CHAPTER 18

Factor–Product Relationship

Factor–product relationship is a basic production relationship between the input and output. This is mainly concerned with resource use and its efficiency. It guides the producer in deciding as to how much to produce. The goal of this relationship is the optimization of resources. This relationship is explained by the law of diminishing returns.

The farmer as a producer has a given goal clearly cut out with the inputs/resources* at his disposal. These resources are put into a process called production**. Through the production process all the inputs get transformed into an output/product***. This production process may pertain to using of the resources or inputs *viz.*, seed, fertilizer, irrigation, human labour *etc.*, to produce a given quantity of output of crop enterprises like paddy, sugarcane, wheat, cotton *etc.*, and similarly employing the resources like human labour, feed and fodder, medicines *etc.*, in the production of milk, meat, eggs, fish, *etc.* Having identified the necessary inputs which facilitate the production process, the question that arises is, whether the farmer possesses the knowledge of the production activity, imbibing the physical relationship between the resources and the expected output. This knowledge is essential because the response of output to input application is at varying magnitude and hence the farmer has to make decisions of how much input to use and how much output to produce. The discussion on factor-product relationship is confined to a single variable input and output.

$$Y = f(X_1 \mid X_2, X_3, \ldots\ldots X_n)$$

Where,

Y = Output from a particular enterprise

X_1 = Variable resource

$X_2 \ldots X_n$ = Fixed resources

| (Vertical bar) = It separates variable resource from fixed resources

PRODUCTION FUNCTION

Production function is a technical and mathematical relationship describing the manner and the extent to which a particular product depends upon the quantities of inputs or services

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* An input/resource is any good or service that goes into production.

** Production may be defined as a process by which inputs are transformed into an output.

*** Output/product is any good or service that comes out of production.

of inputs, used at a given level of technology and in a given period of time. It shows the quantity of output that can be produced using different levels of inputs.

Here, we need to understand certain concepts, which figure in the analysis of this relationship.

Total Physical Product (TPP)

It is the total amount of output obtained by using different units of inputs, measuring in physical units like quintals, *kgs, etc.*

Average Physical Product (APP)

It is the average amount of output produced by each corresponding unit of input. It is obtained by dividing the total output at a given level by the number of units of input applied at the corresponding level. APP reflects the efficiency of the variable input (technical efficiency).

$$APP = \frac{\text{Total physical product}}{\text{Input level}} = \frac{Y}{X}$$

Marginal Physical Product (MPP)

MPP is the additional quantity of output, added by an additional unit of input i.e., the change in output as a result of change in the variable input. It is calculated as

$$MPP = \frac{\text{Change in total physical product}}{\text{Change in input level}} = \frac{\Delta Y}{\Delta X}$$

Elasticity of Production (E_p)

It is defined as percentage change in output as a result of percentage change in input.

$$E_p = \frac{\text{Percentage change in output}}{\text{Percentage change in input}}$$

The elasticity of production can also be defined in terms of the relationship between MPP and APP as given below:

$$E_p = \frac{\left(\frac{\Delta Y}{Y}\right)}{\left(\frac{\Delta X}{X}\right)}$$

It can be written as,

$$E_p = \frac{\left(\frac{\Delta Y}{\Delta X}\right)}{\left(\frac{Y}{X}\right)}$$

We know that,

$\frac{\Delta Y}{\Delta X} = MPP$ and that $\frac{Y}{X} = APP$

Therefore,

$$E_p = \frac{MPP}{APP}$$

Relationship between TPP, MPP and APP

As long as MPP is increasing, TPP is increasing at an increasing rate. TPP goes on increasing at an increasing rate till the point of maximum MPP. After the point of maximum MPP, TPP increases at a decreasing rate. When MPP becomes zero, TPP attains its maximum. Negative MPP results in decreasing TPP. When TPP is increasing, MPP is positive, when TPP is maximum MPP is zero and when TPP declines MPP becomes negative.

