



Type 1:

$$\text{RHS} = e^{ax}$$

Replace  $D$  by  $a$ .

Q. Solve  $(D^2+1)y = e^{-x}$

Soln.

The Auxiliary eqn. is  $m^2+1=0$   
 $m^2=-1$   
 $m=\pm i$

$\therefore$  The roots are imaginary.

$$\text{CF} = e^{0x} [A \cos x + B \sin x]$$

$$\text{CF} = A \cos x + B \sin x$$

$$\text{PI} = \frac{1}{D^2+1} e^{-x}$$

$$= \frac{1}{(-1)^2+1} e^{-x} \quad \text{Replace } D \rightarrow a = -1$$

$$= \frac{1}{2} e^{-x}$$

$$\text{PI} = \frac{e^{-x}}{2}$$

$\therefore$  The soln. is

$$y = \text{CF} + \text{PI}$$
$$y = A \cos x + B \sin x + \frac{e^{-x}}{2}$$



2]. Solve  $(D^2 + 4D + 4)y = 11e^{-2x}$

Soln.

The auxiliary eqn. is,  $m^2 + 4m + 4 = 0$   
 $(m+2)^2 = 0$

$m = -2, -2$

The roots are real and same.

CF =  $(A + Bx)e^{-2x}$

PI =  $\frac{1}{D^2 + 4D + 4} 11e^{-2x}$

=  $11 \frac{1}{(-2)^2 + 4(-2) + 4} e^{-2x}$

=  $11 \frac{1}{4 - 8 + 4} e^{-2x}$

=  $11x \frac{1}{2D + 4} e^{-2x}$

=  $11x \frac{1}{2(-2) + 4} e^{-2x}$

=  $11x^2 \frac{1}{2} e^{-2x}$

=  $\frac{11x^2}{2} e^{-2x}$

Replace  
 $D \rightarrow a = -2$

[If:  $Dx \rightarrow 0$ , then multiply  $x$  in the Nr. and differentiate the Dx. w.r. to  $D$ ]

∴ The general soln. is,

$y = CF + PI$   
 $= (A + Bx)e^{-2x} + \frac{11x^2}{2} e^{-2x}$

3]. Solve  $(D^2 - 2D + 1)y = \cos bx$

Soln.

Given  $(D^2 - 2D + 1)y = \frac{e^x + e^{-x}}{2}$

=  $\frac{1}{2} [e^x + e^{-x}]$

=  $\frac{e^x}{2} + \frac{e^{-x}}{2}$





AE

$$m^2 - 2m + 1 = 0$$
$$m = 1, 1$$

CF

$$CF = (A + Bx)e^x$$

$$PI_1 = \frac{1}{D^2 - 2D + 1} \frac{e^x}{2}$$

$$= \frac{1}{2} \frac{1}{D^2 - 2D + 1} e^x \quad D \rightarrow 1$$

$$= \frac{x}{2} \frac{1}{2D - 2} e^x$$

$$= \frac{x^2}{2} \frac{1}{2} e^x$$

$$PI_1 = \frac{x^2}{4} e^x$$

$$PI_2 = \frac{1}{D^2 - 2D + 1} \frac{e^{-x}}{2}$$

$$= \frac{1}{2} \frac{1}{(D-1)^2 - 2(D-1) + 1} e^{-x}$$

$$= \frac{1}{2} \frac{1}{1 + 2 + 1} e^{-x}$$

$$PI_2 = \frac{1}{8} e^{-x}$$

The general soln. is

$$y = CF + PI_1 + PI_2$$

$$y = (A + Bx)e^x + \frac{x^2}{4} e^x + \frac{1}{8} e^{-x}$$



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Type 2:

$$\text{RHS} = \sin(ax+b)$$

$$\text{or}$$

$$\cos(ax+b)$$

Replace  $D^2 \rightarrow -a^2$

J. solve  $(D^2 + 3D + 2)y = \sin 3x$

Soln.

