SNS COLLEGE OF TECHNOLOGY

Coimbatore-27
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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

19ECT212 – CONTROL SYSTEMS

II YEAR/ IV SEMESTER

UNIT IV – STABILITY ANALYSIS

TOPIC 4.7,8 APPLICATION OF ROOT LOCUS DIAGRAM,NYQUIST STABILITY CRITERION

OUTLINE

- REVIEW ABOUT PREVIOUS CLASS
- APPLICATION OF ROOT LOCUS DIAGRAM
- ACTIVITY
- NYQUIST STABILITY CRITERION
- NYQUIST STABILITY CRITERION-PHASE,GAIN- MARGIN,CROSS OVER FREQ.
- NYQUIST STABILITY CRITERION-PROBLEM
- SUMMARY

APPLICATION OF ROOT LOCUS DIAGRAM

1.the stability of the system,

2.the effect of variation of parameters so that the limits of these para-meters can be determined for stable operation.

The method is particularly suitable in the design stage of the system when there is some latitude in the choice of system parameters.

APPLICATION OF ROOT LOCUS DIAGRAM

The Root Locus Plot technique can be applied to determine the dynamic response of the system.
This method associates itself with the transient response of the system and is particularly useful in the investigation of stability characteristics of the system.

It can also be used to determine the stability boundaries of the system. Selection of suitable parameters may be made using the root locus analysis.

It is a well established fact that the condition of roots of the characteristic equation governs the transient behavior and hence the stability of the system. In the Root Locus Plot technique which is essentially a graphical procedure, the locus of the roots on the s-plane is afforded by the variation of the suitable parameters.

APPLICATIONS OF ROOT LOCUS DIAGRAM

•The Root Locus Plot are drawn in the complex plane, as the roots may be real or complex.
•In this technique also the open loop transfer function may be used to simplify the procedure

• The plots drawn and the information obtained from these plots pertain to a closed loop system. The plots clearly illustrate the effects of variation of parameters on system stability and the nature of response.

•The branch of a root locus shows all the possible values of one root when a pa-rameter is varied. All the separate branches of the Root Locus Plot give the effect of variation of parameters.

The Root Locus Plot technique is essentially a time domain technique.

•Frequency response data can be obtained from the Root Locus Plot. Using this technique the open loop poles and zeros can be modified to satisfy the requirements to be met by closed loop poles and zeros.

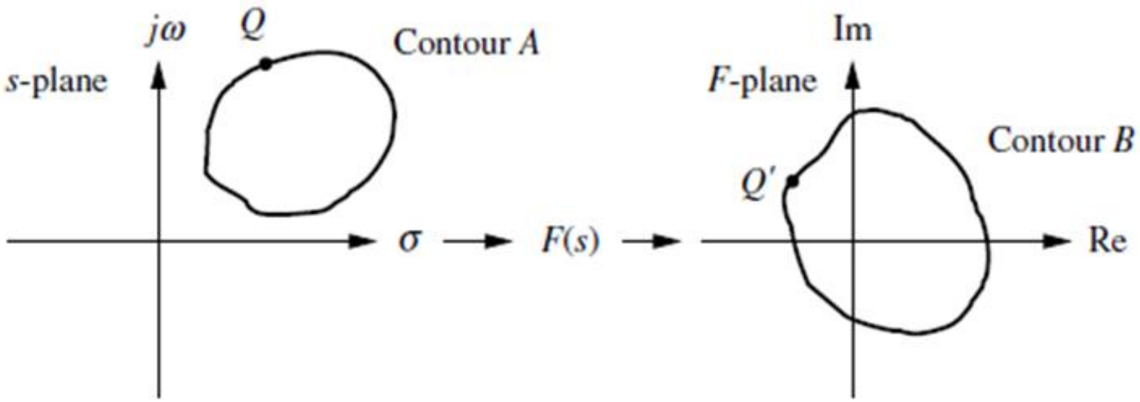
•It may be noted here that a slight change in the pole zero configuration may cause a significant changes in the root locus configuration.

