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UNIT 3

PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGER

CONTENTS

- Condensation with shear edge on bank of tubes
- Boiling – pool and flow boiling
- Heat Exchanger – NTU approach and design procedure
- Compact heat exchangers

CONDENSATION WITH SHEAR EDGE ON BANK OF TUBES

- Condensation within tube banks is subject to the combined effects of *vapor shear* and *falling condensate* from upper tubes. The latter is called *condensate inundation*. Tube orientation may be vertical or horizontal.
- However, horizontal orientation is more common when a pure vapor is to be condensed. the factors that affect condensation heat transfer of pure vapors in a horizontal tube bank.

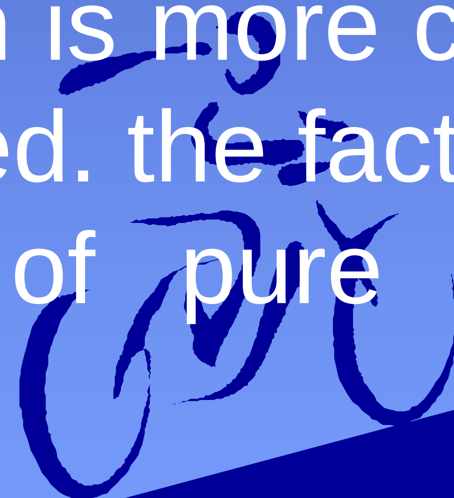


Table 1. Factors that affect condensation of pure vapors in horizontal tube banks

Item	Factor	
Tube	Geometry	Plain Finned
	Layout	In-line
		Staggered
	Number of tube rows	Horizontal
		Vertical
	Pitch-to-diameter ratio	
Vapor	Velocity	
	Flow direction	
Condensate	Inundation rate	
	Mode of inundation	

- The mode of condensate inundation depends on the inundation rate and vapor velocity.
- At low vapor velocities, condensate drains in discrete drops (Figure 1a), then in condensate columns (Figure 1b), and then in a condensate sheet (Figure 1c) as the inundation rate increases. In a closely packed staggered tube bank, side drainage may occur depending on the condition (Figure 1d)

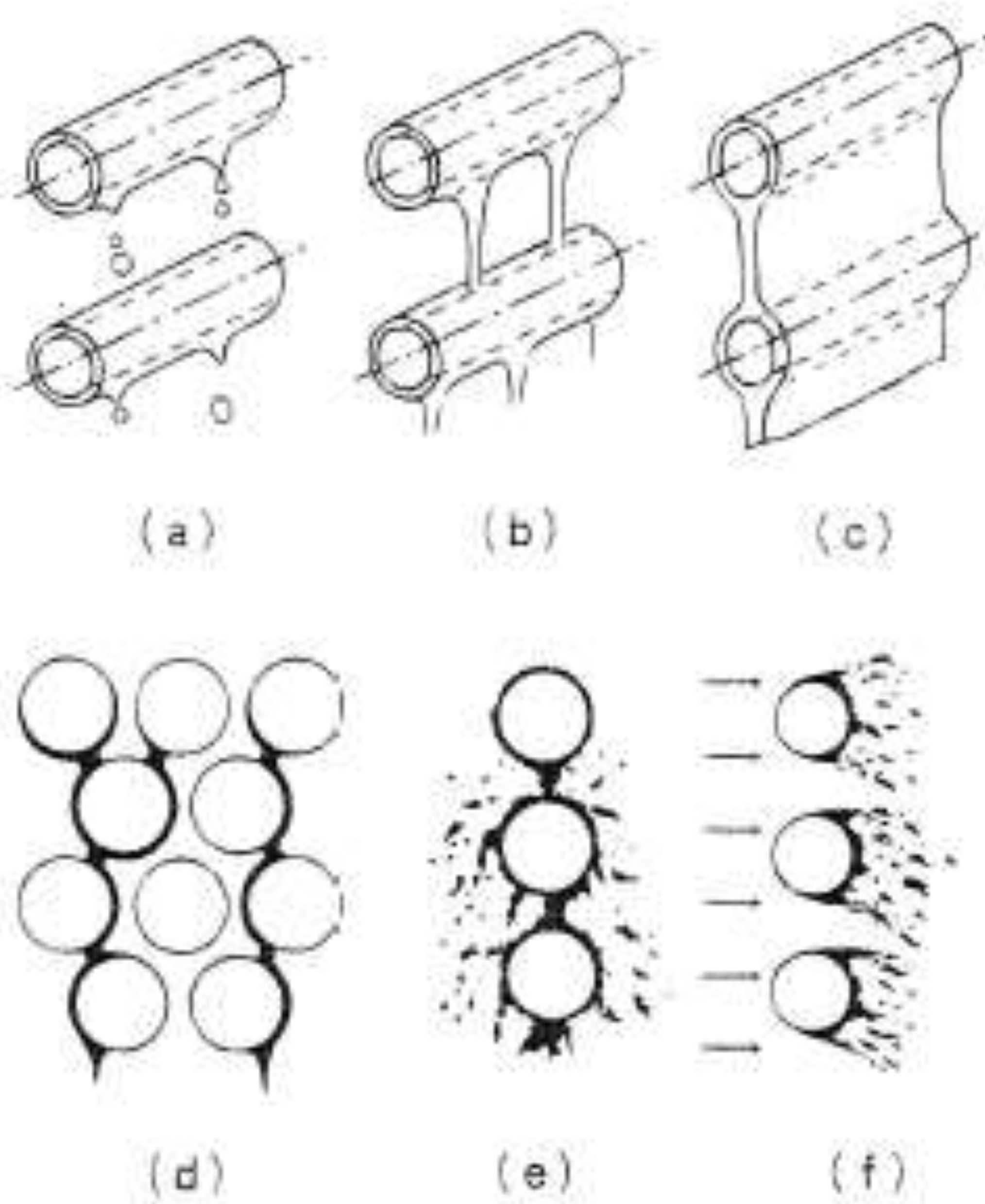


Figure 1. Modes of condensate inundation in horizontal tube banks.

- The condensate impinging on the lower tube causes splashing, ripples and turbulence on the condensate film (Figure 1e). At high vapor velocities, the condensate leaving the tube is disintegrated into small drops and impinges on the other tubes (Figure 1f). A wide variety of condensate drainage mode shown in Figure 1 results in a wide breadth of experimental data regarding the effect of condensate inundation.

FILM CONDENSATION ON HORIZONTAL TUBES

- Nusselt's analysis for laminar filmwise condensation on horizontal tubes leads to the following relations:

$$\bar{h} = 0.0725\left[\frac{\rho_l\,(\rho_l - \rho_v)k^3\, g\, h_{fg}}{\mu_l\,(t_{sat} - t_s)\,D}\right]^{1/4}$$

For single horizontal tube

$$\bar{h} = 0.0725\left[\frac{\rho_l\,(\rho_l - \rho_v)k^3\, g\, h_{fg}}{N\mu_l\,(t_{sat} - t_s)\,D}\right]^{1/4}$$

For horizontal tube bank with N tubes placed directly over one another in the vertical direction

Where D is outer diameter of the tube

FILM CONDENSATION INSIDE HORIZONTAL TUBES

- The phenomena inside tubes are very complicated because the overall flow rate of vapor strongly affects the heat transfer rate and also the rate of condensation on the walls

$$\bar{h} = 0.555\left[\frac{\rho_l(\rho_l-\rho_v)k^3 g h'_{fg}}{\mu_l D(t_{sat}-t_s)}\right]^{1/4}$$

where,

$$h'_{fg} = h_{fg} + \frac{3}{8}c_{pl}(t_{sat}-t_s)$$

- The h'_{fg} equation is restricted to low vapour Reynolds number such that

$$Re_v = \left(\frac{\rho_v \, u_{m,v} \, D}{\mu_v} \right) < 3500$$

Where Re_v is evaluated at inlet condition to the tubes

PROBLEM

1. *A steam condenser consisting of a square array of 625 horizontal tubes, each 6mm in diameter, is installed at the exhaust hood of a steam turbine. The tubes are exposed to saturated steam at a pressure of 15 kPa. If the tube surface temperature is maintained at 25°C, calculate:*

   (i) The heat transfer coefficient, and

   (ii) The rate at which steam is condensed per unit length of the tubes.

   Assume film condensation on the tubes and absence of non-condensable gases.

Solution. *Given*: $D = 6$ mm $= 0.006$ m, $t_s = 25°C$.

Corresponding to 15 kPa pressure, the properties of vapour (from the table) are:

$$t_{sat} = 54°C,\ \rho_v = 0.098\ \text{kg/m}^3,\ h_{fg} = 2373\ \text{kJ/kg}.$$

The properties of saturated water at film temperature $t_f = \frac{54 + 25}{2} = 39.5°C$ are:

$$\rho_l = 992\ \text{kg/m}^3;\ \mu = 663 \times 10^{-6}\ \text{Ns/m}^2;\ k = 0.631\ \text{W/m°C}$$

Since the tubes are arranged in square array, therefore, the number of horizontal tubes in vertical column is : $N = \sqrt{625} = 25$

(*i*) The heat transfer coefficient, $\bar{h}$:

The average heat transfer coefficient for steam condensing on bank of horizontal tubes is given by

$$\bar{h} = 0.725 \left[\frac{\rho_l (\rho_l - \rho_v) k^3 g h_{fg}}{N \mu_l (t_{sat} - t_s) D} \right]^{1/4} \quad \text{...[Eqn. (9.41)]}$$

or,

$$\bar{h} = 0.725 \left[\frac{992 (992 - 0.098) \times (0.631)^3 \times 9.81 \times (2373 \times 10^3)}{25 \times 663 \times 10^{-6} (54 - 25) \times 0.006} \right]^{1/4}$$

$$= 0.725 \left(\frac{5.7548 \times 10^{12}}{2.884 \times 10^{-3}} \right)^{1/4} = \mathbf{4845.6\ W/m^2 °C\ (Ans.)}$$

(ii) The rate at which steam is condensed per unit length, *m* :

The rate of condensation for the single tube of the array per metre length is

$$m_1 = \frac{Q}{h_{fg}} = \frac{\bar{h}\,\pi\,D\,(t_{sat} - t_s)}{h_{fg}}$$

$$= \frac{4845.6 \times \pi \times 0.006\,(54 - 25)}{2373 \times 10^3} = 1.116 \times 10^{-3} \text{ kg/s.m}$$

