Solving of the first order PDE

General form of 1st order pde is

$$f(x, y, z, p, q) = 0$$

where $p = \frac{\partial z}{\partial x}$; $q = \frac{\partial z}{\partial y}$.

Solutions: * C.I / C.S.

* Particular Integral
* Singular Integral
* General Integral.

Solution of PDE in Simple Cases

1) Solve $\frac{\partial z}{\partial y} = \sin x$.

Soln: $\frac{\partial z}{\partial y} = \sin x$

Int w.r. to y

$$\int \frac{\partial z}{\partial y} dy = \int \sin x \, dy$$

$$\int \partial z = \sin x \, y + f(x)$$

$$z = \sin x \cdot y + f(x) \quad [f(x) \text{ is arbitrary}]$$

2) Solve $\frac{\partial^2 z}{\partial x^2} = 0$.

Soln: $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = 0$

Integrating,

$$\partial\left(\frac{\partial z}{\partial x}\right) = 0$$

Again integrating,

$$\int \partial \frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} + f(y) = 0.$$

$$\partial z = f(y) \partial x$$

Again, $\int \partial z = \int f(y) dx.$

$$z = x f(y) + g(y)$$

[$f(y)$ & $g(y)$ are a.f]

3) Solve $\frac{\partial^2 z}{\partial x^2} = \sin x$.

Soln:

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \sin x.$$

$$\partial\left(\frac{\partial z}{\partial x}\right) \sin x \, \partial x.$$

Integrating, $\int \partial\left(\frac{\partial z}{\partial x}\right) = \int \sin x \, \partial x.$

$$\frac{\partial z}{\partial x} = -\cos x + f(y)$$

Int, $\int \frac{\partial z}{\partial x} = \int -\cos x \, \partial x + \int f(y) \partial x$

$$z = -\sin x + x f(y) + g(y)$$

($f(y)$ & $g(y)$ are arbitrary functions)