



J. Solve  $x^2 y'' + 2xy' = 0$

Soln

Given  $(x^2 D^2 + 2x D)y = 0 \rightarrow (1)$

Take  $x = e^z$

$z = \log x$

$x D = D'$

$x^2 D^2 = D'(D' - 1) = D'^2 - D'$

Subs. the above in (1),

$[D'^2 - D' + 2D']y = 0$

$[D'^2 + D]y = 0$

AE

$m^2 + m = 0 \quad D' \rightarrow m$

$m(m+1) = 0$

$m = 0, m = -1$

CF =  $Ae^{0z} + Be^{-z}$

$= A + Be^{-z}$

$\therefore$  The soln. is,  $y = CF = A + Be^{-\log x} = A + \frac{B}{x}$

Q]. Solve  $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} + 4y = x \sin(\log x)$

Soln.

Given  $(x^2 D^2 - 3x D + 4)y = x \sin(\log x) \rightarrow (1)$

Take  $x = e^z$

$z = \log x$

$x D = D'$

$x^2 D^2 = D'(D' - 1) = D'^2 - D'$

Subs. the above in (1)

$[D'^2 - D' - 3D' + 4]y = e^z \sin z$

$[D'^2 - 4D' + 4]y = e^z \sin z$

AE

$m^2 - 4m + 4 = 0$

$(m - 2)^2 = 0$

$m = 2, 2$





$$CF = (A + Bx) e^{2x}$$

$$CF = [A + B \log x] x^2$$

$$PI = \frac{1}{D'^2 - 4D' + 4} e^x \sin x$$

$$= e^x \frac{1}{(D'+1)^2 - 4(D'+1) + 4} \sin x \quad \begin{array}{l} D' \rightarrow D'+a \\ = D'+1 \end{array}$$

$$= e^x \frac{1}{D'^2 + 1 + 2D' - 4D' - 4 + 4} \sin x$$

$$= e^x \frac{1}{D'^2 - 2D' + 1} \sin x$$

$$= e^x \frac{1}{-1 - 2D' + 1} \sin x \quad \begin{array}{l} D'^2 \rightarrow -b^2 = -1^2 \\ = -1 \end{array}$$

$$= e^x \frac{1}{-2D'} \sin x$$

$$= \frac{e^x}{-2} \left[ \frac{1}{D'} \sin x \right]$$

$$= \frac{e^x}{-2} [-\cos x]$$

$$PI = \frac{e^x \cos x}{2} \Rightarrow PI = \frac{x \log(\log x)}{2}$$

The soln. is,

$$y = CF + PI$$

$$y = [A + B \log x] x^2 + \frac{x \cos(\log x)}{2}$$



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