The rate of condensation for the complete array is

$$m = 625 \times m_1 = 625 \times 1.116 \times 10^{-3} = \mathbf{0.6975\ kg/s.m\ (Ans.)}$$

COMPACT HEAT EXCHANGERS(CHE)

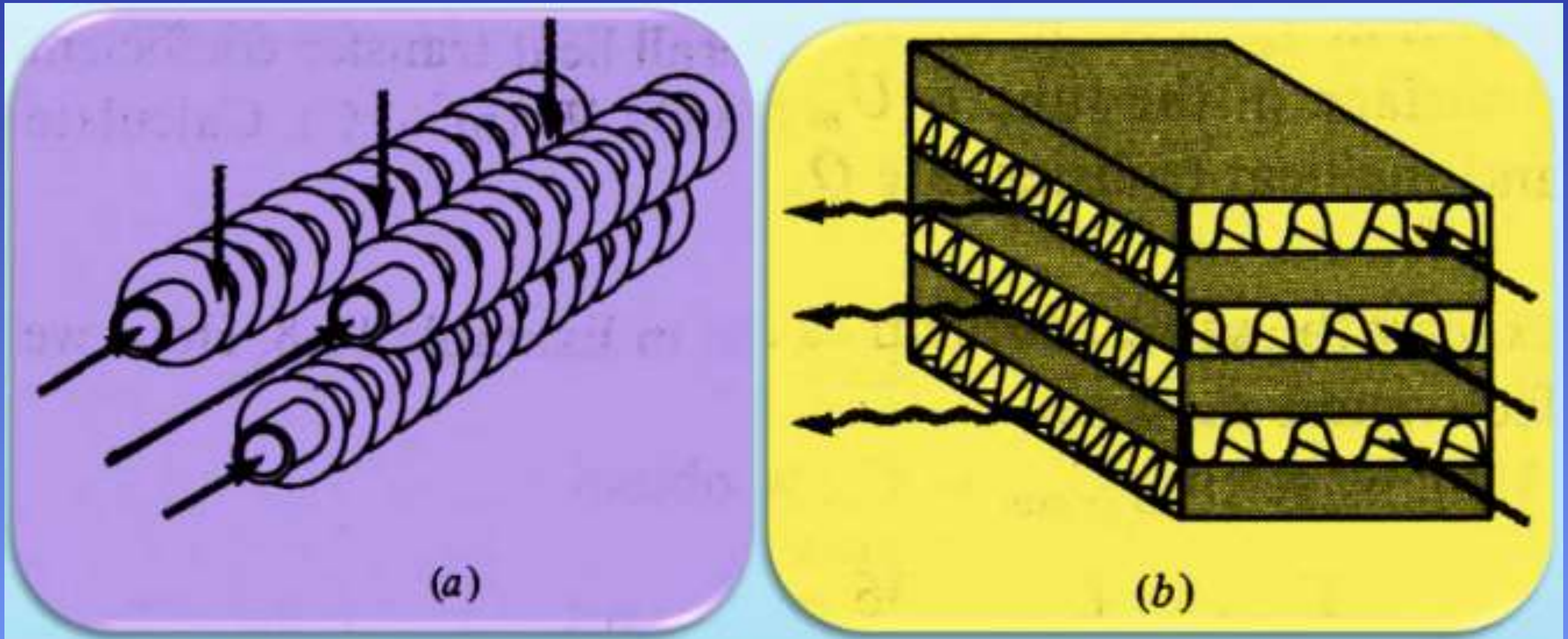
1) Compact heat exchangers are used when a large heat transfer surface area per unit volume is desired. This means that the smallness of weight and size is important.

2) Compact heat exchangers are used when at least one of the fluids is a gas. Hence the heat transfer coefficient is low.

3) A heat exchanger having a surface area density greater than about 700 m^2/m^3 is quite referred to as a compact heat exchanger.

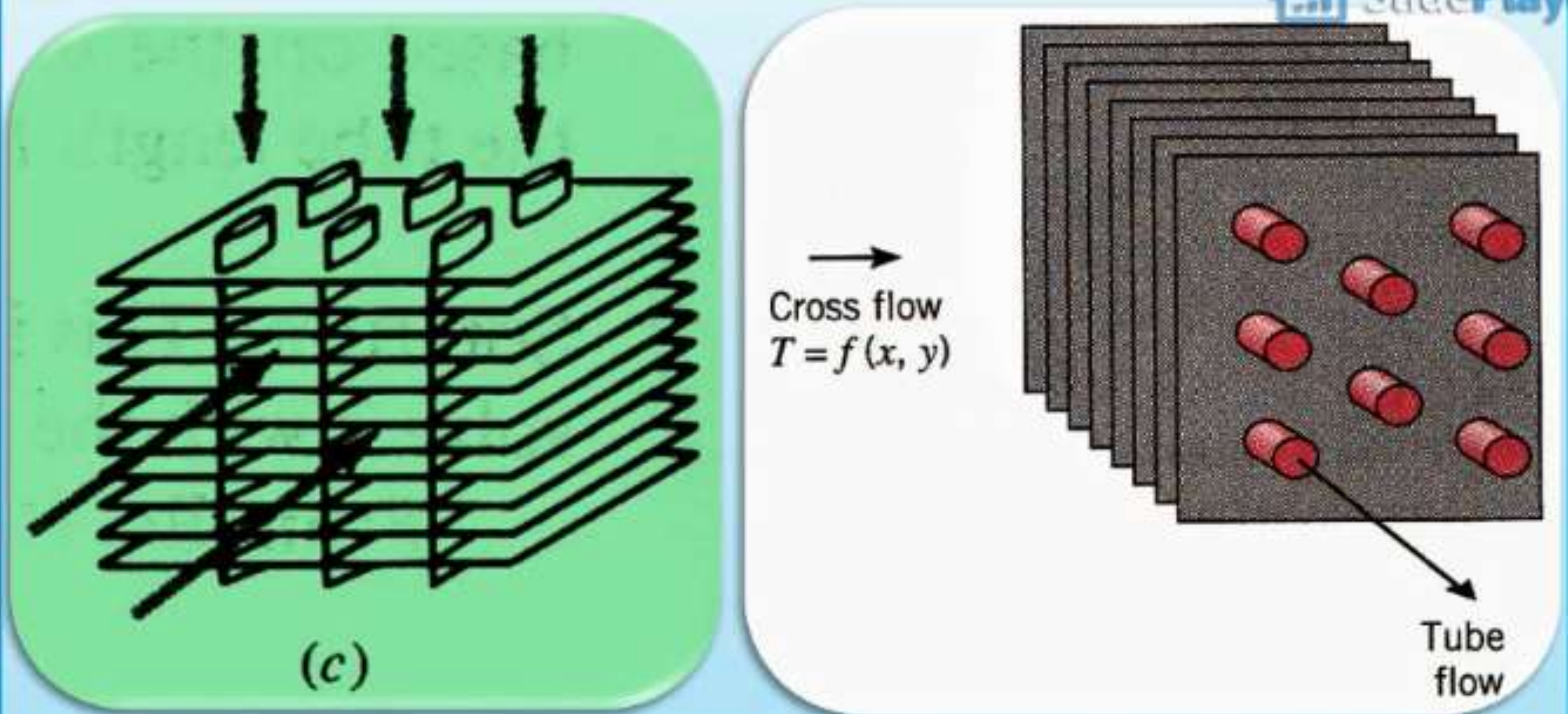
4) Compact heat exchangers are available in a wide variety of configurations. Many different tubular and plate configurations have been considered, where differences are due to fin design and arrangements.

HEAT TRANSFER MATRIX - CHE



a) Fin-tube (circular tubes, circular fins)

b) Plate-fin (single pass)



c) Fin-tube (flat tubes, continuous plate fins)

d) Fin-tube (circular tubes, continuous plate fins)

Heat transfer results are correlated in terms of Colburn j factor and Reynold's number where,

Colburn j factor, $j = St\ Pr^{2/3}$

St = Stanton number

$$St = \frac{Nu}{Re\ Pr} = \frac{\left(\frac{hD_h}{k}\right)}{\left(\frac{\rho V_{max} D_h}{\mu}\right) \times \left(\frac{\mu c_p}{k}\right)} = \frac{h}{\rho V_{max} c_p} = \frac{h}{G c_p}$$

$$Re = \frac{\rho V_{max} D_h}{\mu} = \frac{G D_h}{\mu}$$

where,

Where,

$$G = mass\ velocity = \rho V_{max} = \frac{\rho V_{max} A_{min}}{A_{min}} = \frac{m}{A_{min}}$$

A_{min} = minimum free-flow cross-sectional area regardless of where this minimum occurs.

$$D_h = hydraulic\ diameter = \frac{4 \times flow\ area}{wetted\ perimeter} = \frac{4A_{min}}{A/L} = \frac{4A_{min}L}{A}$$

A = total heat transfer area.

L = flow length of the heat exchanger matrix.

$$\therefore j = St\ Pr^{2/3} = \frac{h}{Gc_p} Pr^{2/3}$$

BOILING

- Occurs at the solid–liquid interface when a liquid is brought into contact with a surface maintained at a temperature T_s sufficiently above the saturation temperature T_{sat} of the liquid
- By Newton's law of cooling,