NYQUIST STABILITY CRITERION

The Nyquist stability criterion determines the stability of a closed-loop system from its open-loop frequency response and open-loop poles.

Mathematical background:

If we take a complex number s = x+jy on the s-plane and substitute it into a function, F(s), another complex number results. This process is called mapping. The collection of points, called a contour.

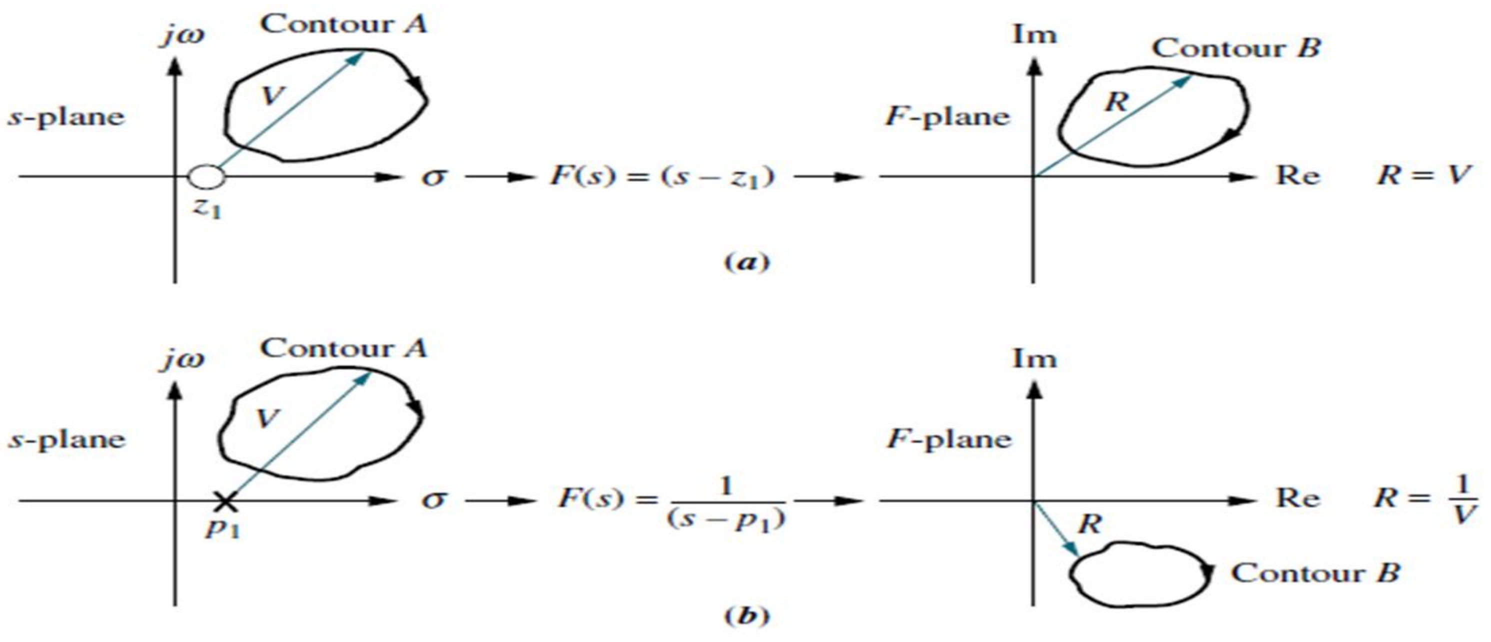


Mapping contour A through function $F(s)$ to contour B

NYQUIST STABILITY CRITERION

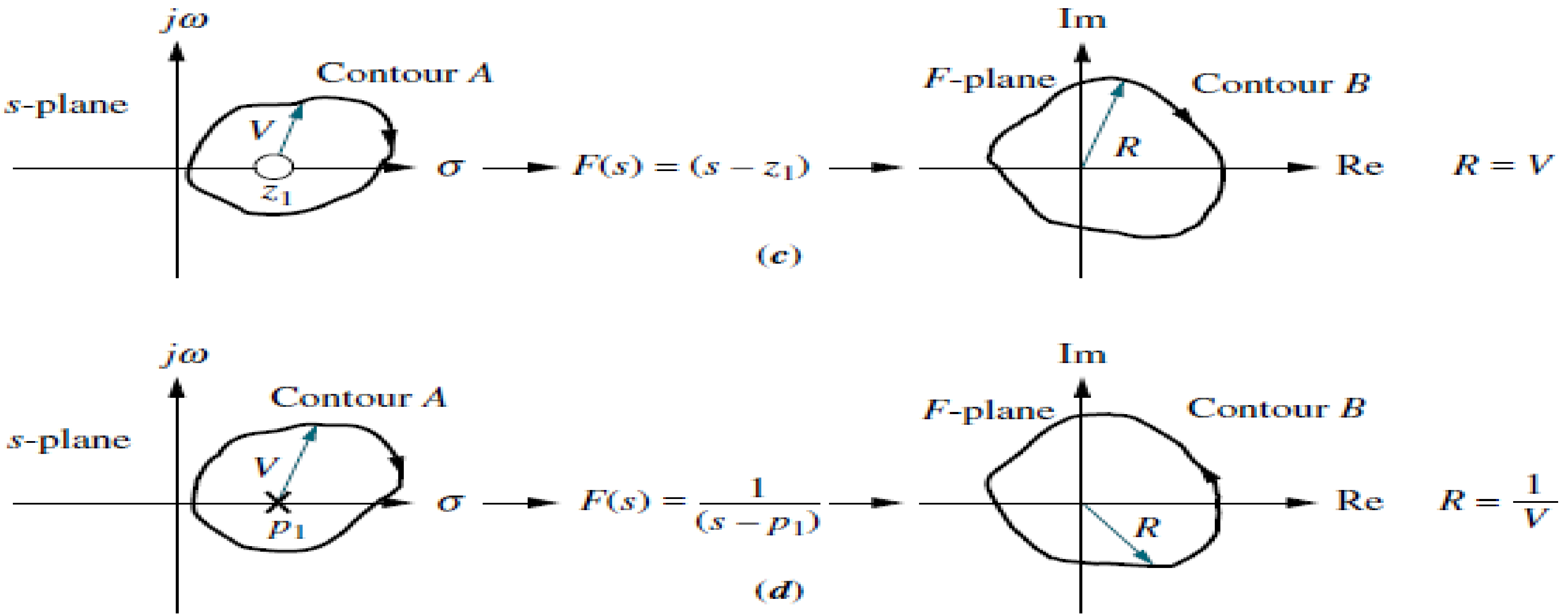
Contour A can be mapped through $F(s)$ into contour B by substituting each point of contour A into the function $F(s)$ and plotting the resulting complex numbers. For example, point Q in Figure 10.21 maps into point Q' through the function $F(s)$.

If we assume a clockwise direction for mapping the points on contour **A**, the contour **B** maps a clockwise direction if **F(s)** has just zeros or has just poles that are not encircled by the contour.



NYQUIST STABILITY CRITERION

The **contour B** maps in a counter clockwise direction if **F(s)** has just poles that are encircled by the contour, Also, you should verify that, if the pole or zero of **F(s)** is enclosed by **contour A**, the mapping encircles the origin.



NYQUIST STABILITY CRITERION

The Nyquist stability test is obtained by applying the Cauchy principle of argument to the complex function.

Cauchy's Principle of Argument

Let F(s) be an analytic function in a closed region of the complex plane given in Figure except at a finite number of points (namely, the poles of F(s)).

It is also assumed that F(s) is analytic at every point on the contour. Then, as travels around the contour in the -plane in the clockwise direction, the function F(s) encircles the origin inthe -plane in the same direction N times (see Figure, with given by

