Unit-5

Friction

When one body slides over another body a Resistance force is developed by the contact surfaces which opposes the motion called friction force.

Types of friction:

1. Dry friction — Static friction; Dynamic friction — Sliding friction; Rolling friction

Limiting friction

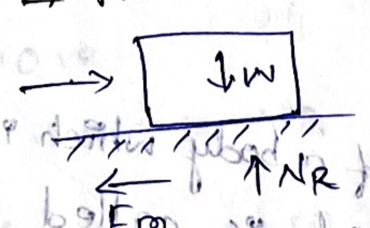
The max resistance offered by the body is called "limiting friction".

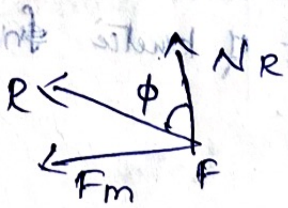
Co-efficient of friction:

It is the ratio of limiting friction to Normal reaction is known co-eff of friction.

$$\mu = \frac{F_m}{N_R}$$

Angle of friction:





$$R = \sqrt{N_R^2 + F_m^2}$$

The angle b/w the resultant 'R' and the normal reaction N_R is called angle of friction.

$$\tan\phi = \frac{F_M}{N_R}$$

$$F_{ms} = M_s \cdot N_R$$

$$\tan\phi = \mu$$

$$F_{mk} = M_k \cdot N_R$$

$$\mu = \frac{F_M}{N_R} \qquad F_m = M \times N_R$$

Laws of dry friction: laws of dry friction.

① Static friction:

i) The frictional force always acts in the opposite direction to ~~meets~~ the body tends move

ii) frictional force does not depend on the shape and area of contact of the bodies.

iii) The frictional force depends on the degree of ________ of the contact area b/w two bodies.

iv) The frictional force is equal to the ~~depends on the~~ force ~~degree of~~ applied to the body, so long as the body is at rest.

V) $F_m \propto N_R$

$F_m = \mu_s \cdot N_R$.

2) laws of Dynamic friction:

i) The frictional force acts in the opposite direction to that body moves.

(ii) The magnitude of dynamic friction bears a constant ratio to the normal reaction b/w two surfaces.

(iii) Coefficient of kinetic friction is less than the co-efficient of static friction.

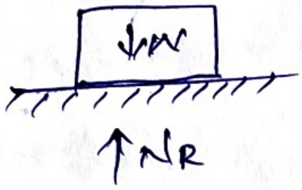
Impending motion:

The state of motion of a body which is just about to move (or) slide is called impending motion of the body.

Contact surfaces		Range of μ_s
Wood on wood	$\longrightarrow$	0.2 - 0.6
leather on wood	$\longrightarrow$	0.2 - 0.5
Metal on ice	$\longrightarrow$	0.03 - 0.05
Leather on metal	$\longrightarrow$	0.3 - 0.6
Mild steel on Mild steel	$\longrightarrow$	0.5 - 0.6
Rubber on pavement	$\longrightarrow$	0.6 - 0.8

Basic Concepts :

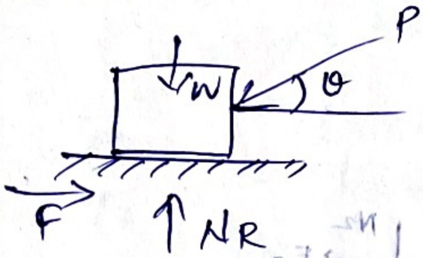
case (i) $F = 0$ Body is in the condition of equilibrium.



$\Sigma V = 0$

$NR = W$

case (ii) $F < Fm$ Body is still in equilibrium



$\Sigma H = 0$, $F = P\cos\theta$

$\Sigma V = 0$ $NR = W + P\sin\theta$

case iii) $F = Fm$

When the limiting friction is attained, then the block is in impending motion, ie. just start to move.

$Fm = \mu \cdot NR$ i) applied.