$$q_{boiling} = h(T_s - T_{sat}) = h\Delta T_{excess} \ (W/m^2)$$

where, $\Delta T_{excess} = (T_s - T_{sat})$-------> Excess temperature

TYPES OF BOILING

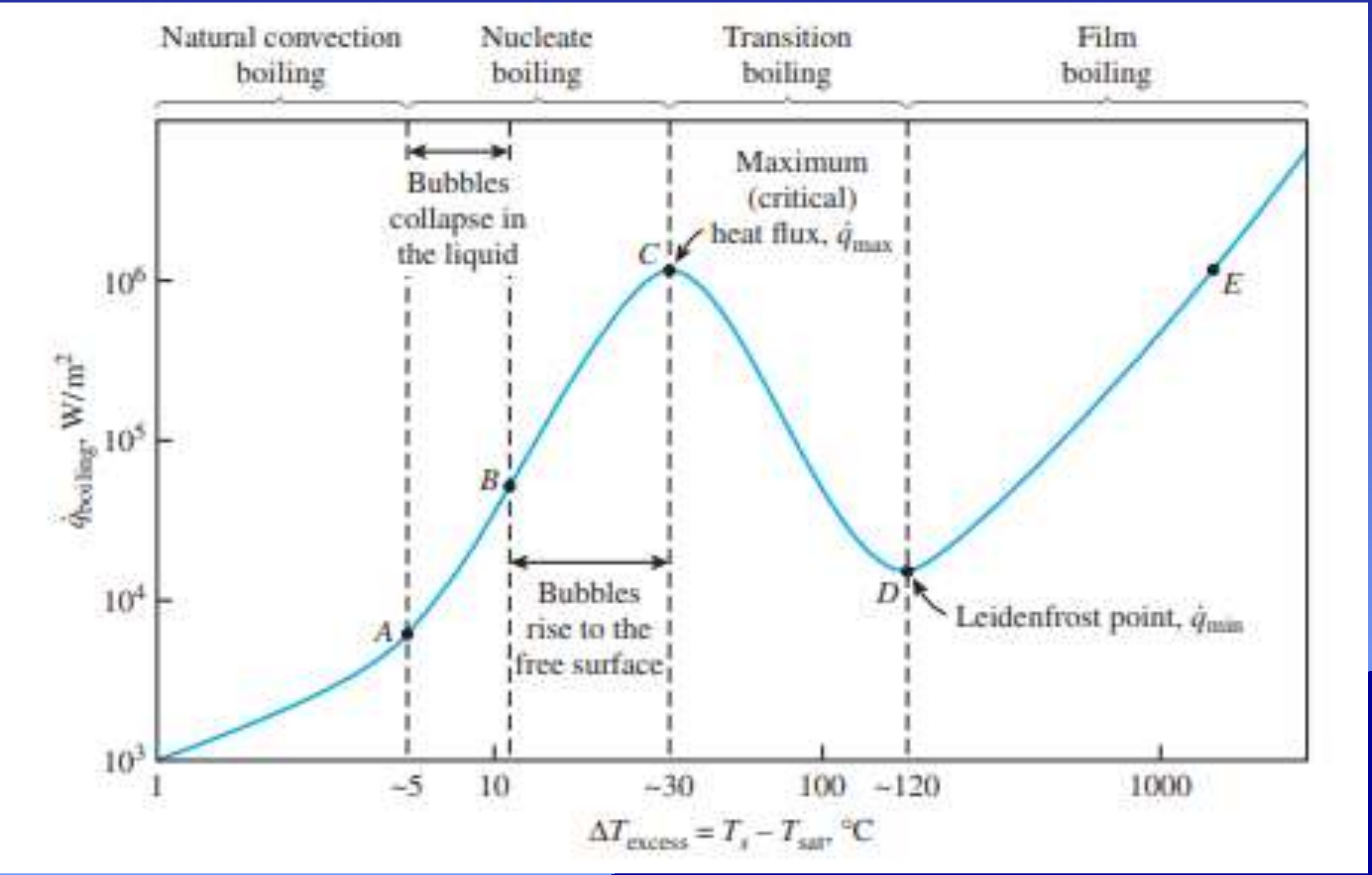
(1) Pool boiling – absence of bulk fluid flow

(2) Flow boiling (or forced convection boiling) – presence of bulk fluid flow

POOL BOILING

- The fluid is not forced to flow by a mover such as a pump, and any motion of the fluid is due to natural convection currents and the motion of the bubbles under the influence of buoyancy.

BOILING REGIMES AND THE BOILING CURVE



(I)Natural Convection Boiling (to Point A on the Boiling Curve)

The fluid motion in this mode of boiling is governed by natural convection currents, and heat transfer from the heating surface to the fluid is by natural convection

(II)Nucleate Boiling (between Points A and C)

Nucleate boiling is the most desirable boiling regime in practice because high heat transfer rates can be achieved in this regime with relatively small values of ΔT_{excess}, typically under 30^0C for water

(III)Transition Boiling (between Points C and D)

In the transition boiling regime, both nucleate and film boiling partially occur. Nucleate boiling at point C is completely replaced by film boiling at point D. Operation in the transition boiling regime, which is also called the unstable film boiling regime, is avoided in practice.

(IV)Film Boiling (beyond Point D)

In this region the heater surface is completely covered by a continuous stable vapor film. Point D, where the heat flux reaches a minimum, is called the Leidenfrost point

HEAT TRANSFER CORRELATIONS IN POOL BOILING

(i) Nucleate boiling regime ($5^0C \le \Delta T_{excess} \le 30^0C$)

$$\dot{q}_{nucleate} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{pl}(T_s - T_{sat})}{C_{sf} h_{fg} \Pr_l^n} \right]^3$$

where

$\dot{q}_{nucleate}$ = nucleate boiling heat flux, W/m^2
μ_l = viscosity of the liquid, kg/m·s
h_{fg} = enthalpy of vaporization, J/kg
g = gravitational acceleration, m/s^2
ρ_l = density of the liquid, kg/m^3
ρ_v = density of the vapor, kg/m^3
σ = surface tension of liquid–vapor interface, N/m
c_{pl} = specific heat of the liquid, J/kg·°C
T_s = surface temperature of the heater, °C
T_{sat} = saturation temperature of the fluid, °C
C_{sf} = experimental constant that depends on surface–fluid combination
$\Pr_l$ = Prandtl number of the liquid
n = experimental constant that depends on the fluid

(ii)Peak Heat Flux

The maximum (or critical) heat flux in nucleate pool boiling was determined by

$$\dot{q}_{\max} = C_{cr}\, h_{fg}[\sigma g \rho_v^2 (\rho_l - \rho_v)]^{1/4}$$

where C_{cr} is a constant whose value depends on the heater geometry

(iii)Minimum Heat Flux

The minimum heat flux for a large horizontal plate,

$$\dot{q}_{\min} = 0.09 \rho_v\, h_{fg} \left[\frac{\sigma g(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

(iv)Film Boiling

The heat flux for film boiling on a horizontal cylinder or sphere of diameter D is given by,

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{g k_v^3 \rho_v (\rho_l - \rho_v)[h_{fg} + 0.4 c_{pv}(T_s - T_{\text{sat}})]}{\mu_v D(T_s - T_{\text{sat}})}\right]^{1/4}(T_s - T_{\text{sat}})$$

where k_v is the thermal conductivity of the vapor in W/mK and

$$C_{\text{film}} = \begin{cases} 0.62 \text{ for horizontal cylinders} \\ 0.67 \text{ for spheres} \end{cases}$$

Treating the vapor film as a transparent medium sandwiched between two large parallel plates and approximating the liquid as a blackbody, radiation heat transfer can be determined from

$$\dot{q}_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sat}^4\right)$$

For $q_{rad} < q_{film}$

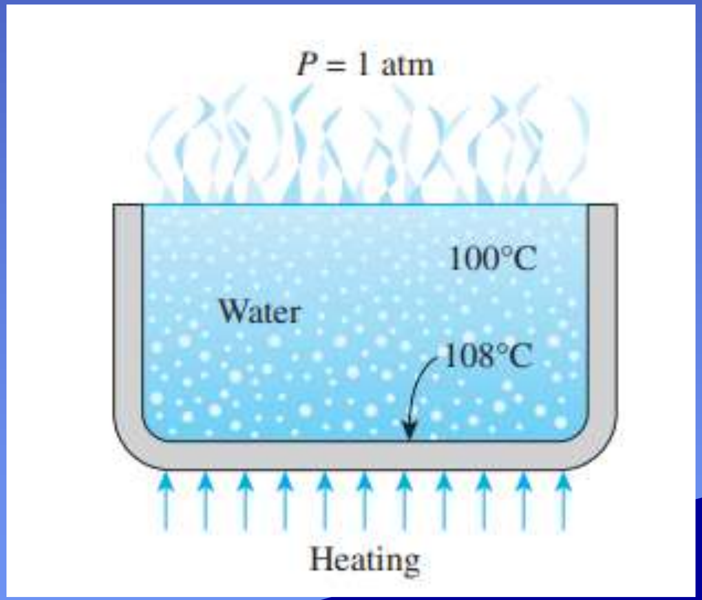
$$\dot{q}_{total} = \dot{q}_{film} + \frac{3}{4}\dot{q}_{rad}$$

PROBLEMS

1. Water is to be boiled at atmospheric pressure in a mechanically polished stainless steel pan placed on top of a heating unit, as shown in Fig. 10–15. The inner surface of the bottom of the pan is maintained at 108°C. If the diameter of the bottom of the pan is 30 cm, determine (*a*) the rate of heat transfer to the water and (*b*) the rate of evaporation of water.

The properties of water at the saturation temperature of 100^0C are σ = 0.0589 N/m and

$\rho_l = 957.9\ kg/m^3$	$h_{fg} = 2257 \times 10^3\ J/kg$
$\rho_v = 0.6\ kg/m^3$	$\mu_l = 0.282 \times 10^{-3}\ kg/m \cdot s$
$Pr_l = 1.75$	$c_{pl} = 4217\ J/kg \cdot K$



for the boiling of water on a mechanically polished stainless steel surface

$C_{sf} = 0.0130$ and $n = 1.0$

(a) The excess temperature in this case $\Delta T_{excess} = (T_s - T_{sat}) = 108 - 100 = 8^0C$ which is relatively low (less than 30^0C).

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \Pr_l^n}\right]^3$$

$$= (0.282 \times 10^{-3})(2257 \times 10^3)\left[\frac{9.81 \times (957.9 - 0.6)}{0.0589}\right]^{1/2}$$

$$\times \left(\frac{4217(108 - 100)}{0.0130(2257 \times 10^3)1.75}\right)^3$$

$$= 7.21 \times 10^4 \text{ W/m}^2$$

The surface area of the bottom of the pan is

$$A = \pi D^2/4 = \pi(0.3 \text{ m})^2/4 = 0.07069 \text{ m}^2$$

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A\dot{q}_{\text{nucleate}} = (0.07069 \text{ m}^2)(7.21 \times 10^4 \text{ W/m}^2) = \mathbf{5097 \text{ W}}$$

(b) The rate of evaporation of water is determined from

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{5097 \text{ J/s}}{2257 \times 10^3 \text{ J/kg}} = \mathbf{2.26 \times 10^{-3} \text{ kg/s}}$$

That is, water in the pan will boil at a rate of more than 2 grams per second.