Tabular presentation of a production function is given in Table 18.1, which indicates the different levels of variable input (human labour) along with the total physical product (TPP), average physical product (APP) and marginal physical product (MPP). Land has been assumed as the fixed resource. The TPP is increasing up to 7th unit and thereafter it begins to decline. The APP up to the application of five units of human labour, increased and thereafter it tended to decline. Marginal product increased from 15 units to 30 units, with the increase in the use of human labour from first unit to fourth unit, and decreased with further increase in the labour input. Marginal product is negative when the variable input level increased to 8th unit and beyond. The production function is illustrated in Figure 18.1.

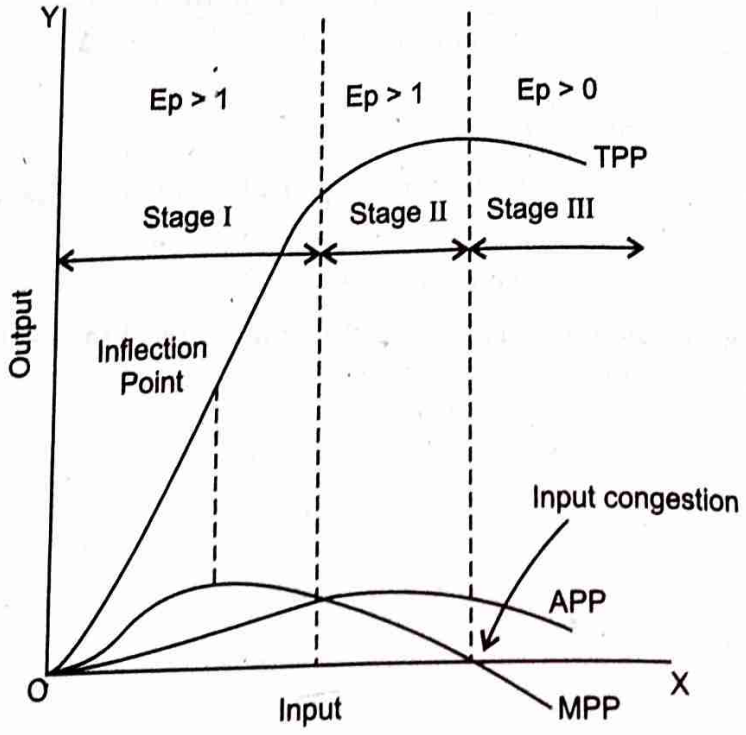

Figure 18.1 Three stages of production function.

THREE STAGES OF PRODUCTION FUNCTION

The classical production function can be divided into three stages or zones or regions. This is done to identify the zone in which production decisions are rational.

Stage I: It starts from the origin and ends at the point where, MPP = APP. In this stage, MPP > APP as a result of which APP is increasing. The MPP attains the

maximum at the point of inflection*, thereafter it begins to decline. TPP increases at an increasing rate till the inflection point and thereafter it increases at decreasing rate. E_p is more than one throughout the stage I of production and E_p is one at the end of this stage. In this stage fixed resources are abundant relative to variable resource. The technical efficiency of the variable resource is increasing as indicated by increasing APP. Also the technical efficiency of fixed resource is increasing as reflected by increasing TPP. This stage is regarded as irrational (sub-optimal) stage of production.

Stage II: It is found from the point of equality of MPP and APP and ends where MPP is zero, at which input use level, TPP is maximum. In this stage, MPP is less than APP. However, both MPP and APP exhibit declining trend. Average productivity derived from each individual unit of the variable resource is on the decline in this stage, though it is at its peak at the beginning of Stage II. TPP increases at a decreasing rate as MPP is declining. E_p is less than one throughout stage II. E_p is zero at the end of this stage. It is rational (optimal) stage of production. The technical efficiency of the variable resource declines as indicated by the declining APP, but the technical efficiency of fixed resource increases as indicated by increasing TPP. In this stage variable resource is abundant relative to fixed resources.

