SNS COLLEGE OF TECHNOLOGY

COIMBATORE-35

DEPARTMENT OF MECHATRONICS ENGINEERING

19MCT203 MECHANICS OF MACHINES

UNIT – I

KINEMATICS OF MECHANICS

Kinematic Link or Element

Each part of a machine, which moves relative to some other part, is known as a ***kinematic link*** (or simply link) or ***element***. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another. For example, in a reciprocating steam engine, as shown in Fig. 1, piston, piston rod and crosshead constitute one link; connecting rod with big and small end bearings constitute a second link; crank, crank shaft and flywheel a third link and the cylinder, engine frame and main bearings a fourth link.

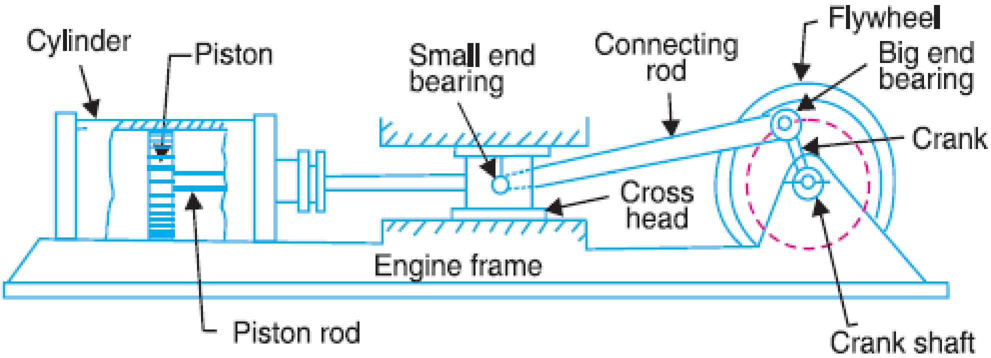


Fig. 1. Reciprocating steam engine.

A link or element need not to be a rigid body, but it must be a ***resistant body***. A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation. Thus a link should have the following two characteristics:

1. It should have relative motion, and

2. It must be a resistant body.

Types of Links

In order to transmit motion, the driver and the follower may be connected by the following three types of links:

1. *Rigid link.* A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. *Flexible link.* A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. *Fluid link*. A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

Structure

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Difference Between a Machine and a Structure

The following differences between a machine and a structure are important from the subject point of view:

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.

2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.

3. The links of a machine may transmit both power and motion, while the members of a structure transmit forces only.

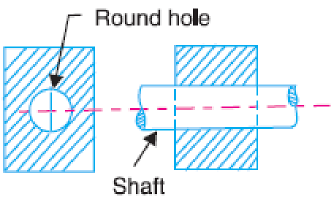
Kinematic Pair

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (*i.e.* in a definite direction), the pair is known as ***kinematic pair***.

Types of Constrained Motions

Following are the three types of constrained motions:

1. *Completely constrained motion*. When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (*i.e.* it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, as shown in Fig. 2.



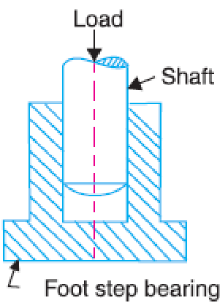
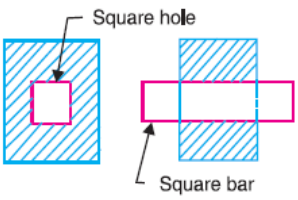


Fig. 2. Square bar in a square hole. **Fig. 3.** Shaft with collars in a circular hole.

The motion of a square bar in a square hole, as shown in Fig.2, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig.3, are also examples of completely constrained motion.

2. *Incompletely constrained motion*. When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig.4, is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.



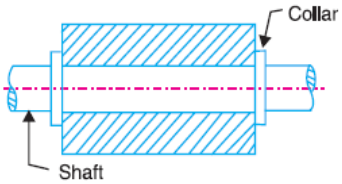


Fig. 4. Shaft in a circular hole. **Fig. 5.** Shaft in a foot step bearing.

3. *Successfully constrained motion.* When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig.5. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

Classification of Kinematic Pairs

The kinematic pairs may be classified according to the following considerations:

1. ***According to the type of relative motion between the elements.*** The kinematic pairs according to type of relative motion between the elements may be classified as discussed below:

(*a*) ***Sliding pair.*** When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a completely constrained motion.