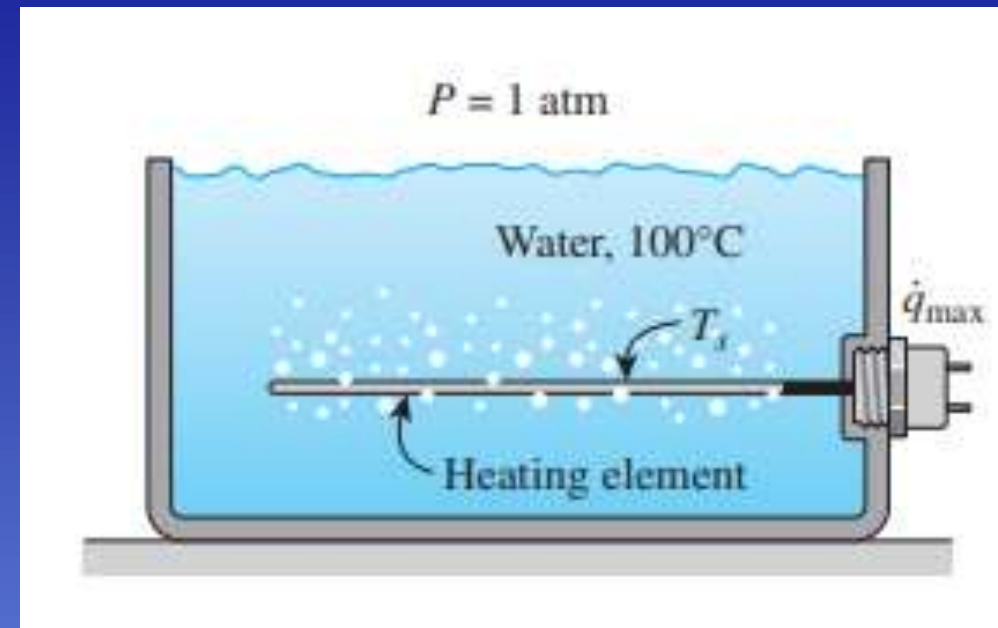
2. Water in a tank is to be boiled at sea level by a 1-cm-diameter nickel plated steel heating element equipped with electrical resistance wires inside, as shown in Fig. 10–16. Determine the maximum heat flux that can be attained in the nucleate boiling regime and the surface temperature of the heater in that case.

The properties of water at the saturation temperature of 100^0C are σ = 0.0589 N/m and

$\rho_l = 957.9 \text{ kg/m}^3$ $\quad h_{fg} = 2257 \times 10^3 \text{ J/kg}$

$\rho_v = 0.6 \text{ kg/m}^3$ $\quad \mu_l = 0.282 \times 10^{-3} \text{ kg/m·s}$

$\text{Pr}_l = 1.75$ $\quad c_{pl} = 4217 \text{ J/kg·K}$



For the boiling of water on a nickel plated surface

$C_{sf} = 0.0060$ and $n = 1.0$

The heating element in this case can be considered to be a short cylinder whose characteristic dimension is its radius. That is, L = r = 0.005 m. The dimensionless parameter L* and the constant C_{cr} are determined from

$$L^* = L\left(\frac{g(\rho_l - \rho_v)}{\sigma}\right)^{1/2} = (0.005)\left(\frac{(9.81)(957.9 - 0.6)}{0.0589}\right)^{1/2} = 2.00 > 1.2$$

which corresponds to $C_{cr} = 0.12$.

Then the maximum or critical heat flux is determined from Eq. 10–3 to be

$$\begin{aligned}\dot{q}_{max} &= C_{cr}\, h_{fg}\, [\sigma g \rho_v^2 (\rho_l - \rho_v)]^{1/4} \\ &= 0.12(2257 \times 10^3)[0.0589 \times 9.81 \times (0.6)^2(957.9 - 0.6)]^{1/4} \\ &= \mathbf{1.017 \times 10^6\ W/m^2}\end{aligned}$$

The nucleate boiling heat flux for a specified surface temperature, can also be used to determine the surface temperature when the heat flux is given

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \Pr_l^n} \right]^3$$

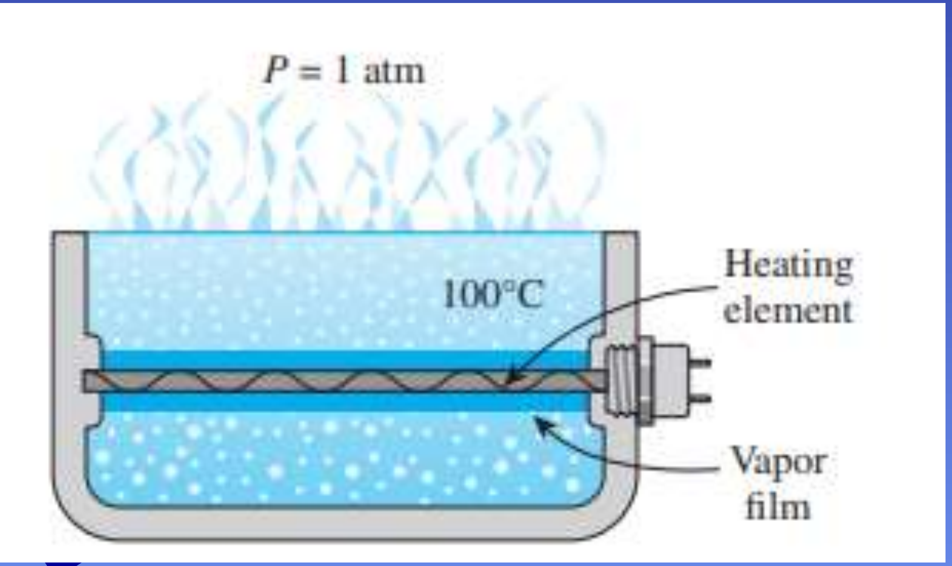
$$1.017 \times 10^6 = (0.282 \times 10^{-3})(2257 \times 10^3) \left[\frac{9.81(957.9 - 0.6)}{0.0589} \right]^{1/2}$$

$$\times \left[\frac{4217(T_s - 100)}{0.0130(2257 \times 10^3)\, 1.75} \right]^3$$

$$T_s = 119°\text{C}$$

3. Water is boiled at atmospheric pressure by a horizontal polished copper heating element of diameter $D = 5$ mm and emissivity $\varepsilon = 0.05$ immersed in water, as shown in Fig. 10–17. If the surface temperature of the heating wire is 350°C, determine the rate of heat transfer from the wire to the water per unit length of the wire.

The properties of water at the saturation temperature of 100^0C are $h_{fg} = 2257 \times 10^3$ J/kg and $\rho_f = 957.9$ kg/m^3. The properties of vapor at the film temperature of $T_f = (T_{sat} + T_s)/2 = (100 + 350)/2 = 225^0$C are



$\rho_v = 0.444$ kg/m^3 $\qquad c_{pv} = 1951$ J/kg·K

$\mu_v = 1.75 \times 10^{-5}$ kg/m·s $\qquad k_v = 0.0358$ W/m·K

The excess temperature in this case is $\Delta T_{excess} = (T_s - T_{sat})$ = 350 – 100 = 250^{0}C which is much larger than 30^{0}C for water. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\dot{q}_{film} = 0.62\left[\frac{gk_v^3\rho_v(\rho_l - \rho_v)[h_{fg} + 0.4c_{pv}(T_s - T_{sat})]}{\mu_v D(T_s - T_{sat})}\right]^{1/4}(T_s - T_{sat})$$

$$= 0.62\left[\frac{9.81(0.0358)^3(0.444)(957.9 - 0.441) \times [(2257 \times 10^3) + 0.4 \times 1951(250)]}{(1.75 \times 10^{-5})(5 \times 10^{-3})(250)}\right]^{1/4} \times 250$$

$$= 5.93 \times 10^4 \text{ W/m}^2$$

The radiation heat flux is determined from

$$\dot{q}_{rad} = \varepsilon\sigma\,(T_s^4 - T_{sat}^4)$$

$$= (0.05)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(350 + 273\ \text{K})^4 - (100 + 273\ \text{K})^4]$$

$$= 372\ \text{W/m}^2$$

Note that heat transfer by radiation is negligible in this case because of the low emissivity of the surface and the relatively low surface temperature of the heating element. Then the total heat flux becomes

$$\dot{q}_{total} = \dot{q}_{film} + \frac{3}{4}\dot{q}_{rad} = 5.93 \times 10^4 + \frac{3}{4} \times 372 = 5.96 \times 10^4\ \text{W/m}^2$$

Finally, the rate of heat transfer from the heating element to the water is determined by multiplying the heat flux by the heat transfer surface area,

$$
\begin{aligned}
\dot{Q}_{\text{total}} &= A\dot{q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}} \\
&= (\pi \times 0.005\ \text{m} \times 1\ \text{m})(5.96 \times 10^4\ \text{W/m}^2) \\
&= 936\ \text{W}
\end{aligned}
$$

FLOW BOILING

- The fluid is forced to move by an external source such as a pump as it undergoes a phase-change process.
- Types: External or internal flow boiling
- **External flow boiling** over a plate or cylinder is similar to pool boiling, but the added motion increases both the nucleate boiling heat flux and the maximum heat flux considerably.
- **Internal flow boiling** commonly referred to as two-phase flow, is much more complicated in nature because there is no free surface for the vapor to escape, and thus both the liquid and the vapor are forced to flow together.

HEAT EXCHANGER

Facilitate the exchange of heat between two fluids that are at different temperatures while keeping them from mixing with each other.

TYPES

1. Nature of heat exchange process
   - (i) Direct contact heat exchangers
   - (ii) Indirect contact heat exchangers
     - (a) Regenerators
     - (b) Recuperators

2. Relative direction of fluid motion
    - (i) Parallel flow heat exchanger
    - (ii) Counter flow heat exchanger
    - (iii) Cross flow heat exchanger

3. Design and constructional features
    - (i) Concentric tubes
    - (ii) Shell and tube
    - (iii) Multiple shell and tube passes
    - (iv) Compact heat exchanger

4. Physical state of fluids
    - (i) Condensers
    - (ii) Evaporators

THE EFFECTIVENESS–NTU METHOD

A heat exchanger can be designed by the LMTD (Logarithmic mean temperature difference) when inlet and outlet conditions are specified. However, when the problem is to determine the inlet or exit temperatures for a particular heat exchanger, the analysis is performed more easily, by using a method based on effectiveness of the heat exchanger and number of transfer units(NTU)

Heat exchanger effectiveness

$$\varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} = \frac{Q}{Q_{max}}$$

The actual heat transfer rate can be determined by writing an energy balance over either side of the heat exchanger

$$Q = \dot{m}_h \, c_{ph} \, (t_{h1} - t_{h2}) = \dot{m}_c \, c_{pc} \, (t_{c2} - t_{c1})$$

$\dot{m}_h \, c_{ph} = C_h$ = Hot fluid capacity rate

$\dot{m}_c \, c_{pc} = C_c$ = Cold fluid capcity rate

C_{min} = The minimum fluid capacity rate (C_h or C_c)

C_{max} = The maximum fluid capacity rate (C_h or C_c).

