SNS COLLEGE OF TECHNOLOGY,COIMBATORE-35

(AN AUTONOMOUS INSTITUTION)

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

19CST202-DATABASE MANAGEMENT SYSTEM

UNIT-III

Database Design

Topic: Armstrong Axioms of FD's

The term Armstrong axioms refer to the sound and complete set of inference rules or axioms, introduced by William W. Armstrong, that is used to test the logical implication of **functional dependencies**. If F is a set of functional dependencies then the closure of F, denoted as F^+, is the set of all functional dependencies logically implied by F. Armstrong's Axioms are a set of rules, that when applied repeatedly, generates a closure of functional dependencies.

Axioms –

1. **Axiom of reflexivity –**
   If $A$is a set of attributes and $B$is subset of A, then Aholds B. If $B \subseteq A$then $A \rightarrow B$This property is trivial property.
2. **Axiom of augmentation –**
   If $A \rightarrow B$holds and $Y$is attribute set, then $AY \rightarrow BY$also holds. That is adding attributes in dependencies, does not change the basic dependencies. If $A \rightarrow B$, then $AC \rightarrow BC$for any C.
3. **Axiom of transitivity –**
   Same as the transitive rule in algebra, if $A \rightarrow B$holds and $B \rightarrow C$holds, then $A \rightarrow C$also holds. $A \rightarrow B$is called as Afunctionally that determines B. If $X \rightarrow Y$and $Y \rightarrow Z$, then $X \rightarrow Z$

Secondary Rules –

These rules can be derived from the above axioms.

1. **Union –**
   If $A \rightarrow B$ holds and $A \rightarrow C$ holds, then $A \rightarrow BC$ holds. If $X \rightarrow Y$ and $X \rightarrow Z$ then $X \rightarrow YZ$
2. **Composition –**
   If $A \rightarrow B$ and $X \rightarrow Y$ holds, then $AX \rightarrow BY$ holds.
3. **Decomposition –**
   If $A \rightarrow BC$ holds then $A \rightarrow B$ and $A \rightarrow C$ hold. If $X \rightarrow YZ$ then $X \rightarrow Y$ and $X \rightarrow Z$
4. **Pseudo Transitivity –**
   If $A \rightarrow B$ holds and $BC \rightarrow D$ holds, then $AC \rightarrow D$ holds. If $X \rightarrow Y$ and $YZ \rightarrow W$ then $XZ \rightarrow W$.

Why armstrong axioms refer to the Sound and Complete ?

By sound, we mean that given a set of functional dependencies F specified on a relation schema R, any dependency that we can infer from F by using the primary rules of Armstrong axioms holds in every relation state r of R that satisfies the dependencies in F.

Secondary Rules

1. **Decomposition**

If A→BC, then A→B and A→C

Proof:

A→BC (given)________________ (i)

BC→B (reflexivity)_____________ (ii)

A→B (transitivity from i and ii)

2. **Composition**

If $A \rightarrow B$ and $C \rightarrow D$ then $AC \rightarrow BD$

Proof

$A \rightarrow B$_________(i)

$C \rightarrow D$_________(ii)

$AC \rightarrow BC$________(iii) (Augmentation of i and C)

$AC \rightarrow B$________(iv) Decomposition of iii)

$AC \rightarrow AD$_______(v) (Augmentation of ii and A)

$AC \rightarrow D$___________(vi) (Decomposition of v)

$AC \rightarrow BD$________ (Union iv and vi)

Union (Notation)

If $A \rightarrow B$ and $A \rightarrow C$ then $A \rightarrow BC$

Proof

$A \rightarrow B$________(i) (given)

$A \rightarrow C$________(ii) (given)

$A \rightarrow AC$_______(iii) (Augmentation of ii and A)

$AC \rightarrow BC$______(iv) (Augmentation of i and C)

$A \rightarrow BC$________(transitivity of iii and ii)

4. **Pseudo transitivity**

If $A \rightarrow B$ and $BC \rightarrow D$ then $AC \rightarrow D$

Proof

$A \rightarrow B$__________(i) (Given)

$BC \rightarrow D$________(ii) (Given)

$AC \rightarrow BC$_______(iii) (Augmentation of i and C)

$AC \rightarrow D$__________(Transitivity of iii and ii)

Self-determination

$A \rightarrow A$ for any given A.

This rule directly follows the Axiom of Reflexivity.

6. **Extensivity**

Extensivity is a particular case of augmentation where C=A

If $A \rightarrow B$, then $A \rightarrow AB$

In the sense that augmentation can be proven from extensivity and other axioms, extensivity can replace augmentation as an axiom.

Proof

$AC \rightarrow A$____(i)

$A \rightarrow B$_________(ii)

$AC \rightarrow B$_________(iii) (Transitivity of i and ii)

$AC \rightarrow ABC$________(iv) (Extensivity of iii)

$ABC \rightarrow BC$_______(v) (Reflexivity)

$AC \rightarrow BC$ (Transitivity of iv and v)