Root locus:

→ A unity feedback control system has an open

1). loop transfer function $G(s) = \dfrac{k}{s(s^2+4s+13)}$

sketch the root locus.

$$s(s^2+4s+13) = 0.$$

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{4^2-(4\times13)}}{2}$$

Poles are lying at $\left.\begin{array}{l} s = -2 \pm j3. \\ s = 0. \end{array}\right\}$ —— (1)

⇒. Poles :- $P_1 = 0$, $P_2 = -2+j3$, $P_3 = -2-j3$

2) To find the root locus on real axis:

* one pole on real axis at origin.
* Total no. of real poles & zeros is one. ⇒ odd no.

3) To find angles of asymptotes & centroid:

$$\text{Angles of asymptotes} = \frac{\pm 180^\circ(2q+1)}{n-m} \quad ; q = 0, 1, \ldots n-m$$

Poles $\boxed{n=3}$ $\quad q = 0, 1, 2, 3.$

Zeros $\boxed{m=0}$ —— (2)

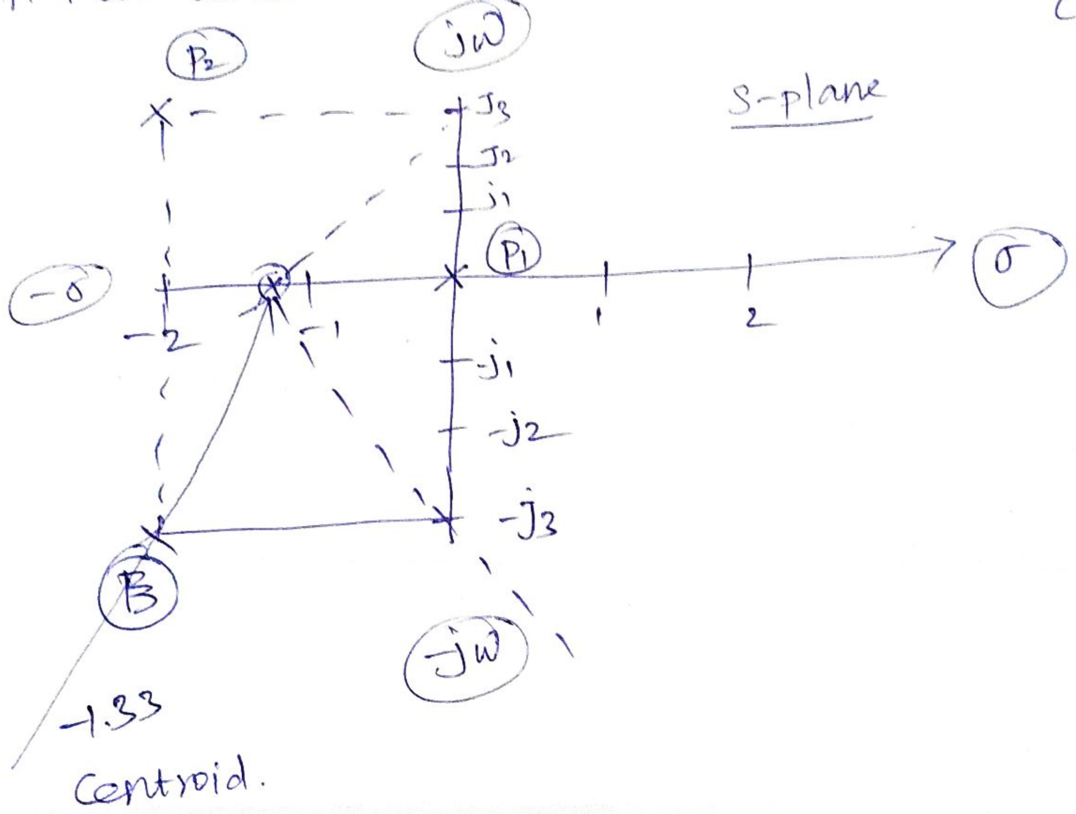
when (q)	Angles	
$q=0$	$\pm 180^\circ/3 = \pm 60^\circ$ (1)	
1	$\pm \frac{180^\circ \times 3}{3} = \pm 180^\circ$ (3)	
2	$\pm \frac{180^\circ \times 5}{3} = \pm 300^\circ$ (5)	$\frac{300}{5} = 60^\circ$
3	$\pm \frac{180^\circ \times 7}{3} = \pm 420^\circ.$ (7)	$\frac{420}{7} = 60^\circ$

Centroid = $\dfrac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$

① ⇒ $= \dfrac{(0-2+j3-2-j3)-0}{3}$

$= -4/3 = -1.33$ ——— ③

* Root locus on real axis & location of poles & centroid.