The maximum rate of heat transfer for parallel flow or counter flow heat exchangers would occur if the outlet temperature of the fluid. With smaller value of C_h or C_c i.e. C_{min} were to be equal to the inlet temperature of the other fluid

$$Q_{max} = C_h\,(t_{h1} - t_{c1}) \text{ or } C_c\,(t_{h1} - t_{c1})$$

Q_{max} is the minimum of these two values, i.e.,

$$Q_{max} = C_{min}\,(t_{h1} - t_{c1})$$

$$\varepsilon = \frac{C_h\,(t_{h1} - t_{h2})}{C_{min}\,(t_{h1} - t_{c1})} = \frac{C_c\,(t_{c2} - t_{c1})}{C_{min}\,(t_{h1} - t_{c1})}$$

Once the effectiveness is known, the heat transfer rate can be very easily calculated by using the equation

$$Q = \varepsilon C_{min} (t_{h1} - t_{c1})$$

PROBLEM

1. Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot-water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at $c_p = 4.18$ kJ/kg·K, determine the maximum heat transfer rate and the outlet temperatures of the cold- and the hot-water streams for this limiting case.

The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (2\ \text{kg/s})(4.18\ \text{kJ/kg·K}) = 8.36\ \text{kW/K}$$

$$C_c = \dot{m}_c c_{pc} = (8\ \text{kg/s})(4.18\ \text{kJ/kg·K}) = 33.4\ \text{kW/K}$$

$$C_{\min} = C_h = 8.36\ \text{kW/K}$$

The maximum heat transfer rate IS

$$\begin{aligned}\dot{Q}_{\max} &= C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) \\ &= (8.36\ \text{kW/K})(70 - 10)^\circ\text{C} \\ &= 502\ \text{kW}\end{aligned}$$

The maximum temperature difference in this heat exchanger is $\Delta T_{max} = T_{h,in} - T_{c,in} = (70 - 10)^0C = 60^0C$. Therefore, the hot water cannot be cooled by more than 60^0C (to 10^0C) in this heat exchanger, and the cold water cannot be heated by more than 60^0C (to 70^0C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) \longrightarrow T_{c,\,out} = T_{c,\,in} + \frac{\dot{Q}}{C_c} = 10^\circ C + \frac{502\ kW}{33.4\ kW/K} = 25^\circ C$$

$$\dot{Q} = C_h(T_{h,\,in} - T_{h,\,out}) \longrightarrow T_{h,\,out} = T_{h,\,in} - \frac{\dot{Q}}{C_h} = 70^\circ C - \frac{502\ kW}{8.38\ kW/K} = 10^\circ C$$

EFFECTIVENESS RELATIONS FOR HEAT EXCHANGER

Effectiveness relations for heat exchangers: NTU $= UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$

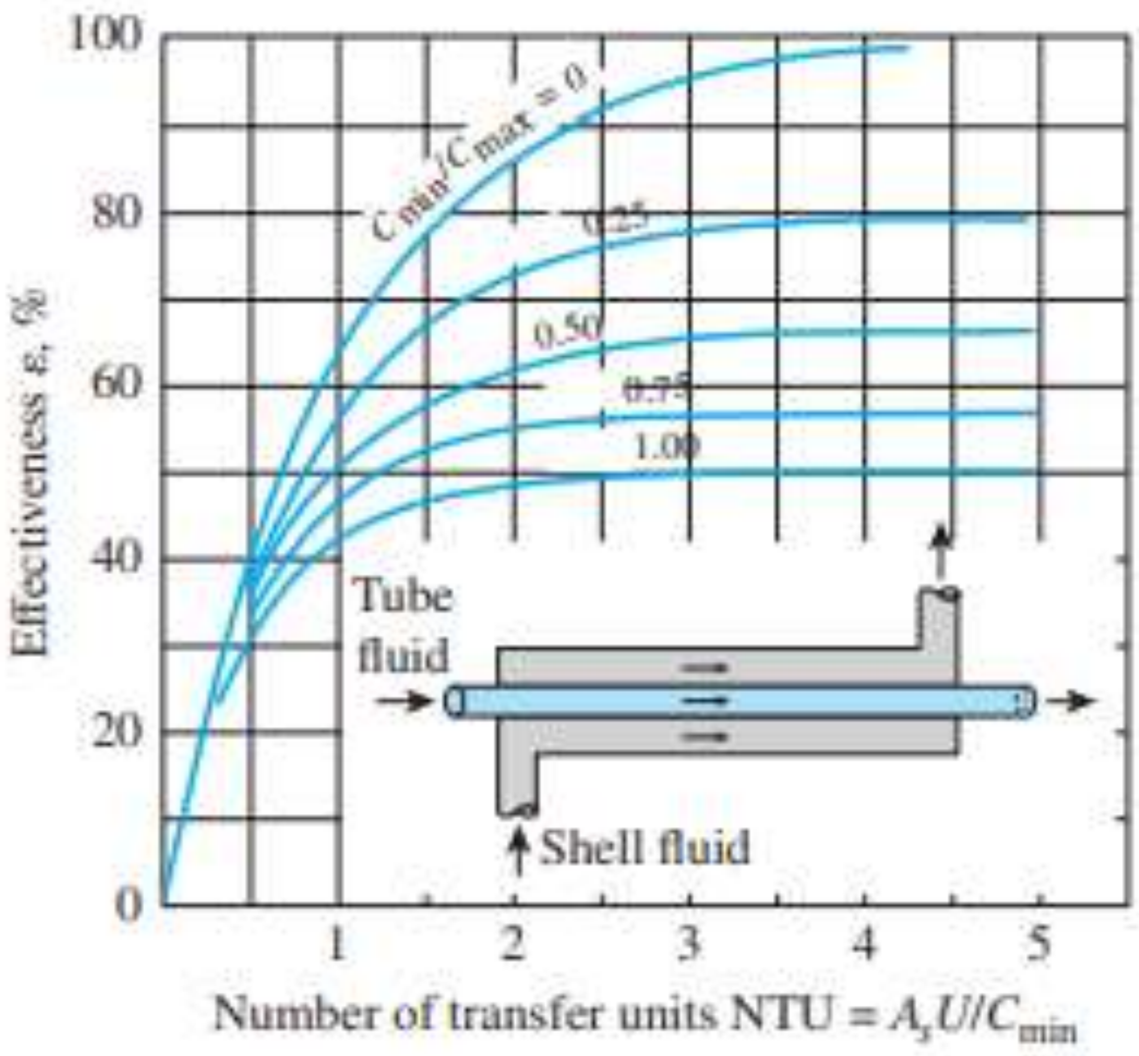
Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1+c)]}{1+c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1-c)]}{1 - c\exp[-\text{NTU}(1-c)]}$ (for $c < 1$)
	$\varepsilon = \dfrac{\text{NTU}}{1+\text{NTU}}$ (for $c = 1$)
2 *Shell-and-tube:* One-shell pass 2, 4,…tube passes	$\varepsilon_1 = 2\left\{1 + c + \sqrt{1+c^2}\dfrac{1+\exp\left[-\text{NTU}_1\sqrt{1+c^2}\right]}{1-\exp\left[-\text{NTU}_1\sqrt{1+c^2}\right]}\right\}^{-1}$
n-shell passes 2*n*, 4*n*, … tube passes	$\varepsilon_n = \left[\left(\dfrac{1-\varepsilon_1 c}{1-\varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1-\varepsilon_1 c}{1-\varepsilon_1}\right)^n - c\right]^{-1}$
3 *Cross-flow (single-pass)* Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{-c[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with* $c = 0$	$\varepsilon = 1 - \exp(-\text{NTU})$

NTU relations for heat exchangers: $NTU = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$