$$N = Z - P$$

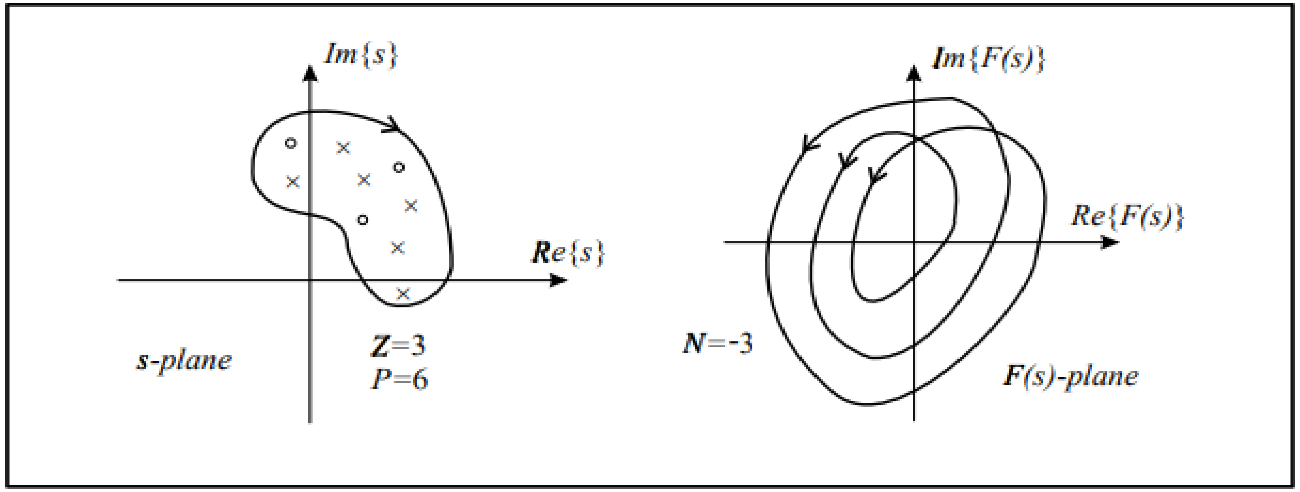
where Z and P stand for the number of zeros and poles (including their multiplicities) of the function $F(s)$ inside the contour.

NYQUIST STABILITY CRITERION

The above result can be also written as

$$\arg\{F(s)\} = (Z - P)2\pi = 2\pi N$$

which justifies the terminology used, "the principle of argument".



Cauchy's principle of argument

NYQUIST STABILITY CRITERION

Nyquist Plot

The Nyquist plot is a polar plot of the function $D(s) = 1+G(s)H(s)$ when s travels around the contour given in Figure 4.7.

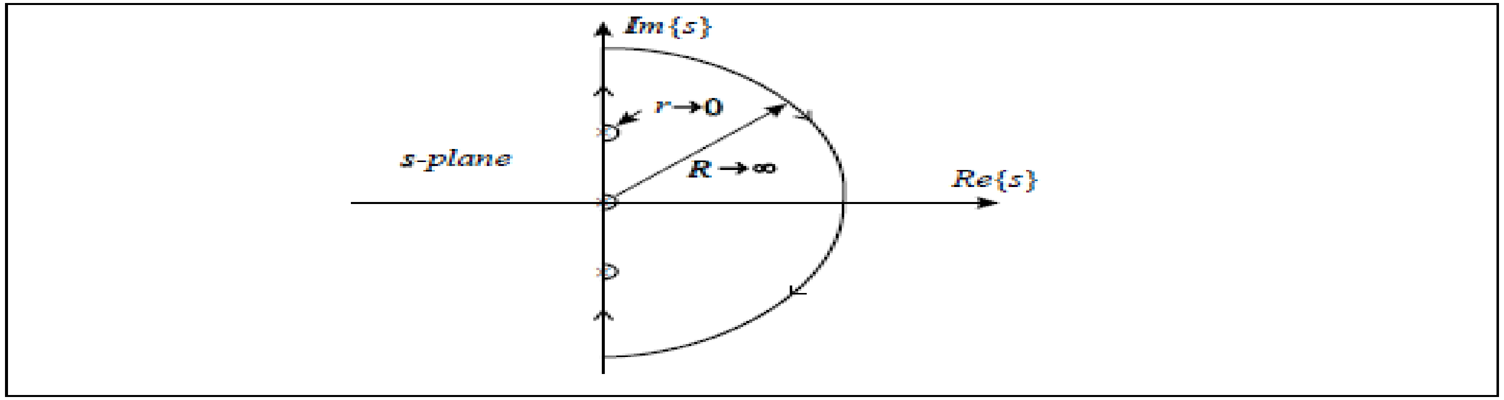


Figure 4.7: Contour in the s-plane

The contour in this figure covers the whole unstable half plane of the complex plane s, $R \longrightarrow \infty$. Since the function $D(s)$, according to Cauchy's principle of argument, must be analytic at every point on the