$\Sigma H = 0$ $Fm = P\cos\theta$

$NR = W + P\sin\theta$

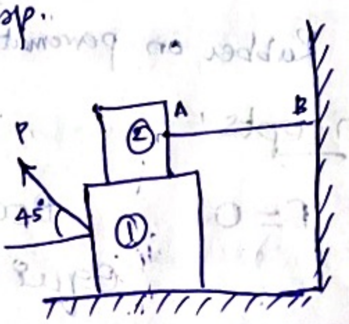
case iv) $F > Fm$

$F = \mu N$ not be applied

$F = \mu_k N$ is applied.

Problem (1).

Block 2 rest on block 1 and is attach by a horizontal rope AB to the wall as shown in ~~fig~~ diagram. What force ~~AB~~ to the P is necessary to cause motion of ~~impend~~ block (1) to impend? The coefficient of friction b/w the blocks is 1/4 and between the floor and block (1) is 1/3. Mass of blocks (1) and (2) are 14 kg and 9 kg resp.



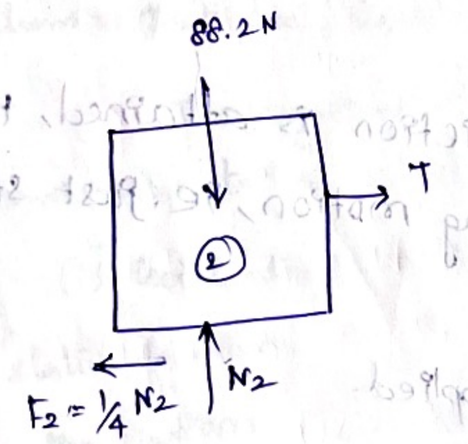
Soln:

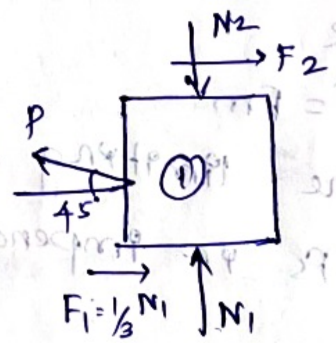
Weight of block (1) $W_1 = 14 \times 9.81$
$= 137.2\ N$

Weight of block (2) $W_2 = 9 \times 9.81$
$= 88.2\ N$

$\mu_1 = 1/3 \quad \mu_2 = 1/4$

$T \rightarrow$ Tension in cable AB





Block (1) moves towards left

Block (2) " " right.

Consider FBD of block 2

$\Sigma V = 0$

$N_2 - 88.2 = 0$

$N_2 = 88.2\,N$

$F_2 = \mu_2 N_2 = \frac{1}{4} \times 88.2$

$= 22.05\,N$

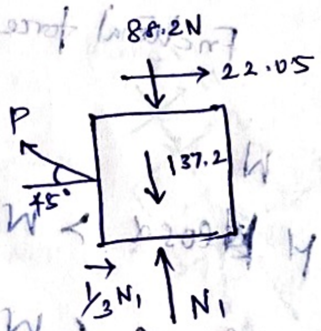
$\Sigma H = 0$

$T - (\mu_2 \cdot N_2) = 0$

$T - (\frac{1}{4} \times N_2) = 0$

$T = 0.25\,N_2$

Consider FBD of block 1



$\Sigma H = 0$

$22.05 + (\frac{1}{3} N_1) - P\cos 45° = 0$

$P\cos 45° = 22.05 + 0.3333 N_1$

$\Sigma V = 0 (\uparrow +)$

$N_1 + P\sin 45° - 88.2 - 137.2 = 0$

$P\sin 45 = 225.4 - N_1$

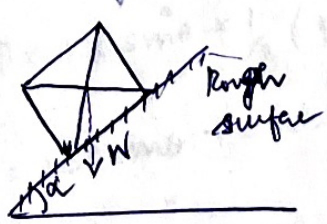
$\frac{②}{①} \Rightarrow \frac{P\sin 45°}{P\cos 45°} = \frac{225.4 - N_1}{22.05 + 0.333 N_1}$

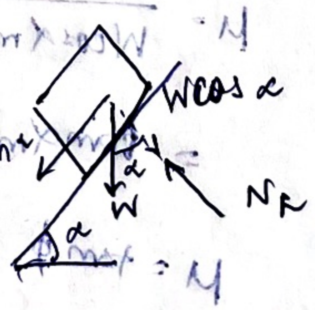
$\tan 45 = 1 = \frac{225.4 - N_1}{22.05 + 0.333 N_1}$