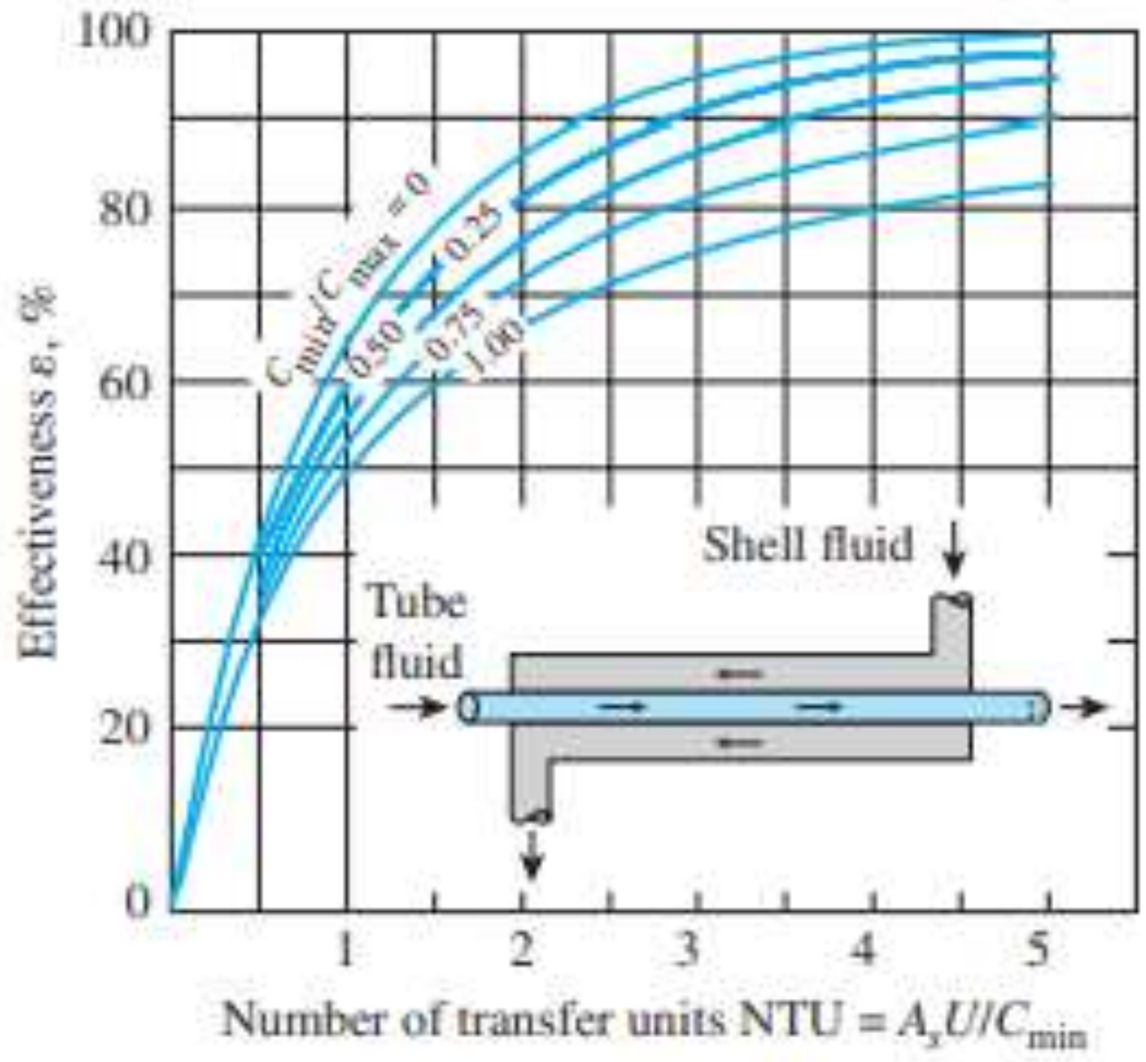
Heat exchanger type	NTU relation
1 *Double-pipe:*	
Parallel-flow	$NTU = -\dfrac{\ln[1 - \varepsilon(1 + c)]}{1 + c}$
Counter-flow	$NTU = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon c - 1}\right)$ (for $c < 1$)
	$NTU = \dfrac{\varepsilon}{1-\varepsilon}$ (for $c = 1$)
2 *Shell and tube:* One-shell pass 2, 4,…tube passes	$NTU_1 = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon_1 - 1 - c - \sqrt{1+c^2}}{2/\varepsilon_1 - 1 - c + \sqrt{1+c^2}}\right)$
n-shell passes $2n$, $4n$,…tube passes	$NTU_n = n(NTU)_1$ To find effectiveness of the heat exchanger with one-shell pass use, $\varepsilon_1 = \dfrac{F-1}{F-c}$ where $F = \left(\dfrac{\varepsilon_n c - 1}{\varepsilon_n - 1}\right)^{1/n}$
3 *Cross-flow (single-pass):* C_{max} mixed, C_{min} unmixed	$NTU = -\ln\left[1 + \dfrac{\ln(1 - \varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$NTU = -\dfrac{\ln[c\ln(1-\varepsilon) + 1]}{c}$
4 *All heat exchangers with $c = 0$*	$NTU = -\ln(1 - \varepsilon)$