NYQUIST STABILITY CRITERION

contour, the poles of $D(s)$ on the imaginary axis must be encircled by infinitesimally small semicircles.

Nyquist Stability Criterion

It states that the number of unstable closed-loop poles is equal to the number of unstable open-loop poles plus the number of encirclements of the origin of the Nyquist plot of the complex function $D(s)$.

This can be easily justified by applying Cauchy's principle of argument to the function $D(s)$ with the s-plane contour given in Figure 4.7. Note that Z and P represent the numbers of zeros and poles, respectively, of $D(s)$ in the unstable part of the complex plane. At the same time, *the zeros of* $D(s)$ *are the closed-loop system poles*, and *the poles of* $D(s)$ *are the open-loop system poles* (closed-loop zeros).

NYQUIST STABILITY CRITERION

The above criterion can be slightly simplified if instead of plotting the function $D(s) = 1 + G(s)H(s)$, we plot only the function $G(s)H(s)$ and count encirclement of the Nyquist plot of $G(s)H(s)$ around the point $(-1, j0)$, so that the modified Nyquist criterion has the following form.

The number of unstable closed-loop poles (Z) is equal to the number of unstable open-loop poles (P) plus the number of encirclements (N) of the point $(-1, j0)$ *of the Nyquist plot of* $G(s)H(s)$, *that is*

$$Z = P + N$$

ACTIVITY

CONNECTIONS….START FROM CONTROL SYSTEMS..U START FROM S..

NYQUIST STABILITY CRITERION

Nyquist plots are the continuation of polar plots for finding the stability of the closed loop control systems by varying ω from −∞ to ∞.

That means, Nyquist plots are used to draw the complete frequency response of the open loop transfer function.

The Nyquist stability criterion works on the **principle of argument**. It states that if there are P poles and Z zeros are enclosed by the ‘s’ plane closed path, then the corresponding $G(s)H(s)$ plane must encircle the origin $P – Z$ times. So, we can write the number of encirclements N as,

$$N=P-Z$$

If the enclosed ‘s’ plane closed path contains only **poles,** then the direction of the encirclement in the $G(s)H(s)$ plane will be opposite to the direction of the enclosed closed path in the ‘s’ plane.

If the enclosed ‘s’ plane closed path contains only **zeros,** then the direction of the encirclement in the $G(s)H(s)$ plane will be in the same direction as that of the enclosed closed path in the ‘s’ plane.

NYQUIST STABILITY CRITERION

Let us now apply the principle of argument to the entire right half of the 's' plane by selecting it as a closed path. This selected path is called the **Nyquist** contour.

We know that the closed loop control system is stable if all the poles of the closed loop transfer function are in the left half of the 's' plane. So, the poles of the closed loop transfer function are nothing but the roots of the characteristic equation.

As the order of the characteristic equation increases, it is difficult to find the roots. So, let us correlate these roots of the characteristic equation as follows.

The Poles of the characteristic equation are same as that of the poles of the open loop transfer function.

The zeros of the characteristic equation are same as that of the poles of the closed loop transfer function.

NYQUIST STABILITY CRITERION

We know that the open loop control system is stable if there is no open loop pole in the the right half of the ‘s’ plane.

i.e.,$P=0 \Rightarrow N=-Z$

We know that the closed loop control system is stable if there is no closed loop pole in the right half of the ‘s’ plane.

i.e.,$Z=0 \Rightarrow N=P$

Nyquist stability criterion states the number of encirclements about the critical point (1+j0) must be equal to the poles of characteristic equation, which is nothing but the poles of the open loop transfer function in the right half of the ‘s’ plane. The shift in origin to (1+j0) gives the characteristic equation plane.

