

Properties of Fourier Sine & Cosine Transforms:

Property 1: Linear Property

$$a) f_s [a f(x) + b g(x)] = a f_s(s) + b g_s(s)$$

$$b) f_c [a f(x) + b g(x)] = a f_c(s) + b g_c(s)$$

Property 2: Modulation Property

$$(i) f_s [f(x) \cos ax] = \frac{1}{2} [f_s(s+a) + f_s(s-a)]$$

$$(ii) f_c [f(x) \cos ax] = \frac{1}{2} [f_c(s+a) + f_c(s-a)]$$

$$(iii) f_s [f(x) \sin ax] = \frac{1}{2} [f_c(s-a) - f_c(s+a)]$$

$$(iv) f_c [f(x) \sin ax] = \frac{1}{2} [f_s(s+a) - f_s(s-a)]$$

$$\text{Property 3: } f_s [f'(x)] = -s f_c(s)$$

$$\text{Property 4: } f_c [f'(x)] = -\sqrt{\frac{2}{\pi}} f(0) + s f_s(s)$$

$$\text{Property 5: } f_s [x \cdot f(x)] = -\frac{d}{ds} f_c [f(x)] \quad (2)$$

$$\text{Property 6: } f_c [x \cdot f(x)] = \frac{d}{ds} f_s [f(x)] \quad (2)$$

$$\text{Property 7: } f_s [f(ax)] = \frac{1}{a} f_s\left(\frac{s}{a}\right)$$

$$f_c [f(ax)] = \frac{1}{a} f_c\left(\frac{s}{a}\right) \quad (2)$$

$$\text{Property 8: } (i) \int_0^{\infty} f_c [f(x)] \cdot g_c [g(x)] ds = \int_0^{\infty} f(x) \cdot g(x) dx \quad (2)$$

$$(ii) \int_0^{\infty} f_s [f(x)] \cdot g_s [g(x)] ds = \int_0^{\infty} f(x) g(x) dx \quad (2)$$

Parseval's Identity:

$$1) \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F(s)|^2 ds \quad 2) \int_{-\infty}^{\infty} |f(x)|^2 dx$$

$$3) \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |F_c(s)|^2 ds$$