CF

$$m^2 + 3m + 2 = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

$$\therefore \text{CF} = A e^{-x} + B e^{-2x}$$

$$\text{PI} = \frac{1}{D^2 + 3D + 2} \sin 3x$$

$$= \frac{1}{-9 - 3D + 2} \sin 3x$$

$$= \frac{1}{-3D - 7} \sin 3x$$

$$= \frac{1}{(-3D - 7)} \frac{(-3D + 7)}{(-3D + 7)} \sin 3x$$

$$= \frac{-3D + 7}{9D^2 - 49} \sin 3x$$

$$= \frac{-3D + 7}{9(-9) - 49} \sin 3x \quad D^2 \rightarrow -9$$

$$= \frac{-3D + 7}{-130} \sin 3x$$

$$= \frac{-1}{130} [(-3D + 7) \sin 3x]$$

$$= \frac{-1}{130} [-3D \sin 3x + 7 \sin 3x]$$



$$\text{Scanned with CamScanner} \quad \frac{-1}{130} [-3(3) \cos 3x + 7 \sin 3x]$$



$$\begin{aligned}
 &= \frac{1}{D^2 - 3D + 2} 2e^x && D \rightarrow a = 1 \\
 &= x \frac{1}{2D - 3} 2e^x \\
 &= x \frac{1}{2(1) - 3} 2e^x \\
 &= \frac{x}{-1} 2e^x \\
 &= -2xe^x
 \end{aligned}$$

∴ the soln. is,

$$\begin{aligned}
 y &= CF + PI_1 + PI_2 \\
 &= Ae^x + Be^{2x} - \frac{1}{20} [6\sin(2x+3) + 2\cos(2x+3)] \\
 &\quad - 2xe^x
 \end{aligned}$$

Ex. Find the PI of  $(D^2 + 5D + 6)y = \sin 3x \cos x$

Soln.

Given that  $(D^2 + 5D + 6)y = \sin 3x \cos x$

$$= \frac{1}{2} [\sin 4x + \sin 2x]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$PI_1 = \frac{1}{D^2 + 5D + 6} \frac{1}{2} \sin 4x$$

$$= \frac{1}{-16 + 5D + 6} \frac{1}{2} \sin 4x \quad D^2 \rightarrow -a^2 = -4^2 = -16$$

$$= \frac{1}{5D - 10} \frac{1}{2} \sin 4x$$

$$= \frac{1}{2} \frac{5D + 10}{25D^2 - 100} \sin 4x$$

$$= \frac{1}{2} \frac{5D \sin 4x + 10 \sin 4x}{25(-16) - 100}$$





$$= \frac{1}{2 \times (-500)} [20 \cos 4x + 10 \sin 4x]$$

$$PI_1 = \frac{-1}{+100} [2 \cos 4x + \sin 4x]$$

$$PI_2 = \frac{1}{D^2 + 5D + 6} \cdot \frac{1}{2} \sin 2x$$

$$= \frac{1}{2} \frac{1}{-4 + 5D + 6} \sin 2x \quad D^2 \rightarrow -a^2 = -4$$

$$= \frac{1}{2} \frac{5D - 2}{25D^2 - 4} \sin 2x$$

$$= \frac{1}{2} \frac{[5D \sin 2x - 2 \sin 2x]}{25(-4) - 4}$$

$$PI_2 = -\frac{1}{104} [5 \cos 2x - \sin 2x]$$

The soln. is,

$$y = CF + PI_1 + PI_2$$

$$= \frac{-1}{100} [2 \cos 4x + \sin 4x] - \frac{1}{104} [5 \cos 2x - \sin 2x]$$



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UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Linear ODE with constant coefficients



Type 3:  $RHS = x^n$

- 1).  $(1-D)^{-1} = 1 + D + D^2 + D^3 + \dots$
- 2).  $(1+D)^{-1} = 1 - D + D^2 - D^3 + \dots$
- 3).  $(1-D)^{-2} = 1 + 2D + 3D^2 + 4D^3 + \dots$
- 4).  $(1+D)^{-2} = 1 - 2D + 3D^2 - 4D^3 + \dots$

Q. Solve  $(D^2+2)y = x^2$

Soln.