RULES FOR DRAWING NYQUIST PLOTS

Follow these rules for plotting the Nyquist plots.

- Locate the poles and zeros of open loop transfer function $G(s)H(s)$ in ‘s’ plane.
- Draw the polar plot by varying ω from zero to infinity. If pole or zero present at s = 0, then varying ω from 0+ to infinity for drawing polar plot.
- Draw the mirror image of above polar plot for values of ω ranging from $-\infty$ to zero (0^- if any pole or zero present at s=0).
- The number of infinite radius half circles will be equal to the number of poles or zeros at origin. The infinite radius half circle will start at the point where the mirror image of the polar plot ends. And this infinite radius half circle will end at the point where the polar plot starts.

After drawing the Nyquist plot, we can find the stability of the closed loop control system using the Nyquist stability criterion. If the critical point (-1+j0) lies outside the encirclement, then the closed loop control system is absolutely stable.

STABILITY ANALYSIS USING NYQUIST PLOTS

From the Nyquist plots, we can identify whether the control system is stable, marginally stable or unstable based on the values of these parameters.

- Gain cross over frequency and phase cross over frequency
- Gain margin and phase margin

Phase Cross over Frequency

The frequency at which the Nyquist plot intersects the negative real axis (phase angle is 180^0) is known as the **phase cross over frequency**. It is denoted by ω_{pc}.

STABILITY ANALYSIS USING NYQUIST PLOTS

Gain Cross over Frequency

The frequency at which the Nyquist plot is having the magnitude of one is known as the **gain cross over frequency**. It is denoted by ω_{gc}.

The stability of the control system based on the relation between phase cross over frequency and gain cross over frequency is listed below.

- If the phase cross over frequency ω_{pc} is greater than the gain cross over frequency ω_{gc}, then the control system is **stable**.
- If the phase cross over frequency ω_{pc} is equal to the gain cross over frequency ω_{gc}, then the control system is **marginally stable**.
- If phase cross over frequency ω_{pc} is less than gain cross over frequency ω_{gc}, then the control system is **unstable**.

STABILITY ANALYSIS USING NYQUIST PLOTS

Gain Margin

The gain margin GM is equal to the reciprocal of the magnitude of the Nyquist plot at the phase cross over frequency.

$$GM=\frac{1}{M_{pc}}$$

Where, M_{pc} is the magnitude in normal scale at the phase cross over frequency.

Phase Margin

The phase margin PM is equal to the sum of 180^0 and the phase angle at the gain cross over frequency.

$$PM=180^0+\phi_{gc}$$

Where, ϕ_{gc} is the phase angle at the gain cross over frequency.

The stability of the control system based on the relation between the gain margin and the phase margin is listed below.

- If the gain margin GM is greater than one and the phase margin PM is positive, then the control system is **stable**.
- If the gain margin GM is equal to one and the phase margin PM is zero degrees, then the control system is **marginally stable**.
- If the gain margin GM is less than one and / or the phase margin PM is negative, then the control system is **unstable**.

Problem: Sketch the NYQUIST plot for the following transfer function and check the stability of the system.

$$GH(s) = \frac{1}{s(s+1)}$$

There is one pole at the origin.

Representing $G(s)H(s)$ in the frequency response form $G(j\omega)H(j\omega)$ by replacing **s = jω**:

$$GH(j\omega) = \frac{1}{j\omega(j\omega+1)}$$

The **magnitude** of $GH(j\omega)$ i.e., $|GH(j\omega)|$, is obtained as;

$$|GH(j\omega)| = \frac{1}{\omega\sqrt{\omega^2+1}}$$

The **phase** of $GH(j\omega)$ denoted by, φ, is obtained as;