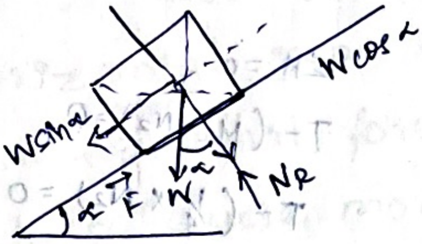
$\boxed{N_1 = 152.55\,N}$

$\boxed{P = 103\,N}$

Angle of Repose: ϕ



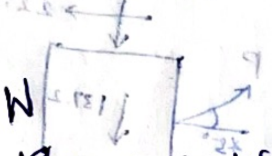




Normal reaction $N_R = W\cos\alpha$

Frictional force $F = \mu N_R$

$= \mu W\cos\alpha$



when $\mu W\cos\alpha > W\sin\alpha$ → Block is at rest

$\mu W\cos\alpha < W\sin\alpha$, the impending downwards motion takes place.

When the angle of plane with horizontal α, is increased, $W\sin\alpha$ will be more than $\mu W\cos\alpha$ and sliding takes place

⇒ "The angle of the inclined plane, at which the body tends to slide down known as angle of repose.

denoted by α_m.

$\mu W\cos\alpha_m \leq W\sin\alpha_m$

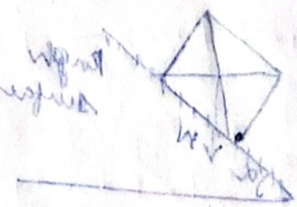
$\mu W\cos\alpha_m = W\sin\alpha_m$

$$\mu = \frac{W\sin\alpha_m}{W\cos\alpha_m}$$

$= \tan\alpha_m \rightarrow (1)$

$\phi \Rightarrow$ angle of static friction.

$\mu = \tan\phi \rightarrow (2)$.

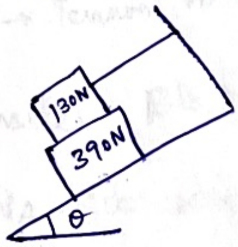


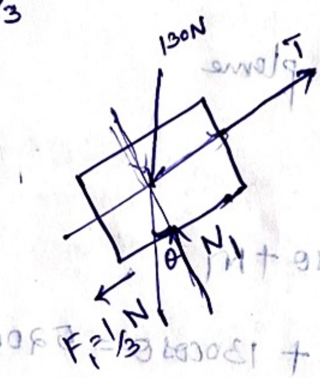
$\tan \alpha_m = \tan \phi \qquad \alpha_m = \phi$

Angle of respose = angle of static friction.

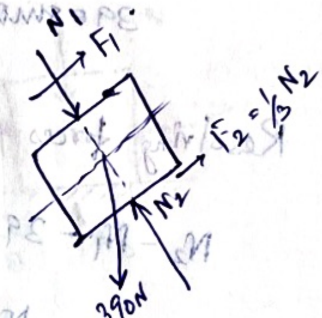
Body on rough Inclined plane

Q What should be the value of the angle θ so that motion of 390N block impends down the plane? The coefficient of friction $\mu = 1/3$.





FBD of upper block



FBD of lower block

FBD of Upper block

$T \rightarrow$ tension of cable.

Resolving forces along plane

$$T - 130\sin\theta - F_1 = 0$$

$$T - 130\sin\theta - \mu N_1 = 0$$

$$T - 130\sin\theta + \frac{1}{3}N_1 \rightarrow (1)$$

$\mu = 1/3$

Resolving the force normal to plane

$$N_1 = 130\cos\theta \rightarrow (2)$$

put (2) in (1)

$$T = 130\sin\theta + \frac{1}{3}(130\cos\theta)$$

$$T = 130\sin\theta + 43.33\cos\theta \rightarrow (3)$$

Consider FBD of lower block

Resolving forces along plane

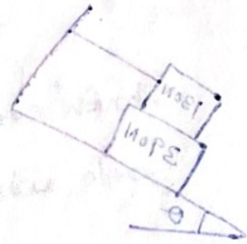
$F_1 + F_2 - 390\sin\theta = 0$

$F_1 + F_2 = 390\sin\theta$

$\mu N_1 + \mu N_2 = 390\sin\theta$

$390\sin\theta = \frac{1}{3}(130\cos\theta + N_2) \rightarrow (4)$

Resolving forces normal to plane



$N_2 - N_1 - 390\cos\theta = 0$

$N_2 = 390\cos\theta + N_1$

$N_2 = 390\cos\theta + 130\cos\theta = 520\cos\theta$

Sub N_2 in eqn (4)

