Type 2: Elimination of two arbitrary functions:

a f = 2 ⇒ order of pde = 2 or > 2

① $z = f(x+ay) + g(x-ay)$ —— ①

Diff ① par. w. r. to x & y.

$$\frac{\partial z}{\partial x} = p = f'(x+ay) + g'(x-ay) \quad \text{—— ②}$$

$$\frac{\partial^2 z}{\partial x^2} = r = f''(x+ay) + g''(x-ay) \quad \text{—— ③}$$

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$$\frac{\partial z}{\partial y} = q = f'(x+ay)(a) + g'(x-ay)(-a) \quad \text{--- (1)}$$

$$\frac{\partial^2 z}{\partial y^2} = t = f''(x+ay)(a^2) + g''(x-ay)(+a^2)$$

$$= a^2\left[f''(x+ay) + g''(x-ay)\right] \quad \text{--- (5)}$$

sub (3) in (5),

$t = a^2 r$ is the req. pde.

HW (2) $z = f(2x+y) + g(3x-y)$ --- (1).

$$p = \frac{\partial z}{\partial x} = f'(2x+y)(2) + g'(3x-y)(3) \quad \text{--- (2)}$$

$$q = \frac{\partial z}{\partial y} = f'(2x+y) + g'(3x-y)(-1) \quad \text{--- (3)}.$$

$$(2) \Rightarrow r = \frac{\partial^2 z}{\partial x^2} = f''(2x+y)(4) + g''(3x-y)(9) \quad \text{--- (4)}$$

$$(3) \Rightarrow t = \frac{\partial^2 z}{\partial y^2} = f''(2x+y) + g''(3x-y)(1) \quad \text{--- (5)}.$$

$$(4) \Rightarrow s = \frac{\partial^2 z}{\partial x \partial y} = 2f''(2x+y)(2) + g'(3x-y)(-3)$$

--- (6).

$r = f''(4) + g''(9)$

$s = f''(2) + g''(-3)$

$t = f''(1) + g''(1)$.

Eliminating f'' & g'' from (4), (5) and (6).

$$\begin{vmatrix} 4 & 9 & r \\ 2 & -3 & s \\ 1 & 1 & t \end{vmatrix} = 0$$

(Using determinant formula.

$4(-3t-s) - 9(2t-s) + r(2+3) = 0$

$-12t - 4s - 18t + 9s + 2r + 3r = 0$

$5r + 5s - 30t = 0$

$r + s - 6t = 0$.

$$\Rightarrow \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$$ is the req. Pde.

3) $z = f(x^3+2y) + g(x^3-2y)$.

Soln: $r = \dfrac{2p}{x} + \dfrac{9tx^4}{4}$.

(1) $z = f(x^2+y) + \phi(x^2-y)$ —— (1)

$p = \frac{\partial z}{\partial x} = 2x f'(x^2+y) + 2x\phi'(x^2-y)$ —— (2)

$q = \frac{\partial z}{\partial y} = f'(x^2+y) + \phi'(x^2-y)(-1)$ —— (3).

(2) $\Rightarrow \frac{\partial^2 z}{\partial x^2} = 2\left[x f''(x^2+y)(2x) + f'(x^2+y)\right]$

$+ 2\left[x \cdot \phi''(x^2-y)(2x) + \phi'(x^2-y)\right]$

$\frac{\partial^2 z}{\partial x^2} = 4x^2 f''(x^2+y) + 2f'(x^2+y) + 4x^2\phi''(x^2-y)$
$+ 2\phi'(x^2-y)$ —— (4)

(3) $\Rightarrow \frac{\partial^2 z}{\partial y^2} = f''(x^2+y) + \phi''(x^2-y)$ —— (5).

(3) $\Rightarrow \frac{\partial^2 z}{\partial x \partial y} = 2x f''(x^2+y) + 2x\phi''(x^2-y)(-1)$ —— (6)

$= 2x f''(x^2+y) - 2x\phi''(x^2-y)$

(4) $\Rightarrow \frac{\partial^2 z}{\partial x^2} = 2\left[f'(x^2+y) + \phi'(x^2-y)\right]$

$+ 4x^2\left[f''(x^2+y) + \phi''(x^2-y)\right]$

Using (2) and (5)

$$\frac{\partial^2 z}{\partial x^2} = 2\left[\frac{1}{2x} \cdot \frac{\partial z}{\partial x}\right] + 4x^2\left[\frac{\partial^2 z}{\partial y^2}\right]$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{x}\frac{\partial z}{\partial x} + 4x^2\frac{\partial^2 z}{\partial y^2}$$

$$x\frac{\partial^2 z}{\partial x^2} - 4x^3\frac{\partial^2 z}{\partial y^2} - \frac{\partial z}{\partial x} = 0.$$

$r - 4x^3 t - p = 0$, is the req pde.

HW: 1) $z = f(x+iy) + g(x-iy)$

$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0 \rightarrow$ ans

2) ~~$z = f(x+y), \phi(x-y)$~~ $z = \phi_1(x) \cdot \phi_2(y)$