ENERGY EQUATION:

Physical Principle: Energy can be neither created nor destroyed, it can only change its form.

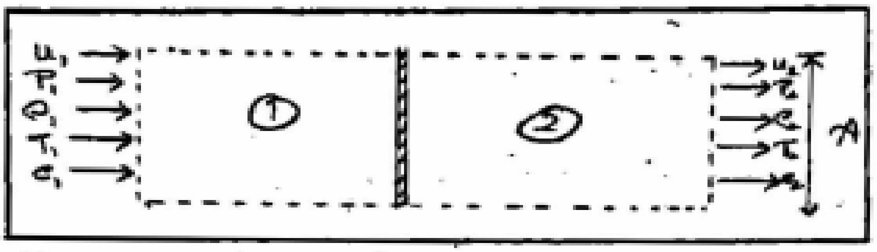


Fig 1: Closed boundary

$$\boxed{\delta w + \delta q = de} \rightarrow (1)$$

where de is change in internal energy.

By First law of thermodynamics:

$$\boxed{B_1 + B_2 = B_3} \rightarrow (2)$$

where

B_1 = Rate of heat added to fluid inside control volume from surroundings

B_2 = Rate of work done on fluid inside the control volume

B_3 = Rate of change of energy of fluid as it flows through control volume.

⇒ Rate of Volumetric heating $\Big\} = \iiint_{V} \dot{q}\,\rho\, dV \longrightarrow (3)$

Where, $\dot{q}$ = heat addition per unit mass $(J/s.kg)$

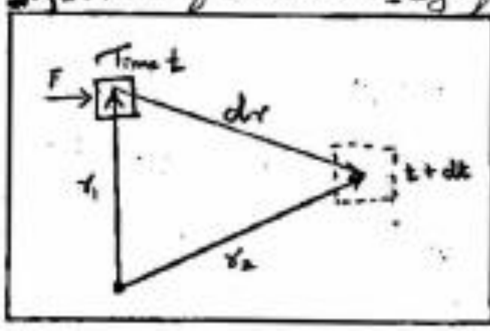
ρdV = mass contained within an elemental volume

dV = elemental volume

$$\boxed{B_1 = \iiint_{V} \dot{q}\,\rho\, dV + \dot{Q}_{viscous}} \longrightarrow (4)$$

⇒ Rate of Work done on moving body = F.V

Fig 2: Rate of work done by force F



Work done $\Big\} = F.dr$

Time rate of W.D $\Big\} = F.\dfrac{dr}{dt}$

W.D on moving body $\Big\} = F.V$

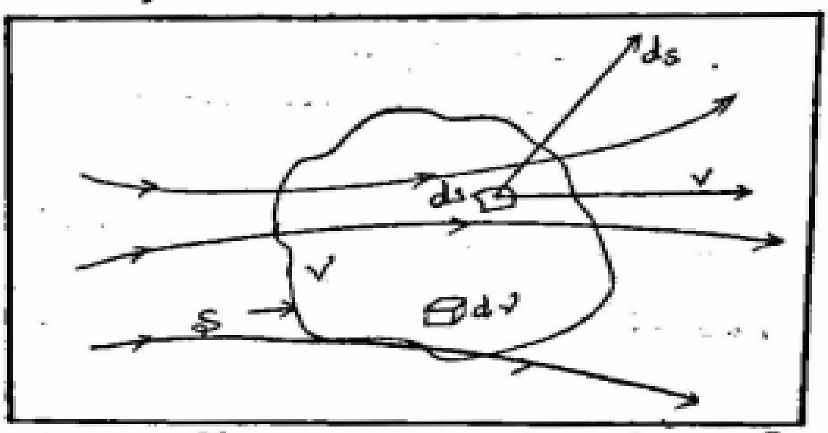
⇒ Rate of work done on a moving body is equal to the product of its velocity & the component of force in the direction of velocity.

Consider the elemental area dS & the pressure force $-pdS$

⇒ Rate of work done on the fluid passing through dS with velocity V is $(-p\,dS).V$

Summing the Control surface,

$$\Rightarrow \left.\begin{array}{l}\text{Rate of Work done on}\\ \text{fluid inside } \mathcal{V} \text{ due to}\\ \text{Pressure force on } S\end{array}\right\} = -\oiint_S (p\,dS)\cdot V \rightarrow (5)$$



$$\left.\begin{array}{l}\text{Work}\\ \text{done}\end{array}\right\} = F.dr$$

Fig 3: Finite Control volume fixed in space

From fig 'f' is the body force per unit mass, the rate of work done on elemental volume is $(\rho f\, d\mathcal{V})\cdot V$

$$\Rightarrow \left.\begin{array}{l}\text{Rate of work done on}\\ \text{fluid inside } \mathcal{V} \text{ due to}\\ \text{body force}\end{array}\right\} = \iiint_{\mathcal{V}} (\rho f\, d\mathcal{V})\cdot V \rightarrow (6)$$

If the flow is Viscous, the shear stress on the control surface will perform work on the fluid as it passes across the surface. Let us denote it as $\dot{W}_{viscous}$.

The total rate of work done on the fluid inside the Control Volume is sum of equ ⑤ & ⑥

$$\boxed{B_2 = \oiint_S p V \cdot ds + \iiint_{\mathcal{V}} \rho (f \cdot V)\, d\mathcal{V} + \dot{W}_{viscous}} \rightarrow ⑦$$

The elemental mass flow across ds is $\rho V \cdot ds$ & elemental flow of total energy across ds is $(\rho V \cdot ds)\left(e + \frac{V^2}{2}\right)$

Summing over the complete control surface,

$$\left.\begin{array}{l}\text{Net rate of flow of total} \\ \text{energy across Control Surface}\end{array}\right\} = \oiint_S (\rho V \cdot ds)\left(e + \frac{V^2}{2}\right) \rightarrow ⑧$$

The total energy contained in the elemental volume $d\mathcal{V}$ is $\rho\left(e + \frac{V^2}{2}\right) d\mathcal{V}$.

The total energy inside the complete control volume at any instant in time is $\iiint_{\mathcal{V}} \rho\left(e + \frac{V^2}{2}\right) d\mathcal{V}$

$$\therefore \left.\begin{array}{l}\text{Time rate of change of} \\ \text{total energy inside } \mathcal{V} \\ \text{due to transient variations} \\ \text{of flow - field variables}\end{array}\right\} = \frac{\partial}{\partial t} \iiint_{\mathcal{V}} \rho\left(e + \frac{V^2}{2}\right) d\mathcal{V} \rightarrow ⑨$$

Summing equation ⑧ & ⑨

$$\boxed{B_3 = \frac{\partial}{\partial t} \iiint_{\mathcal{V}} \rho\left(e + \frac{V^2}{2}\right) d\mathcal{V} + \oiint_S (\rho V \cdot ds)\left(e + \frac{V^2}{2}\right)} \rightarrow ⑩$$