$390\sin\theta = \frac{1}{3}(130\cos\theta + 520\cos\theta)$

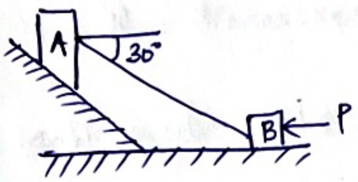
$390\sin\theta = \frac{1}{3}(650\cos\theta)$

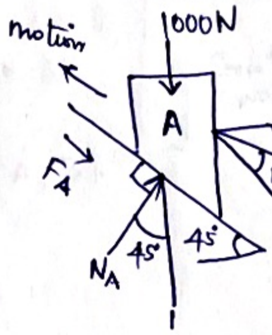
$390\sin\theta = 216.67\cos\theta$

$\tan\theta = 216.67/390$

$\theta = \tan^{-1}\left(\frac{216.67}{390}\right) = 29°$

(P) Block A weighing 100N rests on a rough inclined plane whose inclination to the horizontal is 45°. It is connected to another block B, weighing 3000N rests on rough horizontal plane by a weightless rigid bar inclined at an angle of 30° to the horizontal as shown. Find the horizontal force required to be applied to the block B just to move block A in upward direction. Assume angle of friction as 15° at all surfaces where there is sliding.

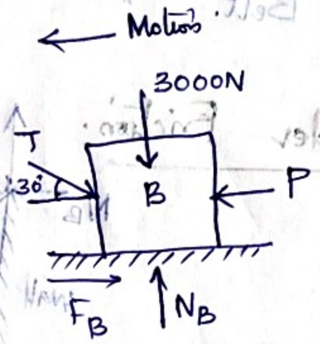




Given

$\phi = 15°$ $\mu = \tan\phi$ $\mu = 0.268$

$T \rightarrow$ Tension in rod,



consider Block A (FBD) $\Sigma V = 0$

$N_A - 1000\cos 45° + T\sin 15 = 0$

$N_A + 0.259T = 707.1 \rightarrow (1)$

$T\cos 15 + F_A + 1000\sin 45° = 0$ $\Sigma H = 0$

$T\cos 15 + (\mu N_A) + 1000\sin 45 = 0$

$0.966T + 0.268 N_A = -707.1 \rightarrow (2)$

solve (1) & (2) $T = -1000\,N$ $N_A = 448.1\,N$

-ve sign indicates rod is compressive.

Consider FBD (Block B)

$\Sigma V = 0$ $N_B - 3000 - T\sin 30 = 0$

$N_B = 3500N$

$\Sigma H = 0$

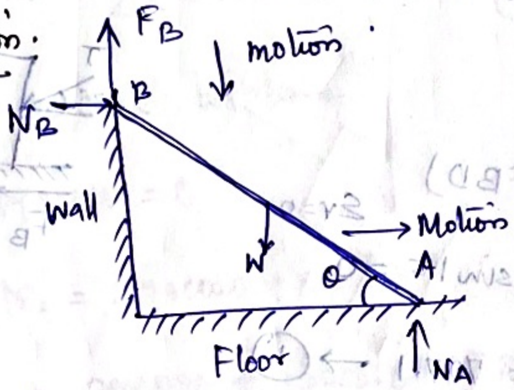
$T\cos 30 + F_B - P = 0$

$P = 1804\,N$

Simple Contact Friction:

1. Ladder Friction
2. Wedge "
3. Screw "
4. Belt "

Ladder Friction:



Ladder Length $= l$

$\theta \Rightarrow$ angle of friction

Frictional force at B, F_B will act upwards

$$\mu_B = \frac{F_B}{N_B} \text{ or } F_B = \mu_B N_B$$

$$\mu_A = \frac{F_A}{N_A}$$

For impending motion of ladder, the equilibrium eqn $\Sigma H = 0, \Sigma V = 0$, $\Sigma M = 0$ and $F = \mu N$.

