

$V_x = 12 m/s \qquad V_y = 32 m/s$

Velocity at 2sec $= V_2 = \sqrt{V_x^2+V_y^2} = \sqrt{12^2+32^2} = 34.17 m/s.$

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 69.4^\circ$

3) Acceleration at $t=0$

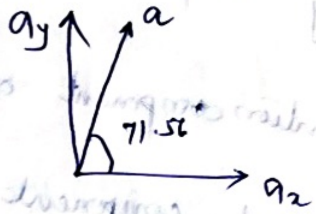
$t=0$ in (3) & (4)

$a_x = 2 m/s^2 \quad a_y = 6 m/s^2$

$t=0$; $a \sqrt{2^2+6^2} = 6.324 \ m/s^2$

let ϕ be the angle of inclination of 'a' with horizontal

$$\tan\phi = \frac{a_y}{a_x} \quad \phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{6}{2}\right) = 71.56^\circ$$

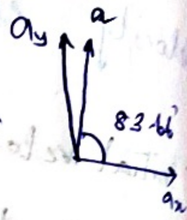


(4) Acceleration at $t = 2\,sec$

sub $t = 2$ in (3) and (4)

$a_x = 2\,m/s^2 \qquad a_y = 18\,m/s^2$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{2^2 + 18^2} = 18.11\,m/s^2$$



$$\phi = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{18}{2}\right) = 83.66^\circ$$

(P) The motion of a body moving on a curved path is given by the eqn $x = 4\sin 3t$ and $y = 4\cos 3t$. Find the acceleration and velocity after 2 sec.

$x = 4\sin 3t$

$y = 4\cos 3t$

<u>Velocity of particle</u>

$V_x = 12\cos 3t$

$V_y = -12\sin 3t$

$\sin^2 a + \cos^2 a = 1$

$$V = \sqrt{V_x^2 + V_y^2} = 12\,m/s$$

velocity of particle after 2 sec

From the above result, the velocity of particle at any time interval is constant i.e. $12\,m/s$.

Acceleration of Particle

$$a_x = \frac{d}{dt}(V_x) = -36 \sin 3t$$

$\sin^2 a + \cos^2 a = 1$

$$a_y = \frac{d}{dt}(V_y) = -36 \cos 3t$$

$$a = \sqrt{a_x^2 + a_y^2} = 36 \, m/s^2$$

Acceleration of particle after 2sec.

From the above result, the acceleration of particle at any time is constant of $36 \, m/s^2$.

Projectile Motion

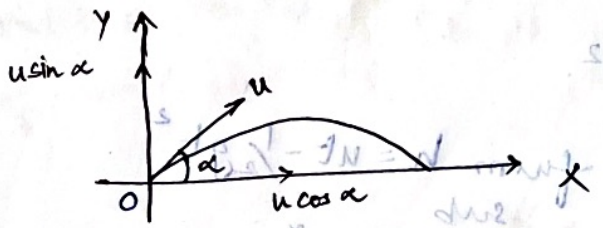
Projectile: — A particle projected in space at an angle to the horizontal plane.

Angle of projection:-

↳ The angle to the horizontal at which the projectile is projected is called angle of projection denoted by 'α'.

Velocity of Projectile:

↳ Velocity with which the projectile is ~~projected~~ thrown into space → denoted by 'u'.



velocity 'u' can be resolved into 2 components along OX and OY axes.

Component of velocity along OX axis $= u \cos \alpha$ → more projectile horizontally

" " " " OY axis $= u \sin \alpha$ → more projectile vertically