Gauss Jordan method:

This method is a modified form of Gaussian elimination method. In this method, the co efficient matrix is reduced to a diagonal matrix or unit matrix, rather than a triangular matrix. Here we get the solution without using the back substitution method.

I. Solve by Gauss-elimination method

$3x+y-z = 3$

$2x-8y+z = -5$

$x-2y+9z = 8$

Soln.

Now, $[A, B] \sim \begin{bmatrix} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -2 & 9 & 8 \\ 2 & -8 & 1 & -5 \\ 3 & 1 & -1 & 3 \end{bmatrix}$ $R_1 \leftrightarrow R_3$

$\sim \begin{bmatrix} 1 & -2 & 9 & 8 \\ 0 & -4 & -17 & -21 \\ 0 & 7 & -28 & -21 \end{bmatrix}$ $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$

$\sim \begin{bmatrix} 1 & -2 & 9 & 8 \\ 0 & -4 & -17 & -21 \\ 0 & 0 & \frac{-231}{7} & \frac{-231}{7} \end{bmatrix}$ $R_3 \rightarrow \frac{4}{7}R_3 + R_2$

Use back substitution, $\frac{-231}{7} z = -\frac{231}{7}$

$\boxed{z = 1}$

$-4y - 17z = -21$

$-4y - 17 = -21$

$-4y = -21+17 = -4$

$\boxed{y = 1}$

and $x - 2y + 9z = 8$

$x - 2 + 9 = 8$

$x = 8 - 7 = 1$

$\boxed{x = 1}$

$\therefore$ The solution is $x = 1,\ y = 1,\ z = 1$

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2). Solve the equations $2x + y + 4z = 12$, $8x - 3y + 2z = 20$, $4x + 11y - z = 33$ by

i). Gauss Elimination method ii). Gauss Jordan Method

Soln.

i). Gauss Elimination method:

$$[A, B] \sim \begin{bmatrix} 2 & 1 & 4 & 12 \\ 8 & -3 & 2 & 20 \\ 4 & 11 & -1 & 33 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & -28 \\ 0 & 9 & -9 & 9 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & -28 \\ 0 & 0 & -27 & -27 \end{bmatrix} \quad R_3 \to R_3 + \frac{9}{7} R_2$$

Use back substitution method

$-27z = -27 \Rightarrow z = 1$

and $-7y - 14z = -28$

$-7y - 14 = -28 \Rightarrow -7y = -28 + 14$

$-7y = -14$

$y = 2$

$2x + y + 4z = 12$

$2x + 2 + 4 = 12 \Rightarrow 2x = 12 - 6 = 6$

$x = 3$

$\therefore$ $x = 3$

$y = 2$

$z = 1$

ii). Gauss Jordan method :

$$[A, B] \sim \begin{bmatrix} 2 & 1 & 4 & 12 \\ 0 & -7 & -14 & -28 \\ 0 & 0 & -27 & -27 \end{bmatrix}$$

$$\sim \begin{bmatrix} 14 & 0 & 14 & 56 \\ 0 & -7 & -14 & -28 \\ 0 & 0 & -27 & -27 \end{bmatrix} \qquad R_1 \to 7R_1 + R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 4 \\ 0 & -7 & -14 & -28 \\ 0 & 0 & +1 & +1 \end{bmatrix} \qquad \begin{matrix} R_1 \to R_1/14 \\ R_3 \to R_3/-27 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & -7 & 0 & -14 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 + 14R_3 \end{matrix}$$

$\therefore \; z = 1$

$-7y + 0z = -14 \Rightarrow -7y = -14$

$y = 2$

$x = 3$

$\therefore$ The solution is, $x = 3, \; y = 2$ and $z = 1$.

5). Solve the system of equations $x + 2y + z = 8$, $2x + 3y + 4z = 20$, $4x + y + 2z = 12$, by Gauss Jordan method.

Soln.

$$[A, B] \sim \begin{bmatrix} 1 & 2 & 1 & 8 \\ 2 & 3 & 4 & 20 \\ 4 & 1 & 2 & 12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & 2 & 4 \\ 0 & -7 & -2 & -20 \end{bmatrix} \qquad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & -16 & -48 \end{bmatrix} \qquad R_3 \to R_3 - 7R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 5 & 16 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & -16 & -48 \end{bmatrix} \quad R_1 \to R_1 + 2R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 5 & 16 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad R_3 \to \frac{R_3}{-16}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \begin{array}{l} R_1 \to R_1 - 5R_3 \\ R_2 \to R_2 - 2R_3 \end{array}$$

$\therefore$ The solution is, $z = 3$

$-y = -2 \Rightarrow y = 2$

$x = 1$

$x = 1,\ y = 2,\ z = 3.$

4). Solve the following system by Gauss elimination method.

$5x_1 + x_2 + x_3 + x_4 = 4$

$x_1 + 7x_2 + x_3 + x_4 = 12$

$x_1 + x_2 + 6x_3 + x_4 = -5$

$x_1 + x_2 + x_3 + 4x_4 = -6$

Soln.

$$[A, B] \sim \begin{bmatrix} 5 & 1 & 1 & 1 & 4 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 1 & 1 & 1 & 4 & -6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 1 & 7 & 1 & 1 & 12 \\ 1 & 1 & 6 & 1 & -5 \\ 5 & 1 & 1 & 1 & 4 \end{bmatrix} \quad R_1 \leftrightarrow R_4$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & -4 & -4 & -19 & 34 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & -4 & -21 & 46 \end{bmatrix} \; R_4 \to R_4 + \frac{2}{3} R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 4 & -6 \\ 0 & 6 & 0 & -3 & 18 \\ 0 & 0 & 5 & -3 & 1 \\ 0 & 0 & 0 & \frac{-117}{5} & \frac{234}{5} \end{bmatrix} \; R_4 \to R_4 + \frac{4}{5} R_3$$

using back substitution method,

$$\frac{-117}{5} x_4 = \frac{234}{5} \Rightarrow x_4 = -2$$

$$5x_3 - 3x_4 = 1$$

$$\Rightarrow 5x_3 - 3(-2) = 1 \Rightarrow 5x_3 = 1 - 6 = -5$$

$$x_3 = -1$$

$$6x_2 + 0x_3 - 3x_4 = 18 \Rightarrow 6x_2 = 18 - 6 = 12$$

$$x_2 = 2$$

$$x_1 + x_2 + x_3 + 4x_4 = -6$$

$$x_1 + 2 - 1 - 8 = -6 \Rightarrow x_1 = -6 + 7 = 1$$

$\therefore$ The solution is $x_1 = 1$, $x_2 = 2$, $x_3 = -1$, $x_4 = -2$.

(3) $x + 3y + 3z = 16$; $x + 4y + 3z = 18$; $x + 3y + 4z = 19$

1, 2, 3

(1)

$x_1 - x_2 + x_3 = 1$

$-3x_1 + 2x_2 - 3x_3 = -6$

$2x_1 - 5x_2 + 4x_3 = 5$

$-2, 3, 6$

$3x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 - x_3 = -5$

$x_1 + x_2 - 6x_3 = -12$

$1, -1, 2$