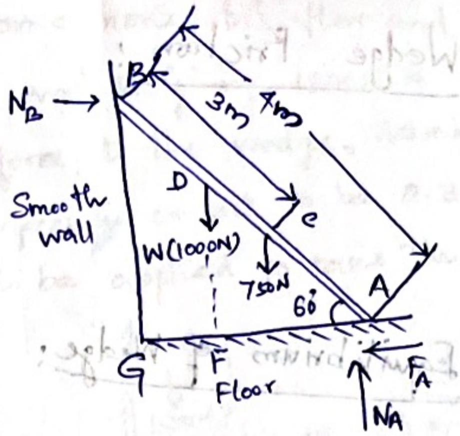
(P) A uniform ladder of weight 1000 N and of length 4 m rests on a horizontal ground and lean against a smooth vertical wall. The ladder makes an angle of 60° with horizontal. When a man of weight 750 N stands on ladder at a distance 3 m from the top of the ladder, the ladder is at the point of sliding. Determine the coefficient of friction between the ladder and the floor.

F_A : Friction force at A = $\mu_A N_A$

$F_B = 0$ (due to smooth wall)

μ_A = coefficient of friction at A

N_A, N_B = Normal reactions at A, B.



Self weight, W = 1000N

weight of man = 750N

Applying $\Sigma V = 0$ (↑+)

$N_A - 1000 - 750 = 0$

$\underline{N_A = 1750N}$

$\Sigma H = 0$ (→+)

$N_B - F_A = 0$

$N_B - \mu_A N_A = 0$ $\quad N_B = \mu_A N_A = \mu_A \times 1750 \rightarrow (1)$

$N_B = \mu_A \times 1750 \rightarrow (1)$

$\Sigma M_A = 0$ (↻+) ⇒ Taking moment of all forces about A

$(N_B \times BG) - (1000 \times AF) - (750 \times AE) = 0$

$AF = 2\cos 60 = 1m$

$BG = 4\sin 60 = 3.464m$

$AE = 1\cos 60 = 0.5m$

$(N_B \times 3.464) - (1000 \times 1) - (750 \times 0.5) = 0$

$N_B = 396.9N$

$\boxed{N_B = 396.9N}$

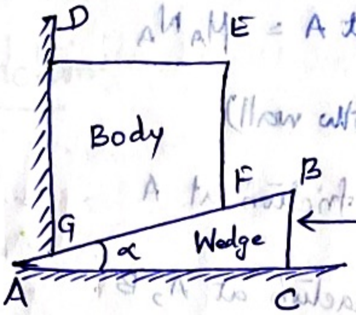
sub N_B in (1)

$N_B = \mu_A \times 1750$

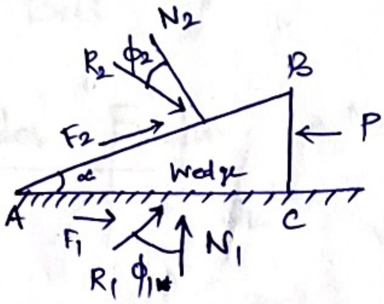
$\mu_A = \frac{N_B}{1750} = \frac{396.9}{1750} = 0.226$

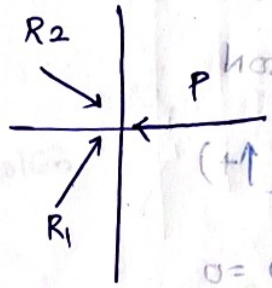
Coefficient of friction b/w ladder & floor $\}$ = 0.226

Wedge Friction:



Equilibrium of Wedge:





$F_2 = \mu_2 N_2$ $\qquad F_1 = \mu_1 N_1$

single Resultant $R = \sqrt{F_R^2 + N_R^2}$

R_1 and R_2 are drawn on wedges

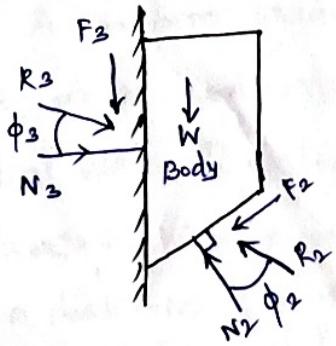
$R_1 = \sqrt{F_1^2 + N_1^2}$ $\qquad R_2 = \sqrt{F_2^2 + N_2^2}$

F_1 and $F_2 \rightarrow$ limiting friction

R_1 and R_2 makes the angles ϕ_1 and ϕ_2 called angle of friction with line of action of respective normal reactions N_1 and N_2.