AE

$$m^2 + 2 = 0$$

$$m^2 = -2$$

$$m = \pm\sqrt{2}i$$

$$\alpha \pm i\beta \Rightarrow \alpha = 0, \beta = \sqrt{2}$$

$$CF = A \cos \sqrt{2}x + B \sin \sqrt{2}x$$

$$PI = \frac{1}{D^2+2} x^2$$

$$= \frac{1}{2 \left[ 1 + \frac{D^2}{2} \right]} x^2$$

$$= \frac{1}{2} \left[ 1 + \frac{D^2}{2} \right]^{-1} x^2$$

$$= \frac{1}{2} \left[ 1 - \frac{D^2}{2} + \frac{D^4}{4} - \dots \right] x^2$$

$$= \frac{1}{2} \left[ 1 - \frac{D^2}{2} \right] x^2$$

$$= \frac{1}{2} \left[ x^2 - \frac{D^2 x^2}{2} \right] = \frac{1}{2} \left[ x^2 - \frac{2}{2} \right]$$

$$= \frac{1}{2} [x^2 - 1]$$

∴ The soln. is,  $y = CF + PI$

$$= A \cos \sqrt{2}x + B \sin \sqrt{2}x + \frac{1}{2} [x^2 - 1]$$







2]. Solve  $(D^2 + 3D + 2)y = x^2$

Soln.

AE  $m^2 + 3m + 2 = 0$

$(m+1)(m+2) = 0$

$m = -1, -2$

CF =  $Ae^x + Be^{-2x}$

PI =  $\frac{1}{D^2 + 3D + 2} x^2$

$= \frac{1}{2 \left[ 1 + \frac{D^2 + 3D}{2} \right]} x^2$

$= \frac{1}{2} \left[ 1 + \left( \frac{D^2 + 3D}{2} \right) \right]^{-1} x^2$

$= \frac{1}{2} \left[ 1 - \left( \frac{D^2 + 3D}{2} \right) + \left( \frac{D^2 + 3D}{2} \right)^2 \right] x^2$

$= \frac{1}{2} \left[ 1 - \frac{D^2}{2} - \frac{3D}{2} + \frac{9D^2}{4} \right] x^2$

$= \frac{1}{2} \left[ x^2 - \frac{D^2 x^2}{2} - \frac{3D x^2}{2} + \frac{9D^2 x^2}{4} \right]$

$= \frac{1}{2} \left[ x^2 - \frac{2}{2} - \frac{3(2x)}{2} + \frac{9(2)}{4} \right]$

$= \frac{1}{2} \left[ x^2 - 1 - 3x + 9/2 \right]$

PI =  $\frac{1}{2} \left[ x^2 - 3x + \frac{7}{2} \right]$

The soln. is,

$y = CF + PI$

$y = Ae^x + Be^{-2x} + \frac{1}{2} \left[ x^2 - 3x + \frac{7}{2} \right]$



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Type-4

RHS =  $e^{ax} \phi(x)$  where  $\phi(x) = \begin{matrix} \sin bx & 0 \rightarrow \\ \cos bx & 0 \rightarrow \\ x^m & \end{matrix}$

Replace  $D \rightarrow D+a$

J. solve  $(D^2 - 4D + 3)y = e^x \cos 2x$

Soln.

$$m^2 - 4m + 3 = 0$$

$$m = 1, 3$$

$$CF = Ae^x + Be^{3x}$$

$$PI = \frac{1}{D^2 - 4D + 3} e^x \cos 2x$$

$$= e^x \frac{1}{(D+1)^2 - 4(D+1) + 3} \cos 2x \quad D \rightarrow D+a = D+1$$

$$= e^x \frac{1}{D^2 + 1 + 2D - 4D - 4 + 3} \cos 2x$$

$$= e^x \frac{1}{D^2 - 2D} \cos 2x$$

~~$$= e^x \frac{1}{-4 - 2D} \cos 2x \quad D^2 \rightarrow -a^2 = -2^2 = -4$$~~

$$= e^x \frac{-2D + 4}{4D^2 - 16} \cos 2x$$

$$= e^x \frac{-2D \cos 2x + 4 \cos 2x}{4(-4) - 16}$$

$$= \frac{e^x}{-32} [4 \sin 2x + 4 \cos 2x]$$

$$= -\frac{e^x}{8} [\sin 2x + \cos 2x]$$

∴ The soln. is  $y = CF + PI$

$$y = Ae^x + Be^{3x} - \frac{e^x}{8} [\sin 2x + \cos 2x]$$





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UNIT-II ORDINARY DIFFERENTIAL EQUATIONS

Linear ODE with constant coefficients



Q7. Find the PI of  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = xe^{-2x}$

Soln.

