Formation of PDE by elimination of a.f.

Type-I: Elimination of 1 arbitrary function

1) Form the pde by eliminating the arbitrary function from (i) $z = f(x^2-y^2)$

~~(ii) $z = f(x+ay) + g(x-ay)$~~.

Soln: (i) Gn: $z = f(x^2-y^2)$ —— ①.

Partially diff ① w.r. to x & y we get

$$\frac{\partial z}{\partial x} = p = f'(x^2-y^2)(2x).$$

$$\Rightarrow f'(x^2-y^2) = \frac{p}{2x}. \quad \text{—— ②}$$

Diff ② partially w.r. to y $\frac{\partial z}{\partial y} = q = f'(x^2-y^2)(-2y)$

$$f'(x^2-y^2) = -\frac{q}{2y} \quad \text{—— ③}.$$

$$② \,\&\, ③ \Rightarrow \frac{p}{2x} = -\frac{q}{2y}$$

$$\Rightarrow py = -qx$$

$$\Rightarrow py + qx = 0$$

is the req. pde.

HW: $z = f(x^3-y^3)$ soln: $py^2 + qx^2 = 0$.

(iii) $z = f(x+y)$ —— (1)

$p = \frac{\partial z}{\partial x} = f'(x+y)$

$q = \frac{\partial z}{\partial y} = f'(x+y)$

$\Rightarrow p = q$ is the req. pde

3) Gn: $z = f\left(\frac{xy}{z}\right)$ —— (1).

Diff w.r.to x & y we get

$$\frac{\partial z}{\partial x} = p = f'\left(\frac{xy}{z}\right) \cdot \left(\frac{z(y) - (xy)\frac{\partial z}{\partial x}}{z^2}\right)$$

$$p = f'\left(\frac{xy}{z}\right)\left(\frac{zy - xyp}{z^2}\right) \quad \text{—— (2)}$$

$$\frac{\partial z}{\partial y} = q = f'\left(\frac{xy}{z}\right)\left(\frac{z(x) - (xy)\frac{\partial z}{\partial y}}{z^2}\right)$$

$$q = f'\left(\frac{xy}{z}\right)\left(\frac{zx - xyq}{z^2}\right) \quad \text{—— (3)}$$

From (2) & (3).

$$\frac{p}{\left(\frac{zy - xyp}{z^2}\right)} = \frac{q}{\left(\frac{zx - xyq}{z^2}\right)}$$

$$p(zx - xyq) = q(zy - xyp)$$

$$pzx - pqxy = qzy - pqxy$$

$$pzx = qzy$$

$\Rightarrow px = qy$ is the req pde.

HW: $z = f(x^2 + y^2 + z^2)$ Soln: $py = qx$.