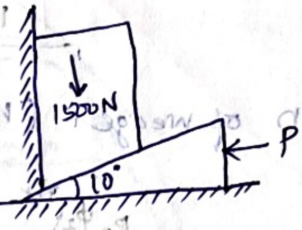
$R_1, R_2, P \rightarrow$ coplanar concurrent forces in equilibrium.

Equilibrium of body



R_2, R_3 and self weight W are coplanar concurrent forces in equilibrium.

(P) A block overlying a 10° wedge on a horizontal floor and ~~leaning~~ leaning against a vertical wall and weighing 1500N is to be raised by applying a horizontal force to the wedge, Assuming co-efficient of friction between all surfaces in contact to be 0.3 determine the minimum horizontal force to be applied to raise the block.



Soln :

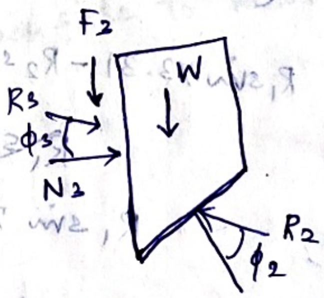
Angle of wedge $= 10^\circ$

$W = 1500N$ $\qquad \mu = 0.3$

$\mu = \tan\phi$

angle of friction, $\phi = \tan^{-1}(\mu) = \tan^{-1}(0.3) = 16.69^\circ$

FBD of block



$\Sigma H = 0 \ (\rightarrow +)$

$R_3 \cos 16.69 - R_2 \cos 63.31 = 0$

$R_3 \cos 16.69 = R_2 \cos 63.31$

$0.9578\, R_3 = 0.4491\, R_2$

$\boxed{R_3 = 0.468\, R_2}$

$\Sigma V = 0 \ (\uparrow +)$

$- R_3 \sin 16.69 - 1500 + R_2 \sin 63.31 = 0$

~~$0.893 R_2$~~

$0.893\, R_2 - 0.287\, R_3 = 1500$

$\boxed{R_3 = 0.468\, R_2}$

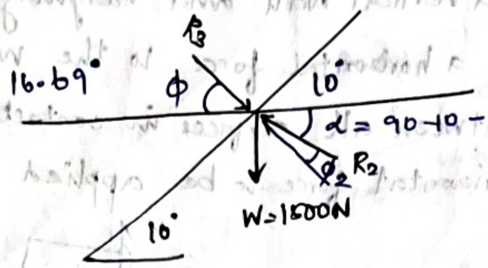
$0.893\, R_2 - 0.287\,(0.468\, R_2) = 1500$

$R_2 = 1977N$

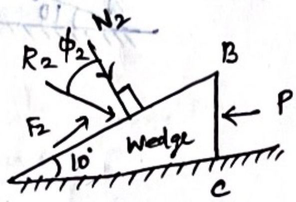
$R_3 = 0.468\, R_2$

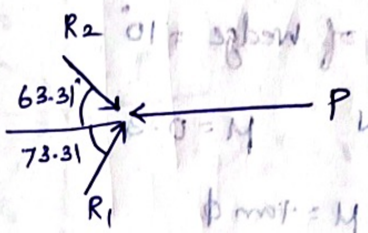
$= 0.468 \times 1977$

$R_3 = 925N$



FBD of wedge. (Wedge)





