Routh Hurwitz

1) By Routh Stability criterion determine the stability of the system represented by the Characteristics eqn. $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$. Comment on the location of roots of Char. eqn.

Given.

Char. eqn. $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$

few coefficients are (−ive) ⇒ Roots lie RHS - splane

So system is unstable.

Roots ⇒ 5 (∵ order ⇒ 5)

					Rows
s^5	:	9	10	−9	— 1
s^4	:	−20	−1	−10	— 2
s^3	:	$\frac{-(20\times 10)+9}{-20}$ = 9.55	$\frac{-20\times -9 - 90}{-20}$ = −13.5	—	
s^2	:	$\frac{9.55\times -1 - (-13.5\times -20)}{9.55}$ = −29.3	−10	$\frac{9.55\times -10 - 0}{9.55}$	
s^1	:	$\frac{-29.3\times -13.5 - (-10\times 9.55)}{-29.3}$ = −16.8			
s^0	:	−10 $\frac{-16.8\times -10}{-16.8}$			

from 1st column. → sign changes

Roots	→	splane-lying
3	→	RHS.
2 remain	→	LHS.

Result:-

→ System is unstable ✓

2) The characteristic polynomial of a system is $s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$. Determine the location of roots on s-plane & hence stability of the system.

<u>Given</u>:- $s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$.

Roots ⇒ 7

s^7 : 1   24   24   23 — (1)

~~s^6 : 9   24   24   15~~

$\div 3$ s^6 : 3   8   8   5 — (2)

~~s^5 : 21.33   21.33   21.33~~ $\frac{24 \times 3 - 8}{3} = 21.33$

$\div 21.33$ s^5 : 1   1   1   0 — (3)

~~s^4 : 5   5   5~~ $\frac{1 \times 8 - 3}{1}$

$\div 1$ s^4 : 1   1   1 — (4)

s^3 : 0   0 — (5) $\frac{1 \times 1 - 1 \times 1}{1}$

Auxiliary polynomial

$$A = s^4 + s^2 + 1$$

$$\frac{dA}{ds} = 4s^3 + 2s$$

s^3 : 4   2

$\div 2$ s^3 : 2   1

s^4 : 1 1 1

s^3 : 2 1 —— (5)

s^2 : $\frac{2\times1-1\times1}{2}$ $\frac{2\times1-(0\times1)}{2}$ —— (6)

0.5 1

s^1 : $\frac{0.5\times1-1\times2}{0.5}$ —— (7)

-3 —

s^0 : $\frac{-3\times1\times(0\times0.5)}{-3}$ —— (8)

1

1^{st} column → 2 sign changes.

2 roots → RHS - s-plane.

5 " → LHS

Row of all zero's → Imaginary axis.

To find roots. $s^4 + s^2 + 1 = 0.$

$s^2 = x$ $x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1-4}}{2}$

$s = \pm\sqrt{x}$

$x = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$\pm\sqrt{1}\angle 120^\circ/2 = \pm\sqrt{1\angle 120^\circ}$

(or)

$= 1\angle 120^\circ$ (or) $1\angle -120^\circ$

$\pm\sqrt{1}\angle -120^\circ/2$ $\pm\sqrt{1\angle -120^\circ}$

Result:
1. unstable
2. 2 - LHS
5. RHS

$\pm1\sqrt{60^\circ}$ (or) $\pm1\sqrt{-60^\circ}$

$\pm(0.5 + j0.866)$ $\pm 0.5 - j0.866)$