CONVOLUTION SUM : [Linear Convolution]

convolution of sum is defined as

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Four steps involved in computing convolution :-

★ Folding ★ Shifting ★ Multiplication ★ Summation

Four Methods available in compute convolution sum :-

1) Definition Method 2) Graphical Method

3) Tabulation Method 4) Multiplication Method

Let M be the total no of samples of $x(n)$ and N be the total no of samples of $h(n)$ then the total no of samples in $y(n) = M+N-1$

Properties :-

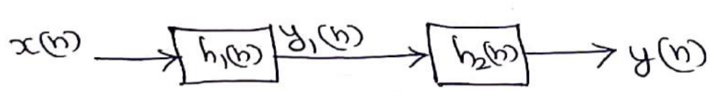
1) commutative property :-

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

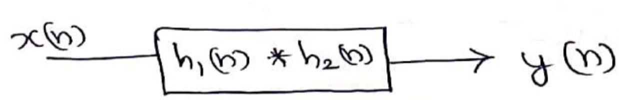
2) Associative property :-

$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

LHS



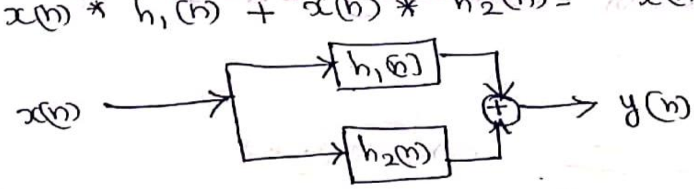
RHS



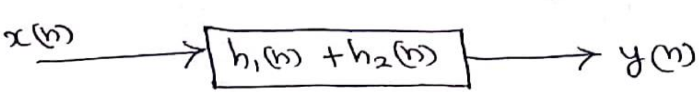
3) Distributive property :-

$$x(n) * h_1(n) + x(n) * h_2(n) = x(n) * [h_1(n) + h_2(n)]$$

LHS



RHS



1) Find the convolution sum of the given sequence :-

$x(n) = \{1, 2, 3, 4, 5\}$ $\quad h(n) = \{6, 7, 8\}$

Multiplication Method :-

$$
\begin{array}{ccccccc}
 & & 1 & 2 & 3 & 4 & 5 \\
 & & & & 6 & 7 & 8 \\
\hline
 & & 8 & 16 & 24 & 32 & 40 \\
 & 7 & 14 & 21 & 28 & 35 & \\
6 & 12 & 18 & 24 & 30 & & \\
\hline
6 & 19 & 40 & 61 & 82 & 67 & 40 \\
\hline
\end{array}
$$

$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$

Tabulation Method :-

	6	7	8
1	6	7	8
2	12	14	16
3	18	21	24
4	24	28	32
5	30	35	40

$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$

Definition Method :-

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$x(n) = \{1, 2, 3, 4, 5\}$ $h(n) = \{6, 7, 8\}$

$x(0) = 1$ $h(0) = 6$

$x(1) = 2$ $h(1) = 7$

$x(2) = 3$ $h(2) = 8$

$x(3) = 4$

$x(4) = 5$

Total No. of Samples $= M + N - 1$

$= 5 + 3 - 1$

$y(n) = 7$ samples [n varies 0 to 6]

$n = 0$

$$y(0) = \sum_{k=0}^{4} x(k)\, h(n-k)$$

$= x(0)\,h(0) + x(1)\,h(-1) + x(2)\,h(-2) + x(3)\,h(-3) + x(4)\,h(-4)$

$= 6$

$n = 1$ $y(1) = \sum_{k=0}^{4} x(k)\, h(1-k)$

$= x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(-1) + x(3)\,h(-2) + x(4)\,h(-3)$

$= 7 + 12 = 19$

$n = 2$

$$y(2) = \sum_{k=0}^{4} x(k)\, h(2-k)$$

$= x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0) + x(3)\,h(-1) + x(4)\,h(-2)$