$\Sigma V = 0 \; (\uparrow +)$

$$R_1 \sin 73.31 - R_2 \sin 63.31 = 0$$

$$\cancel{R_1 \sin 73.3}$$

$$R_1 \sin 73.31 = R_2 \sin 63.31$$

$$R_1 \sin 73.31 = 1977 \sin 63.31$$

$$\boxed{R_1 = 1846\text{ N}}$$

$\Sigma H = 0 \; (\rightarrow +)$

$$R_2 \cos 63.31 + R_1 \cos 73.31 - P = 0$$

$$1977 \cos 63.31 + 1846 \cos 73.31 = P$$

$$\boxed{P = 1431\text{ N}}$$

Belt Friction:

$$\frac{T_2}{T_1} = e^{\mu\theta} \qquad T_2 > T_1$$

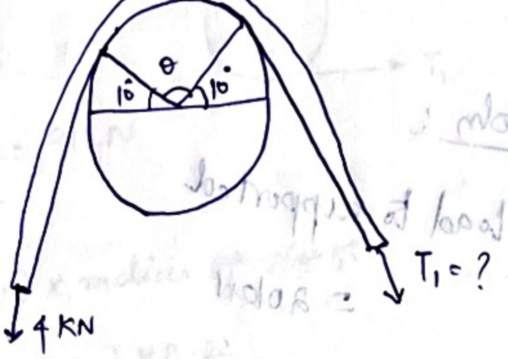
θ = angle of contact

μ = coeff. of friction.

Torque = $(T_2 - T_1)$ x radius of shaft

Power Transmitted = $(T_2 - T_1)$ x velocity of belt.

(P) A rope is wound over a pulley as shown. If the tension which pulls the belt on one end is 4 kN, determine the necessary tension on other side of the belt to resist?
Take $\mu = 0.25$



Tension, $T = 4$ KN

$\theta = 180 - (10+10) = 160°$

$\theta = \left(\frac{160 \times \pi}{180}\right)$ radians

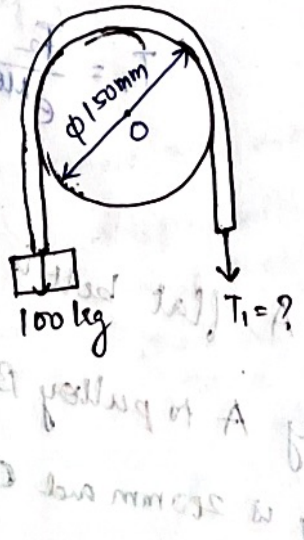
$= 2.792$ rad

T_1 is the necessary tension on the other side to resist

$$T_2 = T_1 e^{\mu\theta} \quad (\text{or}) \quad T_1 = \frac{T_2}{e^{\mu\theta}}$$

$$T_1 = \frac{4}{e^{(0.25 \times 2.792)}} = 2 \text{KN}$$

(P) A 100 kg mass is lifted by a rope rolling on a cylinder of 150 mm dia as shown. Determine the force required on the other side if the coeff of friction is 0.20



$T_2 = 100$ kg $= 100 \times 9.81$ N

$T_1 = ?$

contact angle $\theta = 180° = \frac{180 \times \pi}{180} = \pi$ rad.

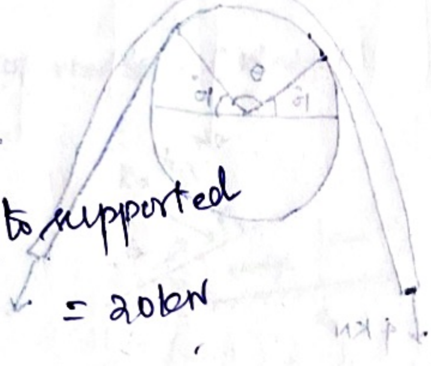
$$T_2 = T_1 e^{\mu\theta}$$

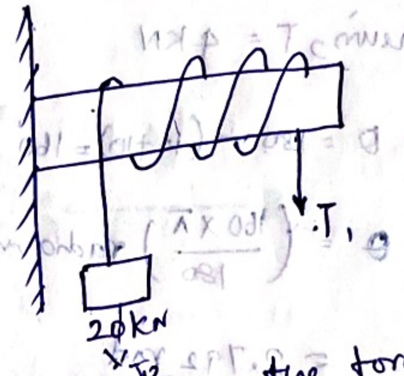
$$T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{100 \times 9.81}{e^{(0.20 \times \pi)}}$$

$= 523.48$ N.

