UNIT5–MultipleIntegrals

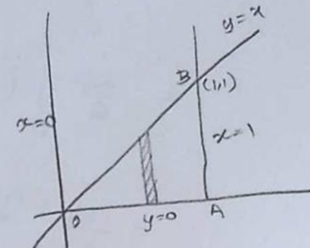
Change of order of integration.

① Change the order of integration for $\int_0^1\int_0^x f(x,y)\,dx\,dy$

Solution:-

Given integral is not in the correct order

let us rearrange it



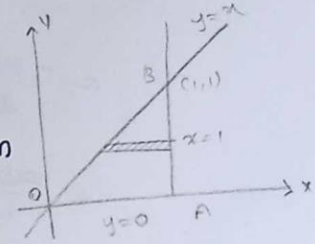

$$I = \int_0^1\int_0^x f(x,y)\,dy\,dx.$$

Given:- $y=0$ to $y=x$

$x=0$ to $x=1$

Inner limit is wrt $y$

$\therefore$ It is a vertical strip




Now to change the order of integration we have to draw a horizontal strip

$x$ limits : $x=y$ to $x=1$

$y$ limits : $y=0$ to $y=1$

$$\therefore I = \int_0^1\int_y^1 f(x,y)\,dx\,dy.$$

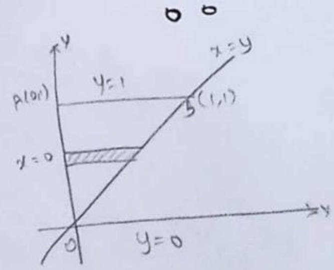
2. Change the order of integration in $\int_0^1\int_0^y f(x,y)\,dx\,dy$

$I = \int_0^1\int_0^y f(x,y)\,dx\,dy$.

Given limits:

$x$ limit : $x=0$ to $x=y$

$y$ limit : $y=0$ to $y=1$




Inner limit is wrt '$x$'

$\therefore$ Given limit is a horizontal strip

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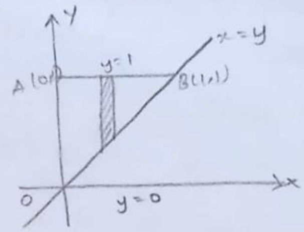
Now to change the order of integration we have to draw a vertical strip

$x$ limit : $x=0$ to $x=1$

$y$ limit : $y=x$ to $y=1$

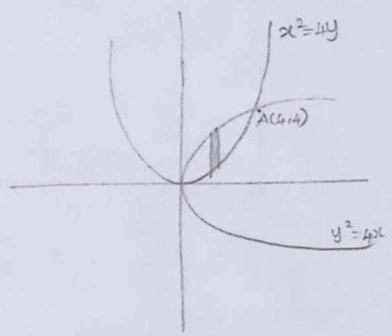
$$I = \int_0^1 \int_x^1 f(x,y)\, dy\, dx.$$




3. Evaluate by changing the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\,dx$

Solution :-

$$I = \int_0^4 \int_{x^2/4}^{2\sqrt{x}} dy\,dx$$




Given limits :

$x$ limits $x=0$ to $x=4$

$y$ limits $y=\frac{x^2}{4}$ to $y=2\sqrt{x}$

ie) $x^2=4y$ to $y^2=4x$

Inner limit is wrt '$y$'

$\therefore$ It is a vertical strip

To find point A:

Solving $\underbrace{x^2=4y}_{①}$ and $\underbrace{y^2=4x}_{②}$

$x^2=4y$

Squaring, $x^4=16y^2$

$x^4=16(4x)$ using ②

$x^4=64x$

$x^3=64$

$\boxed{x=4}$

A(4,4).

Sub $x$ in ②

$y^2=4x$

$y^2=4(4)$

$y^2=16$

$\boxed{y=4}$

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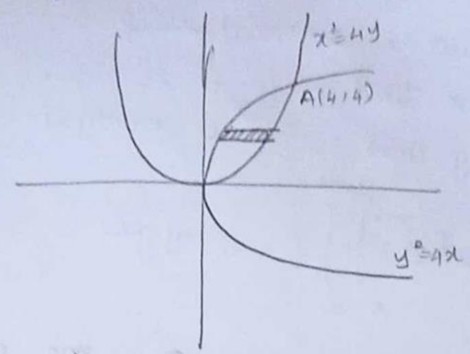
Now to change the order of integration draw a horizontal strip




$x$ limits :

$x = \frac{y^2}{4}$ to $x = 2\sqrt{y}$

$y=0$ to $y=4$

$$\therefore I = \int_0^4 \int_{y^2/4}^{2\sqrt{y}} dx\, dy$$

$$= \int_0^4 \left[x\right]_{y^2/4}^{2\sqrt{y}} dy \qquad = \int_0^4 \left[2\sqrt{y} - y^2/4\right] dy$$

$$= \left[\frac{2y^{3/2}}{3/2} - \frac{1}{4}\frac{y^3}{3}\right]_0^4$$

$$= \left[\frac{2(4)^{3/2}}{3/2} - \frac{1}{4}\frac{(4)^3}{3}\right]$$

$$= \left[\frac{4}{3}(8) - \frac{16}{3}\right] = \frac{32-16}{3}$$

$$\underline{I} = \frac{16}{3}$$

4. Change the order of integration in $\int_0^a \int_{x^2/a}^{2a-x} xy\, dy\, dx$ and then evaluate. $\frac{3}{8}a^4$

The region of integration R is bounded by the curve $y = \frac{x^2}{a}$ i.e. the parabola $x^2 = ay$, the line $y = 2a - x$, ie $x + y = 2a$ and the lines $x=0$ and $x=a$.