Torque of a three phase induction motor is proportional to flux per stator pole, rotor current and the power factor of the rotor.

$T \propto \phi I_2 \cos\phi_2$ OR $T = k \phi I_2 \cos\phi_2$.

where, ϕ = flux per stator pole,

I_2 = rotor current at standstill,

ϕ_2 = angle between rotor emf and rotor current,

k = a constant.

Now, let E_2 = rotor emf at standstill

we know, rotor emf is directly proportional to flux per stator pole, i.e. $E_2 \propto \phi$.

therefore, $T \propto E_2 I_2 \cos\phi_2$ OR $T = k_1 E_2 I_2 \cos\phi_2$.

Starting Torque

The torque developed at the instant of starting of a motor is called as starting torque. Starting torque may be greater than running torque in some cases, or it may be lesser.
We know, $T = k_1 E_2 I_2 \cos\phi_2$.

let, R2 = rotor resistance per phase
X2 = standstill rotor reactance

$$Z_2 = \sqrt{(R_2^2 + X_2^2)} \quad = \text{rotor impedence per phase at standstill}$$

then,

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \quad \text{and} \quad \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}}$$

Therefore, starting torque can be given as,

$$Tst = k_1 E_2 \frac{E_2}{\sqrt{(R_2^2 + X_2^2)}} \times \frac{R_2}{\sqrt{(R_2^2 + X_2^2)}} = \frac{k_1 E_2^2 R_2}{R_2^2 + X_2^2}$$

The constant k1 = 3 / 2πNs

$$Tst = \frac{3}{2\pi Ns} \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Maximum Torque Condition for Three-Phase Induction Motor

In the equation of torque,
$$T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi n_s}$$

The rotor resistance, rotor inductive reactance and synchronous speed of induction motor remain constant. The supply voltage to the three phase induction motor is usually rated and remains constant, so the stator emf also remains the constant. We define the transformation ratio as the ratio of rotor emf to that of stator emf. So if stator emf remains constant, then rotor emf also remains constant.
If we want to find the maximum value of some quantity, then we have to differentiate that quantity concerning some variable parameter and then put it equal to zero. In this case, we have to find the condition for maximum torque, so we have to differentiate torque concerning some variable quantity which is the slip, s in this case as all other parameters in the equation of torque remains constant.

So, for torque to be maximum
$$\frac{dT}{ds} = 0$$

$$T = K\,sE_2^2 \frac{R_2}{R_2^2 + (sX_2)^2}$$
Now differentiate the above equation by using division rule of differentiation.
On differentiating and after putting the terms equal to zero we get,
$$s^2 = \frac{R_2^2}{X_2^2}$$
Neglecting the negative value of slip we get
$$s^2 = \frac{R_2^2}{X_2^2}$$

So, when slip s = R_2 / X_2, the torque will be maximum and this slip is called maximum slip Sm and it is defined as the ratio of rotor resistance to that of rotor reactance.

NOTE: At starting S = 1, so the maximum starting torque occur when rotor resistance is equal to rotor reactance.

Equation of Maximum Torque

The equation of torque is

$$T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

The torque will be maximum when slip s = R_2 / X_2

Substituting the value of this slip in above equation we get the maximum value of torque as,

$$T_{max} = K\frac{{E_2}^2}{2X_2} \quad N-m$$

In order to increase the starting torque, extra resistance should be added to the rotor circuit at start and cut out gradually as motor speeds up.

Conclusion

From the above equation it is concluded that

1. The maximum torque is directly proportional to square of rotor induced emf at the standstill.
2. The maximum torque is inversely proportional to rotor reactance.
3. The maximum torque is independent of rotor resistance.
4. The slip at which maximum torque occur depends upon rotor resistance, R_2. So, by varying the rotor resistance, maximum torque can be obtained at any required slip.