(P) A rope is wrapped 3 times around a rod as shown. Determine the force required on the free end of the rope to support a load of 20 kN weight. The coeff. of friction between the rope & rod is 0.30

Soln :





Load to supported

= 20kN

The rope is wrapped around the rod, hence the force required to balance on the ~~force~~ free end will be less than 20 kN

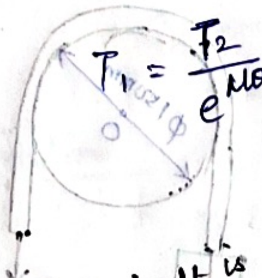
$T_2 = 20\,kN$

T_1 = Unknown force

Contact angle of belt = angle of one turn x no. of turns

$\theta = (360 \times 3) \times \frac{\pi}{180}$

$= 6\pi$ rad.

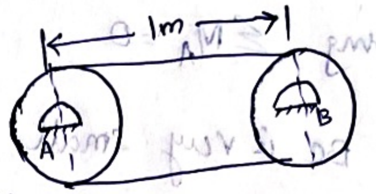


$T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{20}{e^{(0.3 \times 6\pi)}} = 0.07\,kN = 70\,N$

(P) A flat belt is used to transmit a torque from pulley A to pulley B as shown. If the radius of the pulley is 200 mm and Coeff of friction is 0.25 Determine the largest torque which can be transmitted if the allowable belt tension is 2000N

Max tension = 2000 N

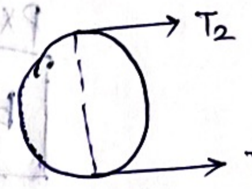
T_2 = 2000 N



T_1 → Tension on the other end of belt

Contact angle, $\theta = 180 \times \frac{\pi}{180}$

$= \pi$ rad



$$T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{2000}{e^{(0.25 \times \pi)}} = 912\,N$$

Torque transmitted = $(T_2 - T_1) \times$ radius of shaft

$= (2000 - 912) \times 0.2$

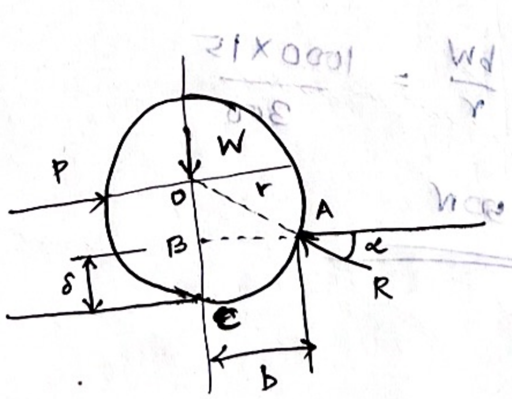
$= 217.6$ Nm

Rolling Resistance:

When one body is made to roll freely over an another body, a resistance is developed in opposite direction known as Rolling Resistance.

→ The resistance helps to roll the body without any slipping or turning of the body.

→ Rolling resistance is developed due to deformation made by rolling body over an another body.



Wheel rolls without slipping with constant velocity the equilibrium condt are satisfied.

Applying $\Sigma M_A = 0$ $\quad P \times OB = W \times BA$

BC is very small $\quad OB$ = radius of wheel, r.

$$P \times r = W \times b$$

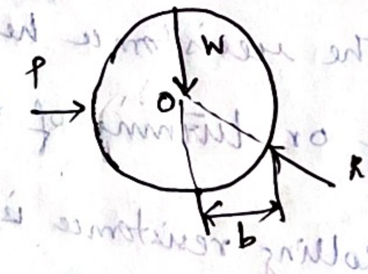
$$\boxed{b = \frac{Pr}{W}}$$

b → Horizontal distance of pt of resistance measured from centre of wheel known as "Co-efficient of Rolling resistance".

$R\cos\alpha \Rightarrow$ Rolling resistance

(P) A wheel of weight 1000 N and dia 600 mm is required to move on a horizontal surface. If the co-efficient of rolling resistance is 15mm. Calculate the force required to roll the wheel without slipping.

Given $\quad W = 1000N$



$b = 15\,mm$

$r = \frac{600}{2} = 300\,mm$

$$b = \frac{Pr}{W}$$

$$P = \frac{bW}{r} = \frac{1000 \times 15}{300}$$

$$P = 50N$$