CHARTS FOR HEAT EXCHANGER PROBLEMS

(a) Parallel flow

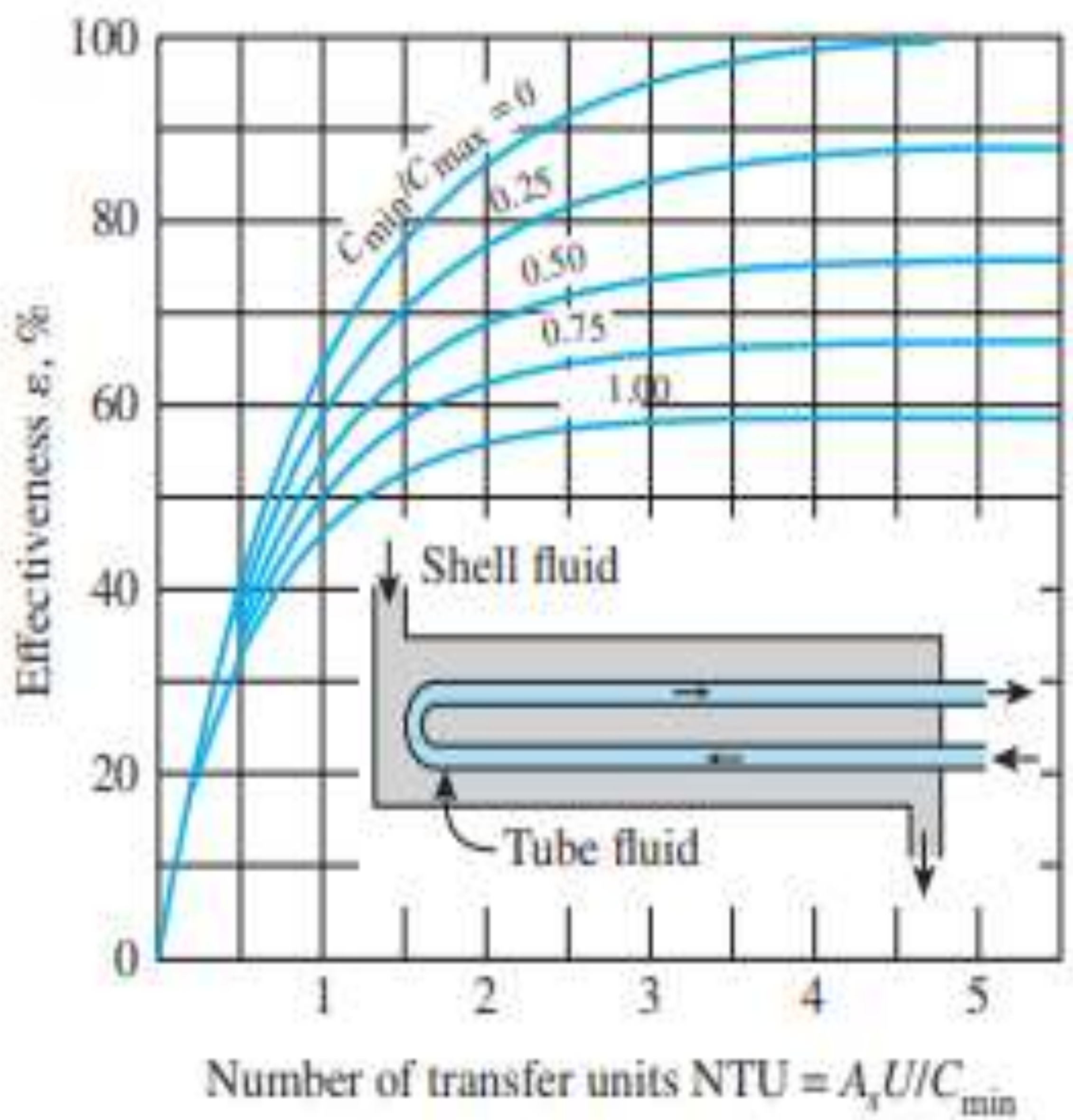


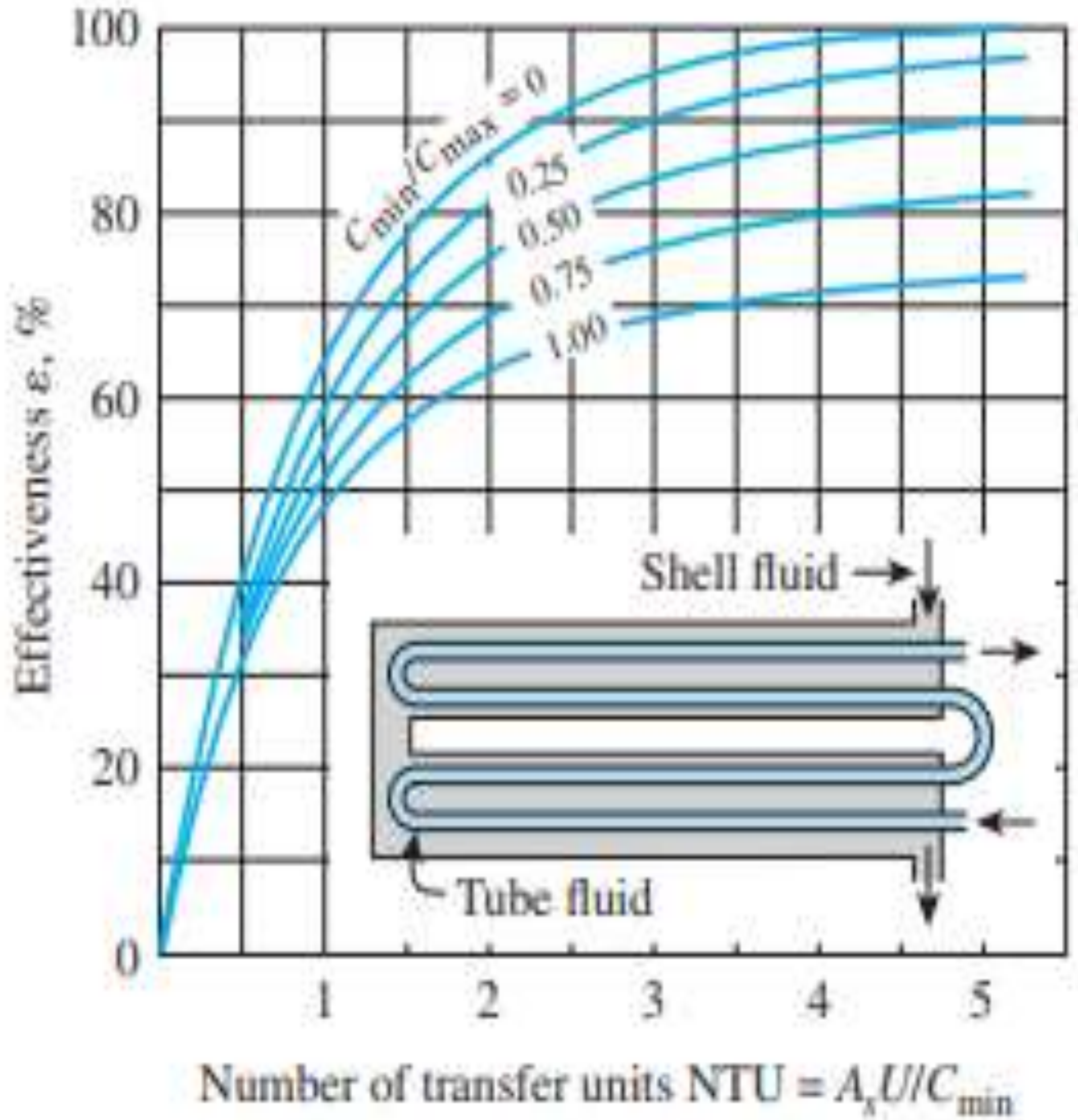
(b) Counter flow



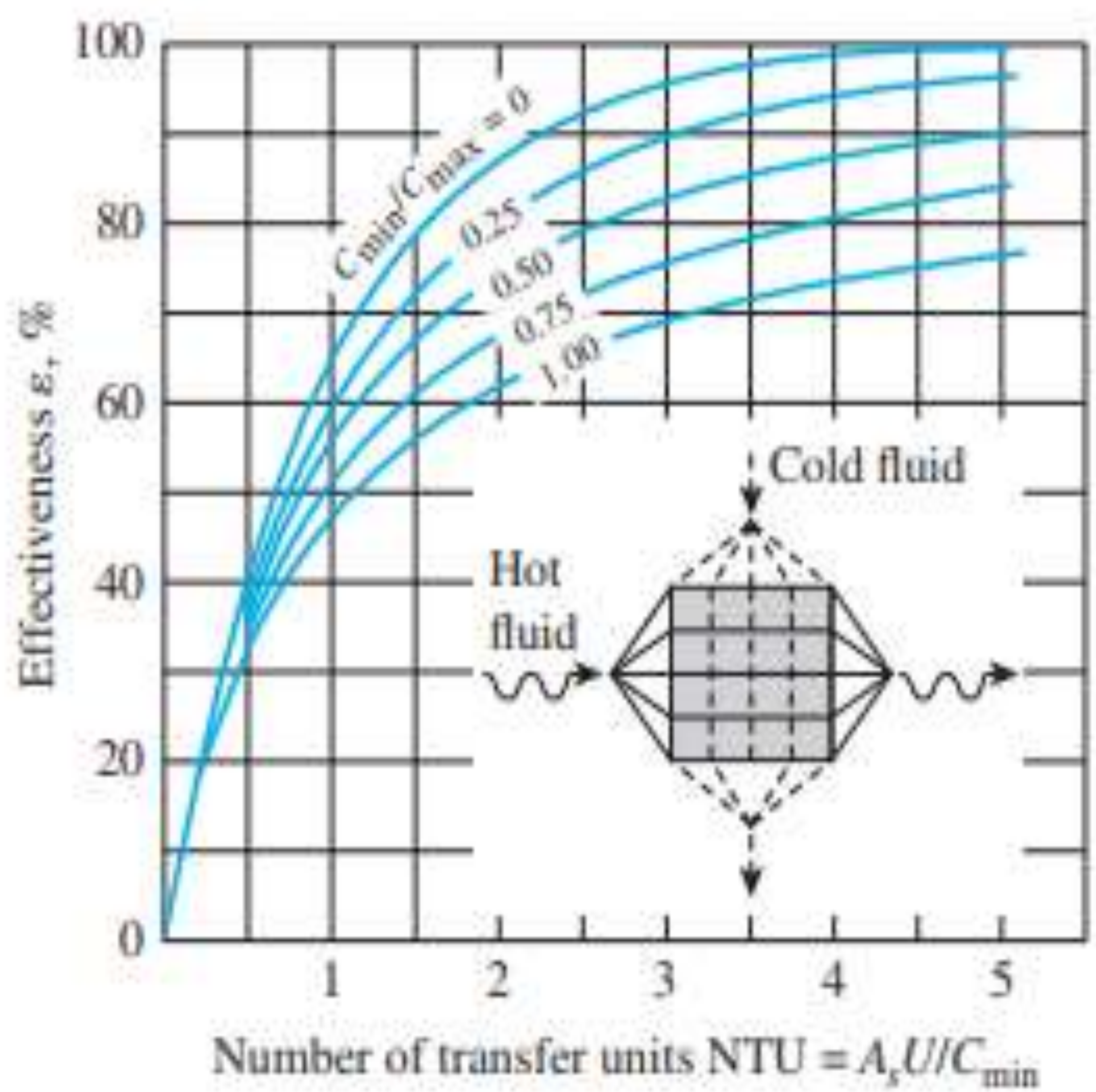
(c) One shell pass and 2, 4, 6, ….tube passes

(d) Two shell passes and 4, 8, 12, …..tube passes

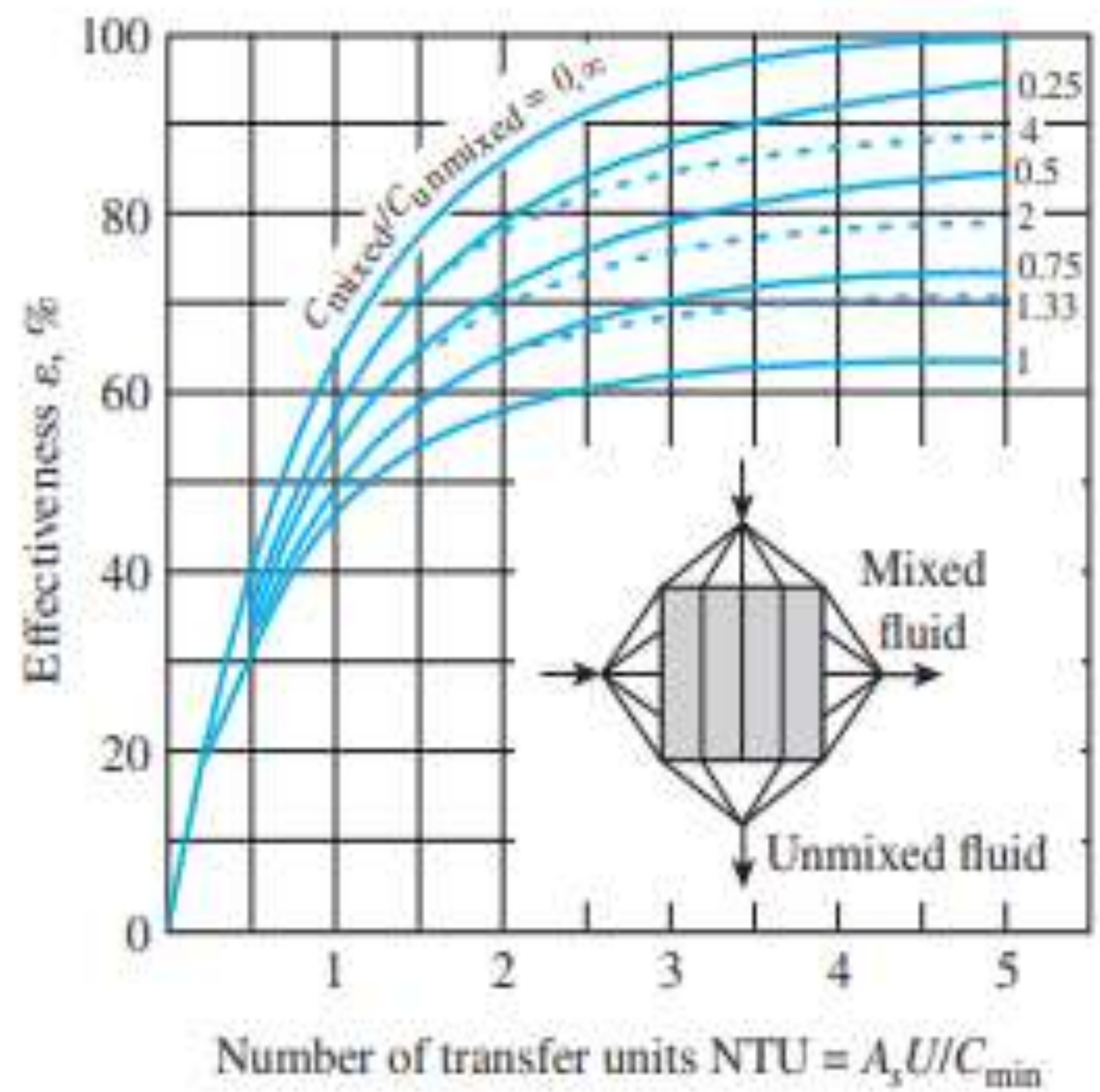




(e) Cross flow with both fluids unmixed



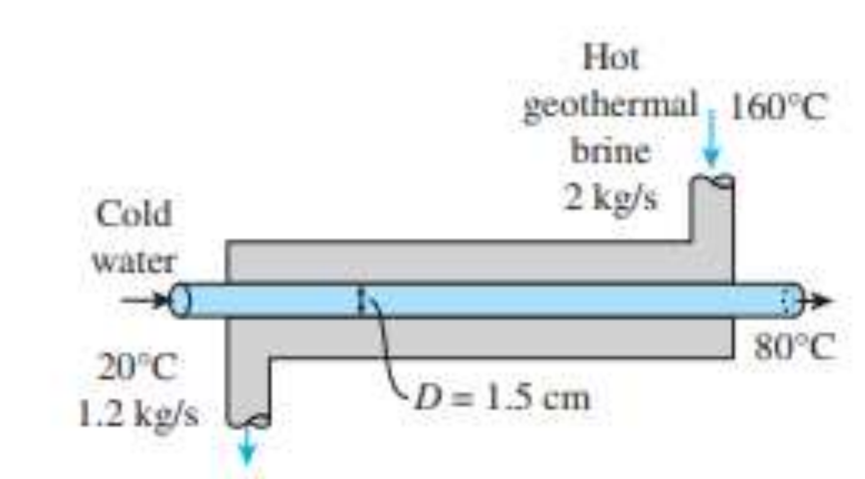
(f) Cross flow with one fluid mixed and the other unmixed



PROBLEMS

1. A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s (Fig. 11–30). The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. The overall heat transfer coefficient of the heat exchanger is 640 W/m²·K. Using the effectiveness-NTU method determine the length of the heat exchanger required to achieve the desired heating.