Given that  $(D^2 + 4D + 4)y = xe^{-2x}$

$$\begin{aligned} PI &= \frac{1}{D^2 + 4D + 4} e^{-2x} x \\ &= e^{-2x} \frac{1}{(D-2)^2 + 4(D-2) + 4} x \quad D \rightarrow D+2 = D-2 \\ &= e^{-2x} \frac{1}{D^2 + 4 - 4D + 4D - 8 + 4} x \\ &= e^{-2x} \frac{1}{D^2} x \end{aligned}$$

$$PI = \frac{e^{-2x} x^3}{6}$$

$$\begin{aligned} \frac{1}{D} x &= \int x = \frac{x^2}{2} \\ \frac{1}{D^2} &= \frac{x^3}{6} \end{aligned}$$

HW Q. Solve  $(D^2 - 4D - 5)y = xe^x$

Q7. Solve  $(D^2 + 4D + 4)y = e^{-2x} x^2$

Q8.  $(D^2 + 4D + 4)y = e^{-2x} \sin x$

TYPE - 5

Case 1: RHS =  $x \phi(x)$  where  $\phi(x) = \sin ax$  or  $\cos ax$

$$PI = x \frac{1}{f(D)} \phi(x) - \frac{f'(D)}{[f(D)]^2} \phi(x)$$

Case 2:

RHS =  $x^n \phi(x)$

i).  $PI = \text{Imaginary part} \left[ \frac{1}{f(D)} x^n e^{iax} \right]$  if  $\phi(x) = \sin ax$

ii).  $PI = \text{Real part} \frac{1}{f(D)} x^n e^{iax}$  if  $\phi(x) = \cos ax$



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11. Solve  $(D^2 + 4)y = x \sin x$

Soln.

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$\alpha = 0, \beta = 2$$

$$CF = A \cos 2x + B \sin 2x$$

$$PI = \frac{1}{D^2 + 4} x \sin x$$

$$= x \frac{1}{D^2 + 4} \sin x - \frac{2D}{(D^2 + 4)^2} \sin x$$

$$= x \frac{1}{-1 + 4} \sin x - \frac{4 \cos x}{(-1 + 4)^2} \quad D^2 \rightarrow -a^2 = -1^2 = -1$$

$$= \frac{x \sin x}{3} - \frac{4 \cos x}{9}$$

∴ The soln. is  $y = CF + PI$

$$y = A \cos 2x + B \sin 2x + \frac{x \sin x}{3} - \frac{4 \cos x}{9}$$

12. Solve  $(D^2 - 2D + 1)y = x e^x \sin x$

Soln.

$$m^2 - 2m + 1 = 0$$

$$m = 1, 1$$

$$CF = (A + Bx)e^x$$

$$PI = \frac{1}{D^2 - 2D + 1} e^x x \sin x$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} x \sin x \quad D \rightarrow D+1$$

$$= D+1$$

$$= e^x \frac{1}{D^2 + 1 + 2D - 2D - 2 + 1} x \sin x$$



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$$= e^x \frac{1}{D^2} x \sin x$$

$$= e^x \left[ x \frac{1}{D^2} \sin x - \frac{2D}{D^4} \sin x \right]$$

$$= e^x \left[ x \frac{1}{-1} \sin x - \frac{2 \cos x}{(-1)^2} \right]$$

$$PI = -x e^x \sin x - 2 e^x \cos x$$

The soln. is,

$$y = CF + PI$$

$$= (A + Bx) e^x - x e^x \sin x - 2 e^x \cos x$$



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