(ii) Method of Multipliers:

1) Solve: $x(y-z)p + y(z-x)q = z(x-y)$ — (1).

Soln:

This is of the form $Pp + Qq = R$. (Lagrange's form).

$P = x(y-z)$; $Q = y(z-x)$; $R = z(x-y)$.

The Auxiliary Equation is gn by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

Equal ratios

$$\frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a+c}{b+d} = \frac{a}{b} = \frac{c}{d}$$

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

choosing 1, 1, 1 as Lagrange's Multipliers, each of the above ratios are equal to .

$$\frac{dx+dy+dz}{x(y-z)+y(z-x)+z(x-y)} = \frac{dx+dy+dz}{0}$$

$$\Rightarrow dx+dy+dz = 0$$

$$\int dx + \cancel{tg} \int dy + dz = 0.$$

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$$\int d(x+y+z) = 0$$

$$\boxed{x+y+z = c_1} \quad \text{(2)}$$

Choosing $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as Lagrange's Multipliers, each of the above is equal to

$$\frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{y-z+z-x+x-y} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\int\frac{dx}{x} + \int\frac{dy}{y} + \int\frac{dz}{z} = 0$$

$$\log x + \log y + \log z = \log c_2$$

$$\log(xyz) = \log c_2$$

$$\boxed{c_2 = xyz} \quad \text{(3)}$$

$\therefore$ The general solution of the given PDE is $\phi(c_1, c_2) = 0$.

$$\phi(x+y+z, xyz) = 0 \;//$$

2) Solve $(mz-ny)p + (nx-lz)q = ly - mx$.

Soln: This is of the form $Pp + Qq = R$.

$P = mz-ny \quad Q = nx-lz \quad R = ly-mx$.

The Auxiliary eqn is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{mz-ny} = \frac{dy}{nx-lz} = \frac{dz}{ly-mx}$$

Choosing l, m, n as Lagrange's Multipliers, each of the following ratios equal to

$$\frac{l\,dx + m\,dy + n\,dz}{l(mz-ny)+m(nx-lz)+n(ly-mx)} = \frac{l\,dx+m\,dy+n\,dz}{0}$$

$$l\,dx + m\,dy + n\,dz = 0.$$

$$\int l\,dx + \int m\,dy + \int n\,dz = 0$$

$$lx + my + nz = c_1 \quad \text{——} (2)$$

Choosing x, y, z as Lagrange's Multipliers, each of the above ratios are equal to

$$\frac{x\,dx + y\,dy + z\,dz}{x(mz-ny)+y(nx-lz)+z(ly-mx)} = \frac{x\,dx+y\,dy+z\,dz}{0}$$

$$\int x\,dx + \int y\,dy + \int z\,dz = 0.$$

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_2$$

$$\Rightarrow x^2+y^2+z^2 = c_2 \quad \text{——} (3).$$

$\therefore$ The general solution of the given PDE is $\phi(c_1, c_2) = 0$

$$\phi(lx+my+nz,\ x^2+y^2+z^2) = 0.$$

3) Find the general solution of

$$x(z^2-y^2)p + y(x^2-z^2)q = z(y^2-x^2).$$

Soln: choose x, y, z as multipliers

$$c_1 = x^2+y^2+z^2$$

choose $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ as multipliers

$$xyz = c_2$$

$\therefore$ The general solution is

$$\phi(x^2+y^2+z^2,\ xyz) = 0$$

1) Solve $(x^2-yz)p+(y^2-zx)q=z^2-xy$

choose x, y, z as multipliers

$$\frac{x\,dx+y\,dy+z\,dz}{x^3+y^3+z^3-3xyz}=\frac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx}.$$

formula: $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$$\therefore \frac{x\,dx+y\,dy+z\,dz}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}=\frac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx}.$$

$$x\,dx+y\,dy+z\,dz=(x+y+z)\,dx+dy+dz.$$

$$\int x\,dx+\int y\,dy+\int z\,dz=\int(x+y+z)\,d(x+y+z)$$

$$\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}=\frac{(x+y+z)^2}{2}+c_1$$

$$x^2+y^2+z^2=(x+y+z)^2+c_1$$

$$x^2+y^2+z^2=x^2+y^2+z^2+2xy+2yz+2zx+c_1$$

$$2xy+2yz+2zx=c_1$$

$$xy+yz+zx=c_1 \quad \text{--- (2)}$$

Now;
$$\frac{dx-dy}{(x^2-yz)-(y^2-zx)}=\frac{dy-dz}{(y^2-zx)-(z^2-xy)}$$

$$\frac{dx-dy}{(x^2-y^2)+z(x-y)}=\frac{dy-dz}{(y^2-z^2)+x(y-z)}$$

$$\frac{dx-dy}{(x+y)(x-y)+z(x-y)}=\frac{dy-dz}{(y+z)(y-z)+x(y-z)}$$

$$\frac{dx-dy}{(x-y)[(x+y+z)]}=\frac{dy-dz}{(y-z)[y+z+x]}.$$

$$\frac{d(x-y)}{(x-y)(x+y+z)} = \frac{d(y-z)}{(y-z)(x+y+z)}$$

$$\int \frac{d(x-y)}{x-y} = \int \frac{d(y-z)}{y-z}$$

$$\log(x-y) = \log(y-z) + \log c_2$$

$$\log(x-y) - \log(y-z) = \log c_2$$

$$\log\left(\frac{x-y}{y-z}\right) = \log c_2$$

$$c_2 = \frac{x-y}{y-z} \quad \text{--- (3)}$$

$\therefore$ Soln is $\phi\left(xy+yz+zx, \frac{x-y}{y-z}\right) = 0.$

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