Type - 3: Elimination of a.f. from $\phi(u,v)=0$

$u, v \rightarrow$ funs of x, y, z.

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$

1) Form the PDE by eliminating a.f from

$\phi(x^2+y^2+z^2, ax+by+cz)=0$.

Let $u = x^2+y^2+z^2$ $\quad v = ax+by+cz$.

diff ① ⓧ

$\frac{\partial u}{\partial x} = 2x+0+2z\frac{\partial z}{\partial x}$ $\quad \frac{\partial v}{\partial x} = a+0+c\cdot\frac{\partial z}{\partial x}$

$\frac{\partial u}{\partial x} = 2x+2zp$ $\quad \frac{\partial v}{\partial x} = a+c\cdot p$

diff ① ⓨ

$\frac{\partial u}{\partial y} = 0+2y+2z\cdot\frac{\partial z}{\partial y}$ $\quad \frac{\partial v}{\partial y} = 0+b+c\cdot\frac{\partial z}{\partial y}$

$\frac{\partial u}{\partial y} = 2y+2zq$ $\quad \frac{\partial v}{\partial y} = b+cq$.

$\phi(u,v)=0;$ $\begin{vmatrix} 2x+2zp & a+cp \\ 2y+2zq & b+cq \end{vmatrix} = 0.$

$(2x+2zp)(b+cq)-(a+cp)(2y+2zq)=0$

$2bx+2cqx+2bzp+2czpq$

$-[2ay+2azq+2cpy+2czpq]=0.$

$2bx+2cqx+2bzp+\cancel{2czpq}-2ay-2azq$

$-2cpy-\cancel{2czpq}=0.$

$-ay+bx+cqx-cpy+bpz-aqz=0$

is the required pde.

(2) Form the PDE by eliminating f, from $f(xy+z^2, x+y+z)=0$.

Soln: Gn: $f(xy+z^2, x+y+z)=0$ ——— (1).

Let $u=xy+z^2$; $v=x+y+z$; $f(u,v)=0$ ——— (2)

Elimination of 'f' from (2) gives,

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0 \quad \text{——— (3).}$$

$$\frac{\partial u}{\partial x} = y + 2z.\frac{\partial z}{\partial x} \qquad \frac{\partial v}{\partial x} = 1+0+\frac{\partial z}{\partial x}$$

$$\frac{\partial u}{\partial x} = y + 2z.p \qquad \frac{\partial v}{\partial x} = 1+p$$

$$\frac{\partial u}{\partial y} = x + 2z.\frac{\partial z}{\partial y} \qquad \frac{\partial v}{\partial y} = 0+1+\frac{\partial z}{\partial y}$$

$$\frac{\partial u}{\partial x} = x+2zq \qquad \frac{\partial v}{\partial y} = 1+q.$$

sub in (3).

$$\begin{vmatrix} y+2zp & 1+p \\ x+2zq & 1+q \end{vmatrix} = 0$$

$$(y+2zp)(1+q) - (1+p)(x+2zq) = 0.$$

$$y+yq+2zp+2zpq-(x+2zq+px+2pzq)=0.$$

$$y+yq+2zp+\cancel{2zpq}-x-2zq-px-\cancel{2pzq}=0.$$

$$(2z-x)p+(y-2z)q = x-y$$ is the required pde.