LTI DT System Analysis using DTFT

(2) A discrete time causal system has a function $H(z) = \frac{1-z^{-1}}{1-0.2z^{-1}-0.15z^{-2}}$. Determine difference equation, Impulse response, pole zero diagram.

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = \frac{1-z^{-1}}{1-0.2z^{-1}-0.15z^{-2}}$$

$$Y(z)\left[1-0.2z^{-1}-0.15z^{-2}\right] = X(z)\left[1-z^{-1}\right]$$

$$Y(z) - 0.2z^{-1}Y(z) - 0.15z^{-2}Y(z) = X(z) - z^{-1}X(z)$$

$$y(n) - 0.2\,y(n-1) - 0.15\,y(n-2) = x(n) - x(n-1)$$

Impulse response :-

$$H(z) = \frac{1-z^{-1}}{1-0.2z^{-1}-0.15z^{-2}}$$

Multiply & divide by z^2

$$H(z) = \frac{z^2-z}{z^2-0.2z-0.15}$$

$$\frac{H(z)}{z} = \frac{z-1}{z^2-0.2z-0.15} \Rightarrow \frac{z-1}{(z+0.3)(z-0.5)}$$

$$\frac{z-1}{(z+0.3)(z-0.5)} = \frac{A}{z+0.3} + \frac{B}{z-0.5}$$

$$z-1 = A(z-0.5) + B(z+0.3)$$

Put $z=-0.3$ $\qquad$ $z=0.5$

$$A = \frac{1.3}{0.8} \qquad B = -0.5/0.8$$

$$\boxed{A = 1.625} \qquad \boxed{B = 0.625}$$

$$\frac{H(z)}{z} = \frac{1.625}{z+0.3} - \frac{0.625}{z-0.5}$$

$$H(z) = 1.625\left(\frac{z}{z+0.3}\right) - 0.625\left(\frac{z}{z-0.5}\right)$$

$$h(n) = 1.625(-0.3)^n u(n) - 0.625(0.5)^n u(n)$$

(3) A discrete time LTI system is described by $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$. Determine system transfer function, Impulse response and Freq response.

Taking z-transform on both sides

$$Y(z) - \frac{3}{4}z^{-1}Y(z) + \frac{1}{8}z^{-2}Y(z) = X(z)$$

$$Y(z)\left[1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}\right] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

Freq response :-

$$H(e^{j\omega}) = \frac{1}{1 - \frac{3}{4}(e^{j\omega})^{-1} + \frac{1}{8}(e^{j\omega})^{-2}}$$

Impulse Response :-

Multiply and divide by z^2

$$H(z) = \frac{z^2}{z^2} \; \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}$$

$$= \frac{z^2}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

$$\frac{Y(z)}{z} = \frac{z}{(z-1/2)(z-1/4)} \Rightarrow \frac{A}{z-1/2} + \frac{B}{z-1/4}$$

$$z = A(z-1/4) + B(z-1/2)$$

$z = 1/4$

$B = -1/4 \times 4$

$\boxed{B = -1}$

$z = 1/2$

$1/2 = (1/4)A + B(0)$

$\boxed{A = 2}$

$$\frac{Y(z)}{z} = \frac{2}{z-1/4} - \frac{1}{z-1/2}$$

$$Y(z) = 2\left(\frac{z}{z-1/4}\right) - \frac{z}{z-1/2}$$

$$\therefore y(n) = 2\left(\frac{1}{4}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(n)$$

HW The o/p $y(n)$ of discrete time LTI system is $2\left(\frac{1}{3}\right)^n u(n)$ when the i/p signal $x(n) = u(n)$. Find the Impulse response of the s/m.

$$H(z) = \frac{2(z-1)}{z-1/3} \qquad \therefore h(n) = 6\delta(n) - 4\left(\frac{1}{3}\right)^n u(n)$$