Homogeneous Linear ODE's with constant Coefficients

Let $(a_0D^2+a_1D+a_2)y = \phi(x)$ be the differential equation.

★ The auxillary equation is given by

$$a_0m^2+a_1m+a_2=0$$

This equation has two roots, say m_1 & m_2

★ The complementary function is given by

i) m_1 & m_2 are real and different

$$Ae^{m_1x} + Be^{m_2x}$$

ii) m_1 & m_2 are real and equal

$$(A+Bx)e^{mx}$$

(iii) m_1 & m_2 are complex numbers, say $\alpha+i\beta$

$$e^{\alpha x}(A\cos\beta x + B\sin\beta x)$$

★ Find particular Integral

★ $y = CF + PI$

Type-1

Solve: $(D^2+3D+2)y = e^{-3x}$ → RHS is in exponential form so (Type I)

AE $m^2+3m+2=0$

$m = -1, -2$

Roots are Real and different

$$CF = Ae^{-x} + Be^{-2x}$$

Particular Integral: $\frac{1}{D^2+3D+2}e^{-3x}$. $D=-3$