Stage III: The starting point of stage III is the end of stage II, at which MPP is zero. In this stage MPP becomes negative. APP continuously declines and TPP which is at its maximum at the end of stage II, begins to decline. E_p is less than zero. It is an irrational (supra-optimal) stage of production. The technical efficiency of variable resource and fixed resource declines. Variable resource is in excessive quantities relative to fixed resource.

Reasons for Increasing, Decreasing and Negative Returns

In stage I, fixed resources are in abundance and these fixed resources are not put to efficient utilization due to lack of sufficient quantity of variable resource, and infact all the fixed resources are slack i.e., they are at the disposal of the farmers. Under this situation application of more quantities of variable resource makes the hither to unutilized fixed resource efficient leading to increasing returns.

As more quantities of variable resource are applied soon we reach the point of maximum marginal physical product. Beyond this point, any further increase in the use of variable resource yields less additional output (decreasing returns). This happens as more quantity of variable resource has to accommodate with less quantity of fixed resources.

Production in general is the integrated effort of fixed and variable resources. Since the proportion of variable resource becomes too large to the very limited fixed resources, the balance which is supposed to exist between fixed resources and variable resource gets disturbed, thereby leading to the negative returns, in the third stage.

RATIONAL AND IRRATIONAL STAGES OF PRODUCTION

Of the three stages, stage I and stage III are irrational, while stage II is rational. In stage I since the average productivity (technical efficiency) of variable resource keeps

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* Inflection point is the point at which MPP is maximum. It is also the point at which TPP curve reverse its shape from convex to concave.

on increasing, it is not judicious to stop the application of the variable resource. Fixed factor which is kept as slack (idle) can be withdrawn and we can reorganize the resources. This reorganization leads to the production of more output. Since it is possible to produce more output from less quantity of resources through reorganization of fixed and variable resources, this stage is called irrational stage.

In stage III, the technical efficiency of variable resource (APP) and fixed resource (TPP) declines. The additional productivity of variable resource (MPP) becomes negative. Given the productivities of variable and fixed resources, no rational producer attempts to operate in this stage, as any attempt to continue production brings him colossal loss in the form of additional costs of variable resource and reduction in total output. At this point a look at Table 18.1 indicates that maximum production is attained with 7^{th} unit of human labour and further increase in this resource is reducing the total output. Withdrawal of the human labour from 10 units to 7 units would increase the output. The application of 7^{th} unit of human labour not only costs the farmer, but also the additional gains in terms of output are zero. Such a situation is called free disposability of input. Application of 8^{th} unit of human labour, not only costs the farmer, but on the other hand, brings him loss, in the form of reduction of output; and such a situation is called weak disposability of input. Since stage III offers the opportunity of reorganization of fixed and variable resources, it is called an irrational stage.

In stage II, the point of optimality, either in the use of input or in the production of output, lies. The boundaries of this stage are the points, at which the technical efficiency of variable resource is maximum and the technical efficiency of fixed resource is maximum. Somewhere the point of optimality lies in this stage and in order to locate the point of optimality, choice indicators are needed. The choice indicators are price ratios i.e., the price per unit of input and price per unit of output. Without knowing the prices, certain generalizations can be made with regard to the use of variable and fixed resources. Assuming that fixed resource is unlimited in quantity and variable resource is scarce, the aim of the producer is to produce maximum output per unit of scarce resource i.e., human labour here. This is achieved when the average productivity (technical efficiency of variable resource) is maximum. This is seen at the beginning of stage II. When variable resources are unlimited and fixed resource is scarce; the objective again is to maximize physical production per unit of scarce resource i.e. fixed resource. This is achieved when the technical efficiency of fixed factor (TPP) is maximum i.e., the point of congestion. This point is found at the end of stage II. In stage II, since there is no possibility of recombining fixed and variable resources, it is a rational stage or optimal stage.

