Type - II : No. of a.c. > No of I.V

$\Rightarrow$ p. d.e. of order 1.

Example 1 : Form a PDE by eliminating a, b, c from

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Soln: Gn: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ — (1)

Diff w.r.to. x & y we get

$$\frac{2x}{a^2} + 0 + \frac{2z}{c^2}\frac{\partial z}{\partial x} = 0$$

$$\frac{2x}{a^2} + \frac{2z}{c^2} p = 0 \quad \text{— (2)}$$

$$0 + \frac{2y}{b^2} + \frac{2z}{c^2} \cdot \frac{\partial z}{\partial y} = 0$$

$$\frac{2y}{b^2} + \frac{2z}{c^2} q = 0 \quad \text{— (3)}$$

Again, diff (2) w. r. to x

$$\frac{2}{a^2} + \frac{2}{c^2}\left(z \cdot \frac{\partial^2 z}{\partial x^2} + \frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial x}\right) = 0.$$

$$\frac{2}{a^2} + \frac{2}{c^2} p^2 + \frac{2z}{c^2}\frac{\partial^2 z}{\partial x^2} = 0.$$

$$\frac{2c^2}{a^2} + 2p^2 + 2z\frac{\partial^2 z}{\partial x^2} = 0.$$

$$p^2 + z\frac{\partial^2 z}{\partial x^2} = -\frac{c^2}{a^2} \quad \text{— (4)}$$

(2) $\Rightarrow$ $\frac{x}{a^2} = \frac{-zp}{c^2}$ $\Rightarrow$ $\frac{-c^2}{a^2} = \frac{zp}{x}$ — (5)

(5) sub in (4)

$$p^2 + z \cdot \frac{\partial^2 z}{\partial x^2} = \frac{zp}{x}.$$

$$\Rightarrow -\frac{zp}{x} + p^2 + z \cdot \frac{\partial^2 z}{\partial x^2} = 0 \Rightarrow -zp + xp^2 + zx\frac{\partial^2 z}{\partial x^2} = 0$$

Ans: $-z\frac{\partial z}{\partial x} + x\left(\frac{\partial z}{\partial x}\right)^2 + zx\left(\frac{\partial^2 z}{\partial x^2}\right) = 0$

2) Form a pde by eliminating the a.c. a, b and c from $z = ax + by + cxy$

Soln: Gn: $z = ax + by + cxy$ — ①

Partially diff w. r. to x & y we get

$$\frac{\partial z}{\partial x} = p = a + cy \quad — ②$$

$$\frac{\partial z}{\partial y} = q = b + cx \quad — ③$$

Diff ② again w. r. to y

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial x \partial y} = c = s \quad — ④$$

Diff ③ again w. r. to y

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial y^2} = t = 0 \quad — ⑤$$

Diff ② again w. r. to x

$$\left(\frac{\partial^2 z}{\partial x^2}\right) = 0 \quad — ⑥$$

$s = 0$ & $t = 0$ are the req. p.d.e.

sub ④ in ③

$q = b + sx \Rightarrow b = -sx + q$ — ⑦

sub ④ in ②

$p = a + sy \Rightarrow a = -sy + p$ — ⑧

sub ⑦ & ⑧ in ①

$z = (-sy + p)x + (-sx + q)y + sxy$

$z = -sxy + px - sxy + qy + sxy$

$\therefore z = px + qy - sxy$ is the req. p.d.e.

1) $z = (x+a)^2 + (y+b)^2 + c^2$ Ans $r = 2x$

HW) 2) $ax + by + cz = 1$ Ans $r = 0$

$t = 0$