4) To find break away & breaking points.

Closed loop tran. fn. $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)}$

$$= \dfrac{\dfrac{K}{s(s^2+4s+3)}}{1+\dfrac{K}{s(s^2+4s+3)}} = \dfrac{K}{s(s^2+4s+3)+K}$$

Characteristic eqn. $s(s^2+4s+13)+K=0$.

$s^3+4s^2+13s+K=0$

(2) R.L

$$k = -s^3 - 4s^2 - 13s.$$

$$\frac{dk}{ds} = -3s^2 - 8s - 13$$

$$\frac{dk}{ds} = 0 = 3s^2 + 8s + 13.$$

$$s = \frac{-8 \pm \sqrt{8^2 - 4 \times 13 \times 3}}{2 \times 3}$$

$$\boxed{s = -1.33 \pm j1.6.}$$ — (4)

5) <u>To find angle of departure</u>

→ Let angles of vectors be θ_1 & θ_2

$$\theta_1 = 180^\circ - \underbrace{\tan^{-1}(3/2)}_{56.3} = 123.7^\circ$$

$\left(\tan^{-1} b/a \quad -2 \pm j3\right)$

$$\boxed{\theta_2 = 90^\circ} \text{ — ?}$$

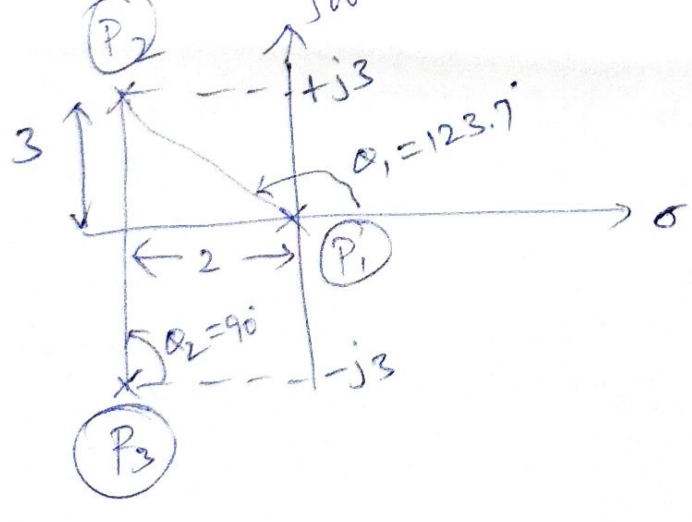
— (5)

Angle of depart from Complex pole: $P_2 = 180^\circ - (\theta_1 + \theta_2)$

$$= 180^\circ - (123.7 + 90^\circ)$$

$$\boxed{P_2 = 33.7.} \qquad \boxed{P_3 = -33.7} \text{ — ?}$$

— (6)



6) To find the crossing point on imaginary axis

char. eqn. $\Rightarrow$ $s^3 + 4s^2 + 13s + k = 0.$

$\boxed{s = j\omega}$ $\quad (j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + k = 0.$

$-j\omega^3 - 4\omega^2 + 13j\omega + k = 0.$

(i) equate imaginary part to zero.

$-\omega^3 + 13\omega = 0.$

$-\omega^2 = -13$ — (7)

$\omega^2 = 13 \Rightarrow \omega = \pm\sqrt{13}$

$\boxed{\omega = \pm 3.6.}$ — (9)

(ii) equate real part to zero.

$-j\omega^3 - 4\omega^2 + 13j\omega + k = 0$

$-4\omega^2 + k = 0$

$k = 4\omega^2 = 4 \times 13 = 52.$ — (8)