

The torque produced by **three phase induction motor** depends upon the following three factors:

Firstly the magnitude of rotor current, secondly the **flux** which interact with the rotor of three phase induction motor and is responsible for producing emf in the rotor part of **induction motor**, lastly the **power factor** of rotor of the three phase induction motor.

Combining all these factors, we get the equation of torque as-

$$T \propto \phi I_2 \cos \theta_2$$

Where, T is the torque produced by the induction motor,

$\phi$  is flux responsible for producing induced emf,

$I_2$  is rotor current,

$\cos \theta_2$  is the power factor of rotor circuit.

The flux  $\phi$  produced by the stator is proportional to stator emf  $E_1$ .

i.e  $\phi \propto E_1$

We know that transformation ratio K is defined as the ratio of secondary **voltage** (rotor voltage) to that of primary voltage (stator voltage).

$$K = \frac{E_2}{E_1}$$

$$\text{or, } K = \frac{E_2}{\phi}$$

$$\text{or, } E_2 = \phi$$

Rotor **current**  $I_2$  is defined as the ratio of rotor induced emf under running condition,  $sE_2$  to total impedance,  $Z_2$  of rotor side,

$$\text{i.e } I_2 = \frac{sE_2}{Z_2}$$

and total impedance  $Z_2$  on rotor side is given by ,

$$Z_2 = \sqrt{R_2^2 + (sX_2)^2}$$

Putting this value in above equation we get,

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

s = slip of **induction motor**

We know that **power factor** is defined as ratio of **resistance** to that of impedance. The power factor of the rotor circuit is

$$\cos \theta_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Putting the value of flux  $\phi$ , rotor current  $I_2$ , power factor  $\cos\theta_2$  in the equation of torque we get,

$$T \propto E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Combining similar term we get,

$$T \propto sE_2^2 \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Removing proportionality constant we get,

$$T = K s E_2^2 \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{This constant } K = \frac{3}{2\pi n_s}$$

Where,  $n_s$  is synchronous speed in r. p. s,  $n_s = N_s / 60$ . So, finally the equation of torque becomes,

$$T = s E_2^2 \times \frac{R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi n_s} N - m$$

Derivation of K in torque equation.

In case of **three phase induction motor**, there occur copper losses in rotor.

These rotor copper losses are expressed as

$$P_c = 3I_2^2 R_2$$

We know that rotor current,

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Substitute this value of  $I_2$  in the equation of rotor copper losses,  $P_c$ . So, we get

$$P_c = 3R_2 \left( \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \right)^2$$

$$\text{On simplifying } P_c = \frac{3R_2 s^2 E_2^2}{R_2^2 + (sX_2)^2}$$

The ratio of  $P_2 : P_c : P_m = 1 : s : (1 - s)$

Where,  $P_2$  is the rotor input,

$P_c$  is the rotor copper losses,

$P_m$  is the mechanical power developed.

$$\frac{P_c}{P_m} = \frac{s}{1 - s}$$

or  $P_m = \frac{(1 - s)P_c}{s}$

Substitute the value of  $P_c$  in above equation we get,

$$P_m = \frac{1}{s} \times \frac{(1-s)3R_2s^2E_2^2}{R_2^2 + (sX_2)^2}$$

On simplifying we get,

$$P_m = \frac{(1-s)3R_2sE_2^2}{R_2^2 + (sX_2)^2}$$

The mechanical power developed  $P_m = T\omega$ ,

$$\omega = \frac{2\pi N}{60}$$

$$\text{or } P_m = T \frac{2\pi N}{60}$$

Substituting the value of  $P_m$

$$\frac{1}{s} \times \frac{(1-s)3R_2s^2E_2^2}{R_2^2 + (sX_2)^2} = T \frac{2\pi N}{60}$$

$$\text{or } T = \frac{1}{s} \times \frac{(1-s)3R_2s^2E_2^2}{R_2^2 + (sX_2)^2} \times \frac{60}{2\pi N}$$

We know that the rotor speed  $N = N_s(1-s)$

Substituting this value of rotor speed in above equation we get,

$$T = \frac{1}{s} \times \frac{(1-s)3R_2s^2E_2^2}{R_2^2 + (sX_2)^2} \times \frac{60}{2\pi N_s(1-s)}$$

$N_s$  is speed in revolution per minute (rpm) and  $n_s$  is speed in revolution per sec (rps) and the relation between the two is

$$\frac{N_s}{60} = n_s$$

Substitute this value of  $N_s$  in above equation and simplifying it we get

$$\text{Toeque, } T = \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi N_s}$$

$$\text{or, } T = K s E_2^2 \frac{R_2}{R_2^2 + (sX_2)^2}$$

Comparing both the equations, we get, constant  $K = 3 / 2\pi n_s$