

Formation of Difference Equation:

1) Form the difference eqn from $y_n = a + b3^n$ by eliminating the constants

Soln: Given: $y_n = a + b3^n$ ——— (1)

$$y_{n+1} = a + b3^{n+1}$$

$$y_{n+1} = a + 3b^n$$
 ——— (2)

$$y_{n+2} = a + b3^{n+2}$$

$$y_{n+2} = a + 9b^n$$
 ——— (3)

Eliminating a and b from (1), (2) and (3).

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 1 & 3 \\ y_{n+2} & 1 & 9 \end{vmatrix} = 0$$

$$y_n(9-3) - 1(9y_{n+1} - 3y_{n+2}) + 1(y_{n+1} - y_{n+2}) = 0$$

$$6y_n - 9y_{n+1} + 27y_{n+2} + y_{n+1} - y_{n+2} = 0$$

$$6y_n - 8y_{n+1} + 26y_{n+2} = 0$$

$$\therefore y_{n+2} - 4y_{n+1} + 3y_n = 0$$

2) $y_n = A2^n + Bn$ (or) $y_x = a2^x + 2^x$

$$\text{Gn: } y_n = A2^n + Bn$$
 ——— (1)

$$y_{n+1} = A2^{n+1} + B(n+1)$$

$$= 2A2^n + B(n+1)$$

$$y_{n+1} = 2 \cdot 2^n A + (n+1)B$$
 ——— (2)

$$y_{n+2} = A2^{n+2} + B(n+2)$$

$$y_{n+2} = 2^2 \cdot 2^n \cdot A + (n+2)B$$
 ——— (3)

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Eliminating A and B we get

$$\begin{vmatrix} y_n & 1 & n \\ y_{n+1} & 2 & n+1 \\ y_{n+2} & 4 & n+2 \end{vmatrix} = 0.$$

$$y_n (2(n+2) - 4(n+1)) - 1((y_{n+1})(n+2) - (n+1)y_{n+2}) + n(4y_{n+1} - 2y_{n+2}) = 0.$$

$$2(n+2)y_n - 4(n+1)y_n - (n+2)y_{n+1} + (n+1)y_{n+2} + 4ny_{n+1} - 2ny_{n+2} = 0.$$

$$(2n+4-4n-4)y_n + (-n+2+4n)y_{n+1}$$

$$+ (n+1-2n)y_{n+2} = 0.$$

$$-2ny_n + (3n-2)y_{n+1} + (1-n)y_{n+2} = 0.$$

$$\Rightarrow (1-n)y_{n+2} + (3n-2)y_{n+1} - 2ny_n = 0.$$

is the required Difference equation.

3) Form a difference eqn by eliminating arbitrary constants from

$$y_n = A + B \cdot 2^n$$

Soln Gn: $y_n = A + B \cdot 2^n$ — (1)

$$y_{n+1} = A + B \cdot 2^{n+1}$$

$$y_{n+1} = A + 2B \cdot 2^n$$
 — (2)

$$y_{n+2} = A + B \cdot 2^{n+2}$$

$$y_{n+2} = A + 4B \cdot 2^n$$
 — (3)

$$\begin{aligned} (1) y_n &= A \cdot 3^n \\ y_{n+1} &= A \cdot 3^{n+1} \\ &= 3A \cdot 3^n \\ &= 3y_n \end{aligned}$$

$$y_{n+1} - 3y_n = 0 //$$

Eliminating A and B we get

$$\begin{bmatrix} y_n & 1 & 1 \\ y_{n+1} & 1 & 2 \\ y_{n+2} & 1 & 4 \end{bmatrix} = 0$$

$$y_n(4-2) - 1(4y_{n+1} - 2y_{n+2}) + 1(y_{n+1} - y_{n+2}) = 0$$

$$2y_n - 4y_{n+1} + 2y_{n+2} + y_{n+1} - y_{n+2} = 0.$$

$$2y_n - 3y_{n+1} + y_{n+2} = 0.$$

$$y_{n+2} - 3y_{n+1} + 2y_n = 0 //$$