Production function presents the consistent relationship between different levels of variable resource applied in a production activity along with the corresponding levels of output. The producer is interested in knowing the level of input use and level of production, at which profits are maximum. To find out optimal level of input and output, we have to know the choice indicators.

DETERMINATION OF OPTIMUM LEVEL OF INPUT

Having identified stage II as rational stage, the extent of variable resource use needs to be studied. This is done by working out and comparing marginal value product (MVP) and marginal input cost (MIC) as presented in Table 18.2.

Marginal Value Product (MVP) of Input

It is the additional income received from using an additional unit of input. It is computed using the following formula.

$$MVP = \frac{\text{Change in the total value product}}{\text{Change in input level}} = \frac{\Delta TR}{\Delta X} = MPP \times P_Q$$

Marginal Input Cost (MIC)

MIC is defined as the change in the total input cost by using an additional unit of input. It is expressed as

$$MIC = \frac{\text{Change in total input cost}}{\text{Change in input level}} = \frac{\Delta TC}{\Delta X} = \text{Price per unit of input}$$

In the first five rows of Table 18.2 MVP is greater than MIC. At the sixth input level, MVP and MIC are exactly equal which is the optimum level of human labour use. Beyond the sixth unit of human labour use, MVP < MIC, indicating the profit reduction as more units of input are used.

DETERMINATION OF OPTIMUM LEVEL OF OUTPUT

Apart from identifying the profit maximizing input level, it is pertinent to find out the level of output, which maximizes profit. To carry on this analysis, it is essential to work out and compare marginal revenue (MR) and marginal cost (MC).

Marginal Cost (MC)

MC is defined as the additional cost incurred for producing an additional unit of output. The expression is as follows:

$$MC = \frac{\text{Change in total input cost}}{\text{Change in total physical product}} = \frac{\Delta TC}{\Delta Q}$$

Marginal Revenue (MR)

MR is defined as the additional income obtained from producing one more unit of output. It is expressed as:

$$MR = \frac{\text{Change in total income}}{\text{Change in total physical product}} = \frac{\Delta TR}{\Delta Q}$$

MR is always equal to price per unit of output. MR and MC values in Table 18.3 are compared to find out the optimal output (profit maximizing) level. In the first five rows of the table, MR is greater than MC. At the sixth input level, MR = MC, which is the optimum level of output. Using beyond sixth unit, it causes MR to fall below MC, implying profit reduction. If MR > MC, the additional unit of output enhances the profit and if MR < MC, the production of additional unit of output decreases the profit.

The analysis reveals that whether it is the equality of MVP and MIC or MR and MC, the result as well as decision-making is the same i.e., there is only one profit maximizing level of input and output for the given information of technical coefficients and prices.

Summary of Three Stages of Production Function:

S. No.	Stage I	Stage II	Stage III
1.	Starts from the origin and ends where MPP = APP	Starts from where APP is maximum and ends where MPP is zero	Starts from where MPP is zero or TPP is maximum
2.	TPP increases at increasing rate	TPP increases at up to the point of inflection	TPP decreases at decreasing rate
3.	APP is increasing throughout this stage	APP decreases	APP decreases
4.	MPP increases up to the point	MPP decreases of inflection	MPP becomes negative
5.	MPP > APP	MPP < APP	-
6.	$E_p > 1$	$E_p < 1$	$E_p < 0$
7.	Technical efficiency of variable and fixed resources increases	Technical efficiency of variable resource decreases but of the fixed resources increases	Technical efficiency of variable and fixed resources decreases
8.	Fixed resources are abundant and variable resource is scarce	Variable resource is abundant and fixed resources are scarce	Variable resource is in excess capacity
9.	Sub-optimal (irrational) stage of production	Optimal (rational) stage of production	Supra-optimal (irrational) stage of production
10.	Scope for the reorganization of resources	No scope for the reorganization	Scope for the reorganization
11.	MVP > MIC	MVP = MIC	MVP < MIC
12.	MR > MC	MR = MC	MR < MC