First to determine the heat capacity rates of the hot and cold fluids and identify the smaller one



$$C_h = \dot{m}_h c_{ph} = (2 \text{ kg/s})(4.31 \text{ kJ/kg·K}) = 8.62 \text{ kW/K}$$
$$C_c = \dot{m}_c c_{pc} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg·K}) = 5.02 \text{ kW/K}$$

$$C_{min} = C_c = 5.02 \text{ kW/K}$$

$$c = C_{min}/C_{max} = 5.02/8.62 = 0.582$$

The maximum heat transfer rate is

$$\begin{aligned}\dot{Q}_{max} &= C_{min}(T_{h,in} - T_{c,in}) \\ &= (5.02 \text{ kW/K})(160 - 20)^{\circ}\text{C} \\ &= 702.8 \text{ kW}\end{aligned}$$

The actual heat transfer rate is

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{water} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg·K})(80 - 20)°C$$
$$= 301.0 \text{ kW}$$

The effectiveness of heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{301.0 \text{ kW}}{702.8 \text{ kW}} = 0.428$$

$$\text{NTU} = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.582 - 1}\ln\left(\frac{0.428 - 1}{0.428 \times 0.582 - 1}\right) = 0.651$$

The heat transfer surface area becomes

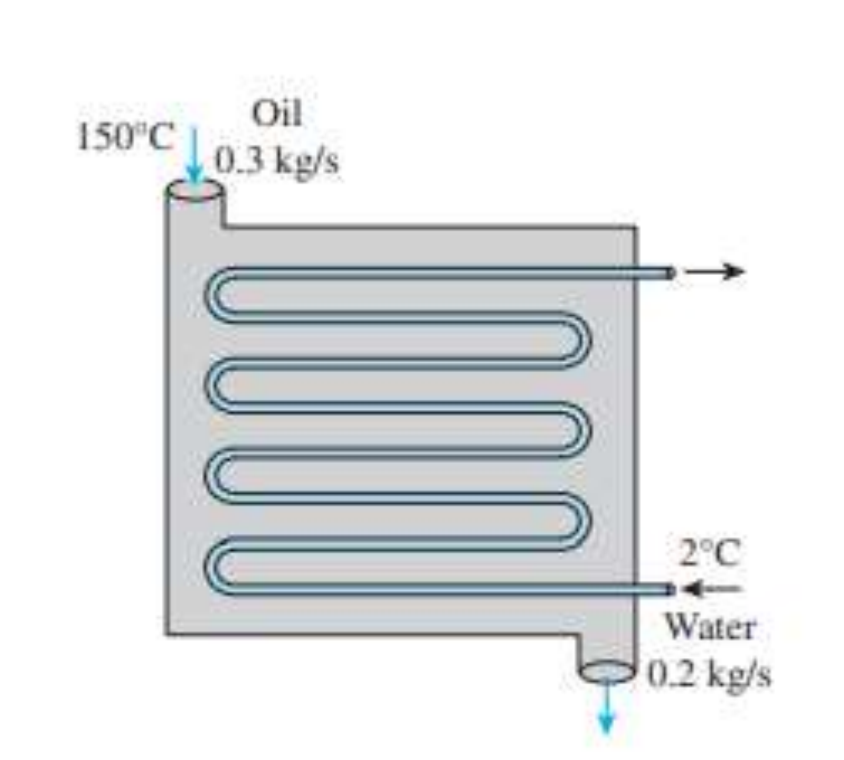
$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU}\,C_{\min}}{U} = \frac{(0.651)(5020\ \text{W/K})}{640\ \text{W/m}^2\cdot\text{K}} = 5.11\ \text{m}^2$$

The length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11\ \text{m}^2}{\pi(0.015\ \text{m})} = 108\ \text{m}$$

2. Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m²·K. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

We take the specific heats of water and oil to be 4.18 and 2.13 kJ/kg^{0}C, respectively.



Now, determining the heat capacity rates of the hot and cold fluids and identify the smaller one

$$C_h = \dot{m}_h c_{ph} = (0.3\ \text{kg/s})(2.13\ \text{kJ/kg}\cdot{}^\circ\text{C}) = 0.639\ \text{kW/K}$$

$$C_c = \dot{m}_c c_{pc} = (0.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C}) = 0.836\ \text{kW/K}$$

$$C_{\min} = C_h = 0.639\ \text{kW/K and } c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

The maximum heat transfer rate is,

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) = (0.639\ \text{kW/K})(150 - 20)^\circ\text{C} = 83.1\ \text{kW}$$

The heat transfer surface area is

$$A_s = n(\pi DL) = 8\pi(0.014\ \text{m})(5\ \text{m}) = 1.76\ \text{m}^2$$

$$NTU = \frac{UA_s}{C_{min}} = \frac{(310\ W/m^2{\cdot}K)(1.76\ m^2)}{639\ W/K} = 0.854$$

The effectiveness of this heat exchanger corresponding to c =0.764 and NTU = 0.854 is (from the chart c)

$$\varepsilon = 0.47$$

The actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{max} = (0.47)(83.1\ kW) = 39.1\ kW$$

The outlet temperatures of the cold and the hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) \longrightarrow T_{c,\,out} = T_{c,\,in} + \frac{\dot{Q}}{C_c}$$

$$= 20°C + \frac{39.1\ kW}{0.836\ kW/K} = 66.8°C$$

$$\dot{Q} = C_h(T_{h,\,in} - T_{h,\,out}) \longrightarrow T_{h,\,out} = T_{h,\,in} - \frac{\dot{Q}}{C_h}$$

$$= 150°C - \frac{39.1\ kW}{0.639\ kW/K} = 88.8°C$$

3. *Steam at atmospheric pressure enters the shell of a surface condenser in which the water flows through a bundle of tubes of diameter 25 mm at the rate of 0.05 kg/s. The inlet and outlet temperatures of water are 15°C and 70°C, respectively. The condensation of steam takes place on the outside surface of the tube. If the overall heat transfer coefficient is 230 W/m^2°C, calculate the following, using NTU method :*

(i) The effectiveness of the heat exchanger,

(ii) The length of the tube, and

(iii) The rate of steam condensation.

Take the latent heat of vaporisation at 100°C = 2257 kJ/kg

Take the latent heat of vaporisation at 100°C = 2257 kJ/kg

Solution. *Given* : $d = 25\text{ mm} = 0.025\text{ m};\ \dot{m}_w = \dot{m}_c = 0.05\text{ kg/s},\ t_{c1} = 15°\text{C},\ t_{c2} = 70°\text{C};$

$U = 230\text{ W/m}^2°\text{C};\ t_{h1} = 100°\text{C}.$

(*i*) The effectiveness of the heat exchanger, ε :

Throughout the condenser the hot fluid (*i.e.*, steam), remains at constant temperature. Hence C_{max} is infinity and thus C_{min} is obviously for cold fluid (*i.e.*, water). Thus $\frac{C_{min}}{C_{max}} \simeq 0.$

When $C_h > C_c$, then effectiveness is given by

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} = \frac{70 - 15}{100 - 15} = \mathbf{0.647} \qquad \textbf{(Ans.)}$$

(ii) The length of the tube, L :

$$C_{min} = \dot{m}_c\, c_{pc} = 0.05 \times 4.18 = 0.209 \text{ kJ/K}$$

For $\dfrac{C_{min}}{C_{max}} (= R) \simeq 0$

$$\varepsilon = 1 - \exp(-\text{NTU}) \quad \ldots$$

or, $$0.647 = 1 - e^{-NTU}$$

or, $$e^{-NTU} = 1 - 0.647 = 0.353$$

or, $$-NTU = \ln(0.353) = -1.04$$

∴ $$NTU = 1.04$$

But, $$NTU = \frac{UA}{C_{min}} = \frac{U \times \pi\, d\, L}{C_{min}}$$

or, $$L = \frac{NTU \times C_{min}}{U\, \pi\, d} = \frac{1.04 \times (0.209 \times 1000)}{230 \times \pi \times 0.025} = \mathbf{12\ m}$$

(iii) The rate of steam condensation, $\dot{m}_h$:

Using the overall energy balance, we get

$$\dot{m}_h \,.\, h_{fg} = \dot{m}_c\, c_{pc}\, (t_{c2} - t_{c1})$$

$$= \dot{m}_h \times 2257 = 0.05 \times 4.18\,(70 - 15)$$

or, $$\dot{m}_h = 0.00509 \text{ kg/s or } \mathbf{18.32\ kg/h} \quad \textbf{(Ans.)}$$

4. *8000 kg/h of air at 100°C is cooled by passing it through a single-pass cross-flow heat exchanger. To what temperature is the air cooled if water entering at 15°C flows through the tubes unmixed at the rate of 7500 kg/h?*

Take : $U = 500$ kJ/h-m^2°C and $A = 20$ m^2

c_p (air) $= 1$ kJ/kg°C and c_p (water) $= 4.2$ kJ/kg°C

Treat both fluids are unmixed.

Solution. *Given :* $\dot{m}_h = \frac{8000}{3600} = 2.22$ kg/s; $c_{ph} = 1$ kJ/kg °C; $t_{h1} = 100$°C;

$$\dot{m}_c = \frac{7500}{3600} = 2.08 \text{ kg/s}; \; c_{pc} = 4.2 \text{ kJ/kg°C}$$

$$U = \frac{500 \times 1000}{3600} = 138.9 \text{ W/m}^2\text{°C}; \; A = 20 \text{ m}^2.$$

$$U = \frac{500 \times 1000}{3600} = 138.9 \text{ W/m}^2 \, ^\circ\text{C}; \; A = 20 \text{ m}^2.$$

$$C_h = \dot{m}_h \, c_{ph} = 2.22 \times (1 \times 1000) = 2220 = C_{min}$$

$$C_c = \dot{m}_c \, c_{pc} = 2.08 \times (4.2 \times 1000) = 8736 = C_{max}$$

$$\therefore \quad \frac{C_{min}}{C_{max}} = \frac{2220}{8736} = 0.254$$

$$NTU = \frac{UA}{C_{min}} = \frac{138.9 \times 20}{2220} = 1.25$$

From the calculated values, $\varepsilon = 0.63$

The effectiveness ε is given by,

$$\varepsilon = \frac{C_h \, (t_{h1} - t_{h2})}{C_{min} \, (t_{h1} - t_{c1})} = \frac{C_c \, (t_{c2} - t_{c1})}{C_{min} \, (t_{h1} - t_{c1})}$$

or,
$$0.63 = \frac{2220 \, (100 - t_{h2})}{2220 \, (100 - 15)} = \frac{8736 \, (t_{c2} - 15)}{2220 \, (100 - 15)}$$

or,
$$0.63 = \frac{100 - t_{h2}}{100 - 15} = 3.935 \left(\frac{t_{c2} - 15}{100 - 15} \right)$$

$$\therefore \quad t_{h2} = 100 - 0.63 \, (100 - 15) = 46.45^\circ\text{C}$$

and,
$$t_{c2} = \frac{0.63 \, (100 - 15)}{3.935} + 15 = 28.6^\circ\text{C}$$

The *air is cooled to a minimum temperature of* **46.45°C. (Ans.)**