



Method of variation of Parameters

The second order linear differential eqn. is

$$\frac{d^2 y}{dx^2} + P \frac{dy}{dx} + Q = X \text{ where } X \text{ is a fn. of } x.$$

CF = $C_1 f_1 + C_2 f_2$, where C_1, C_2 are constants
 f_1, f_2 are functions of x .

PI = $P f_1 + Q f_2$

where $P = - \int \frac{f_2 X}{f_1 f_2' - f_1' f_2} dx$

$Q = \int \frac{f_1 X}{f_1 f_2' - f_1' f_2} dx$

$\int \tan x dx = + \log (\sec x)$

$\int \cot x dx = \log (\sin x)$

$\int \operatorname{cosec} x dx = -\log [\csc x + \cot x]$

$\int \sec x dx = \log (\sec x + \tan x)$

J. Solve $\frac{d^2 y}{dx^2} + 4y = 4 \tan 2x$ using method of

variation of parameters.

Soln.

Given $(D^2 + 4)y = 4 \tan 2x$ where $X = 4 \tan 2x$

AE

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$CF = C_1 \cos 2x + C_2 \sin 2x$$

$$PI = P f_1 + Q f_2$$

Here $f_1 = \cos 2x$

$$f_1' = -2 \sin 2x$$

$$f_2 = \sin 2x$$

$$f_2' = 2 \cos 2x$$

Now $w = f_1 f_2' - f_2 f_1'$

$$= \cos 2x [2 \cos 2x] - \sin 2x [-2 \sin 2x]$$

$$= 2 \cos^2 2x + 2 \sin^2 2x$$

$$= 2 [\cos^2 2x + \sin^2 2x]$$

$$= 2(1) = 2$$





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UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Method of variation of parameters

$$\begin{aligned}
P &= - \int \frac{f_2 x}{w} dx \\
&= - \int \frac{\sin 2x + \tan 2x}{2} dx \\
&= - \frac{1}{2} \int \sin 2x + \frac{\sin 2x}{\cos 2x} dx \\
&= - \frac{1}{2} \int \frac{\sin^2 2x}{\cos 2x} dx \\
&= - \frac{1}{2} \int \frac{1 - \cos^2 2x}{\cos 2x} dx = - \frac{1}{2} \left[\int \frac{1}{\cos 2x} dx - \int \cos 2x dx \right] \\
&= - \frac{1}{2} \left[\int \sec 2x dx - \int \cos 2x dx \right] \\
&= - \frac{1}{2} \left[\frac{\log(\sec 2x + \tan 2x)}{2} + \frac{\sin 2x}{2} \right]
\end{aligned}$$

$$P = - \log(\sec 2x + \tan 2x) + \sin 2x$$

$$\begin{aligned}
Q &= \int \frac{f_1 x}{w} dx \\
&= \int \frac{\cos 2x + \tan 2x}{2} dx \\
&= \frac{1}{2} \int \cos 2x + \frac{\sin 2x}{\cos 2x} dx \\
&= \frac{1}{2} \int \sin 2x dx = \frac{1}{2} \left[-\frac{\cos 2x}{2} \right]
\end{aligned}$$

$$Q = - \cos 2x$$

$$\begin{aligned}
PI &= Pf_1 + Qf_2 \\
&= \left[\log(\sec 2x + \tan 2x) + \sin 2x \right] \cos 2x \\
&\quad - \cos 2x \sin 2x
\end{aligned}$$

$$PI = - \log(\sec 2x + \tan 2x) \cos 2x$$

$$\therefore y = CF + PI = c_1 \cos 2x + c_2 \sin 2x - \log(\sec 2x + \tan 2x) \cos 2x$$



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