$$\varphi = \angle GH(0) = -90^\circ - \tan^{-1}\left(\frac{\omega}{1}\right)$$

Point 1: The start of plot where ω = 0

$$|GH(0)| = \frac{1}{0\sqrt{0+1}} = \infty$$

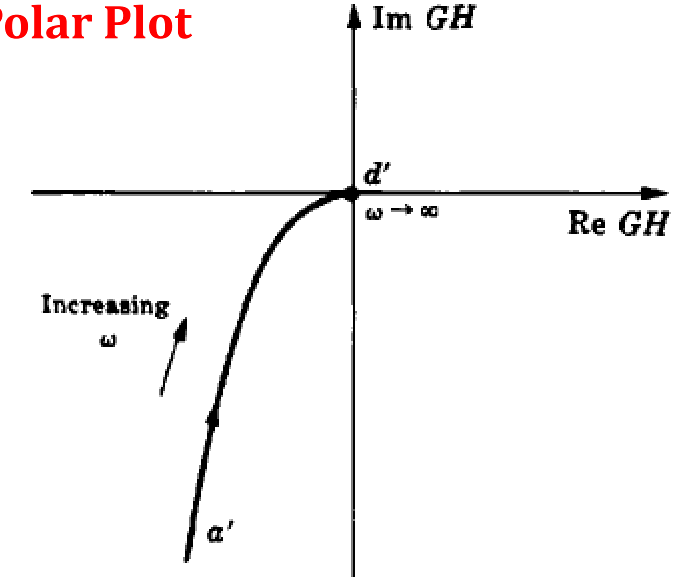
$$\varphi = \angle GH(0) = -90° - \tan^{-1}\frac{0}{1} = -90^{\circ}$$

Point 2: The end of plot where ω = ∞

$$|GH(\infty)| = \frac{1}{\infty\sqrt{\infty+1}} = 0$$

$$\varphi = \angle GH(\infty) = -90 - \tan^{-1}\frac{\infty}{p} = -90° - 90° = -180°$$