$= 8 + 14 + 18$

$= 40$

$n = 3$

$$\therefore y(3) = \sum_{k=0}^{4} x(k)\, h(3-k)$$

$$= x(0)\,h(3) + x(1)\,h(2) + x(2)\,h(1) + x(3)\,h(0) + x(4)\,h(-1)$$

$$= 16 + 21 + 24$$

$$= 61$$

$n = 4$

$$y(4) = \sum_{k=0}^{4} x(k)\, h(4-k)$$

$$= x(0)\,h(4) + x(1)\,h(3) + x(2)\,h(2) + x(3)\,h(1) + x(4)\,h(0)$$

$$= 24 + 28 + 30$$

$$= 82$$

$n = 5$

$$y(5) = \sum_{k=0}^{4} x(k)\, h(5-k)$$

$$= x(0)\,h(5) + x(0)\,h(4) + x(2)\,h(3) + x(3)\,h(2) + x(4)\,h(3)$$

$$= 32 + 35$$

$$= 67$$

$n = 6$

$$y(6) = \sum_{k=0}^{4} x(k)\, h(6-k)$$

$$= x(0)\,h(6) + x(1)\,h(5) + x(2)\,h(4) + x(3)\,h(3) + x(4)\,h(2)$$

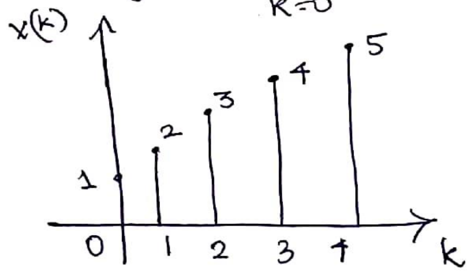
$$= 40$$

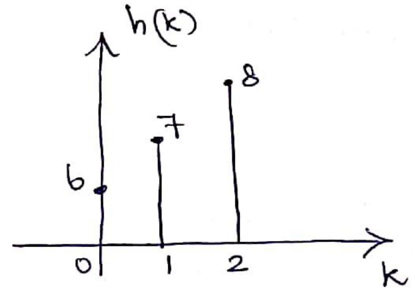
$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$

Graphical Method :-

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

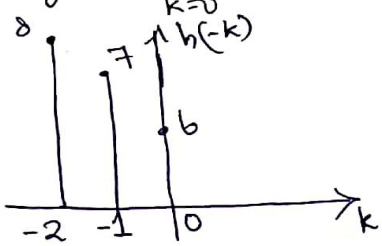
$$y(n) = \sum_{k=0}^{4} x(k)\, h(n-k)$$





n = 0

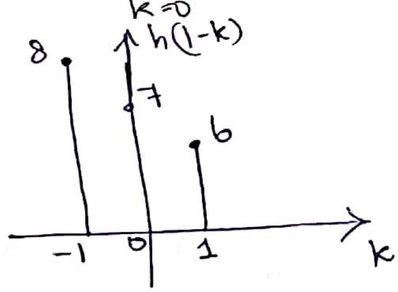
$$y(0) = \sum_{k=0}^{4} x(k)\, h(-k)$$



$y(0) = 1 \times 6 = 6$

n = 1

$$y(1) = \sum_{k=0}^{4} x(k)\, h(1-k)$$

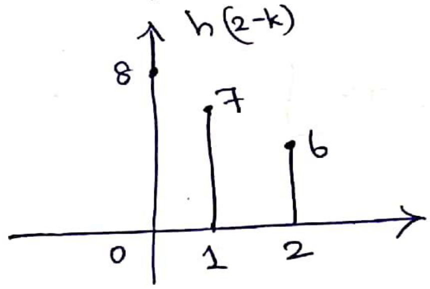


$y(1) = 7 + 12$

$= 19$

n = 2

$$y(2) = \sum_{k=0}^{4} x(k)\, h(2-k)$$

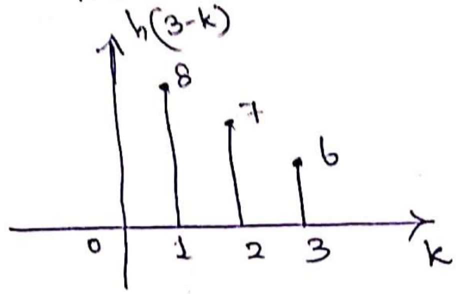


$y(2) = 8 + 14 + 18$

$= 40$

$n=3$

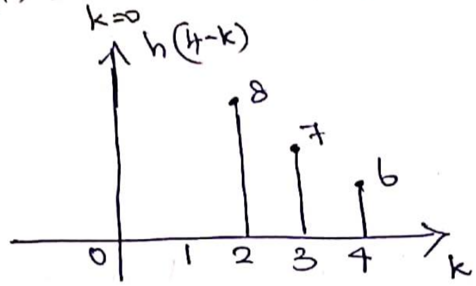
$$y(3) = \sum_{k=0}^{4} x(k)\, h(3-k)$$



$$y(3) = 16+21+24$$
$$= 61$$

$n=4$

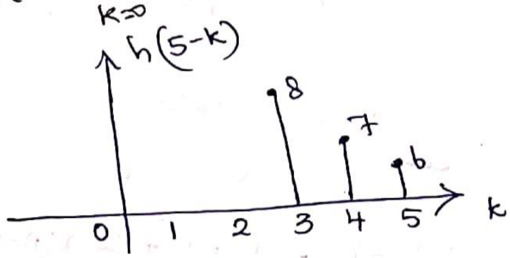
$$y(4) = \sum_{k=0}^{4} x(k)\, h(4-k)$$



$$y(4) = 24+28+30$$
$$= 82$$

$n=5$

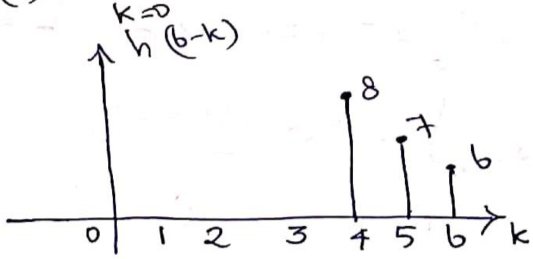
$$y(5) = \sum_{k=0}^{4} x(k)\, h(5-k)$$



$$y(5) = 32+35$$
$$= 67$$

$n=6$

$$y(6) = \sum_{k=0}^{4} x(k)\, h(6-k)$$



$$y(6) = 40$$

$$y(n) = \{6, 19, 40, 61, 82, 67, 40\}$$