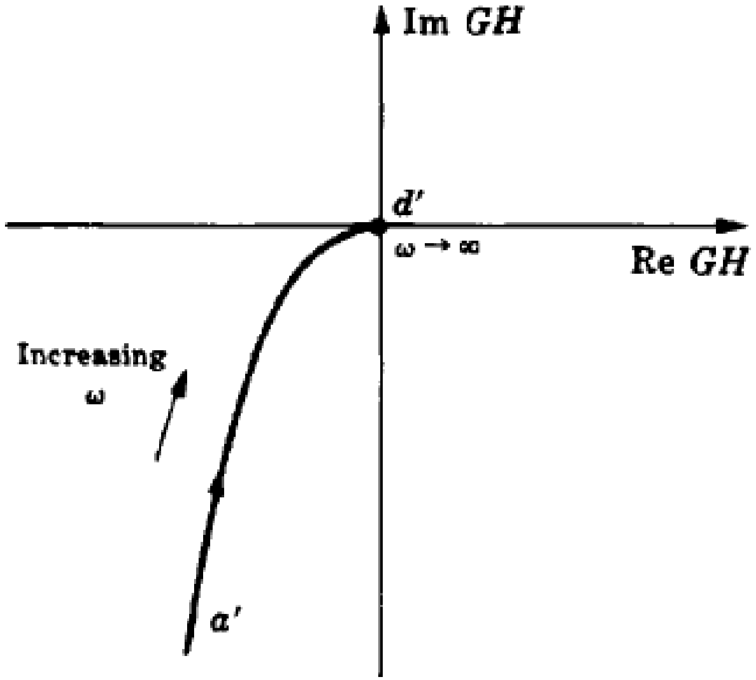
Polar Plot



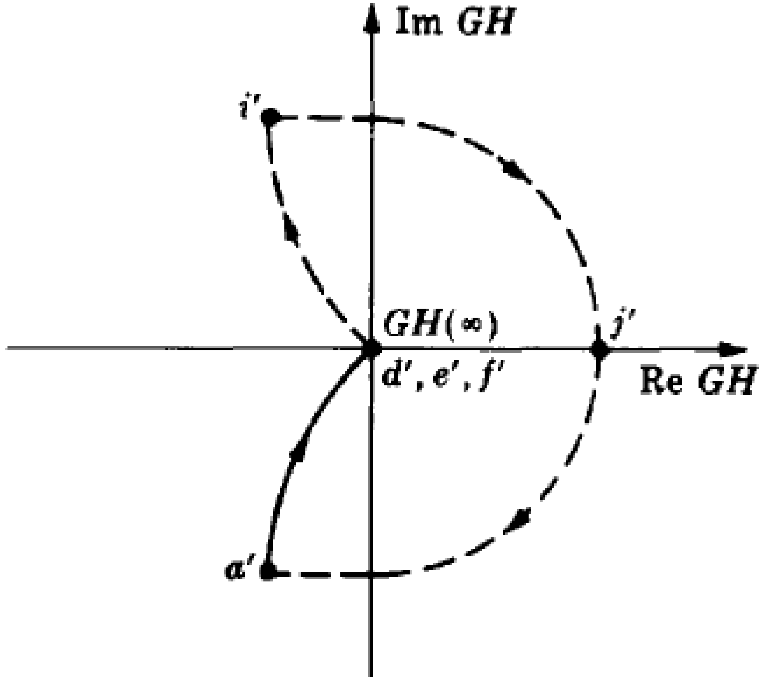
	|GH(jω)|	∠GH(jω)
ω = 0	∞	-90°
ω = ∞	0	-180°

Problem: Sketch the NYQUIST plot for the following transfer function and check the stability of the system.

Nyquist or Polar Plot

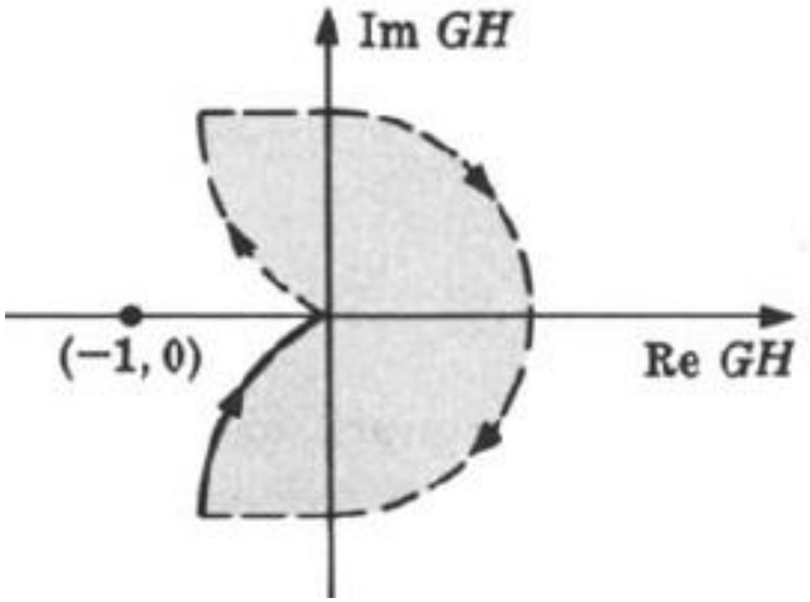


Nyquist Diagram



Problem: Sketch the NYQUIST plot for the following transfer function and check the stability of the system.

- The region to the right of the contour has been shaded.
- Clearly, the **(-1,0)** point is not in the shaded region; therefore it is not enclosed by the contour and so $N \leq 0$.
- The poles of $GH(s)$ are at **s = 0** and **s = -1**, neither of which are in the right-hand-plane RHP; hence $P_o = 0$. Thus $N = -P_o = 0$, and the system is absolutely stable.



STABILITY CONDITIONS

In examining the stability of linear control systems using the Nyquist stability criterion, we see that three possibilities can occur:

1. There is no encirclement of the $-1+j0$ point.This implies that the system is stable if there are no poles of $G(s)H(s)$ in the right-half s plane; otherwise, the system is unstable.

2. There are one or more counterclockwise encirclements of the $-1+j0$ point. In this case the system is stable if the number of counterclockwise encirclements is the same as the number of poles of $G(s)H(s)$ in the right-half s plane; otherwise, the system is unstable.

3. There are one or more clockwise encirclements of the $-1+j0$ point. In this case the system is unstable.

SUMMARY