(*b*) ***Turning pair***. When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

(*c*) ***Rolling pair***. When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

(*d*) ***Screw pair.*** When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut are examples of a screw pair.

(*e*) ***Spherical pair***. When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.

2. ***According to the type of contact between the elements***. The kinematic pairs according to the type of contact between the elements may be classified as discussed below:

(*a*) ***Lower pair***. When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

(*b*) ***Higher pair***. When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

3. ***According to the type of closure***. The kinematic pairs according to the type of closure between the elements may be classified as discussed below:

(*a*) *Self closed pair*. When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self-closed pair. The lower pairs are self-closed pair.
(*b*) *Force - closed pair*. When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

Kinematic Chain

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (*i.e.* completely or successfully constrained motion), it is called a ***kinematic chain.*** In other words, a kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained. For example, the crankshaft of an engine forms a kinematic pair with the bearings which are fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms a fourth pair. The total combination of these links is a kinematic chain.

If each link is assumed to form two pairs with two adjacent links, then the relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation :

$l = 2p - 4$ . . . **(*i*)**

Since in a kinematic chain each link forms a part of two pairs, therefore there will be as many links as the number of pairs.

Another relation between the number of links (l) and the number of joints (j) which constitute a kinematic chain is given by the expression :

$j = 3/2l - 2$ ...**(*ii*)**

The equations **(*i*)** and **(*ii*)** are applicable only to kinematic chains, in which lower pairs are used. These equations may also be applied to kinematic chains, in which higher pairs are used. In that case each higher pair may be taken as equivalent to two lower pairs with an additional element or link.

Let us apply the above equations to the following cases to determine whether each of them is a kinematic chain or not.

1. Consider the arrangement of three links *AB*, *BC* and *CA* with pin joints at *A*, *B* and *C* as shown in Fig.6. In this case,

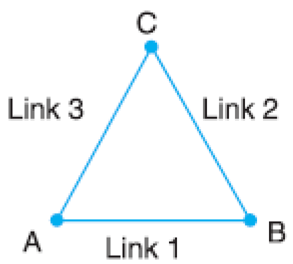
Number of links, $l = 3$

Number of pairs, $p = 3$

and number of joints, $j = 3$

From equation **(*i*)**, $l = 2p - 4$

or $3 = 2 \times 3 - 4 = 2$



i.e. L.H.S. > R.H.S.

Now from equation ***(ii)*,**

$j = 3/2l - 2$ or 3 = 3/2X3-2 = 2.5

L.H.S. > R.H.S.

Fig. 6. Arrangement of three links

Since the arrangement of three links, as shown in Fig.6, does not satisfy the equations **(*i*)** and **(*ii*)** and the left hand side is greater than the right hand side, therefore it is not a kinematic chain and hence no relative motion is possible. Such type of chain is called ***locked chain*** and forms a rigid frame or structure which is used in bridges and trusses.

2. Consider the arrangement of four links *AB*, *BC*, *CD* and *DA* as shown in Fig. 7. In this case

$l = 4$, $p = 4$, and $j = 4$

From equation **(*i*),** $l = 2p - 4$

$4 = 2 \times 4 - 4 = 4$

i.e. L.H.S. = R.H.S.

From equation **(*ii*),**

$j = 3/2l - 2$

4=3/2X4-2=4

i.e. L.H.S. = R.H.S.

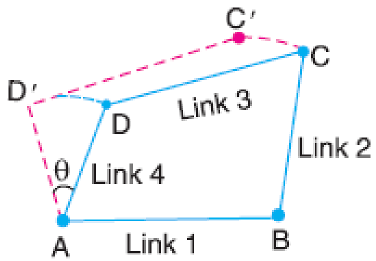


Fig. 7. Arrangement of four links.

Since the arrangement of four links, as shown in Fig. 5.7, satisfy the equations **(*i*)** and **(*ii*),** therefore it is a ***kinematic chain of one degree of freedom***.

A chain in which a single link such as *AD* in Fig.7 is sufficient to define the position of all other links, it is then called a kinematic chain of one degree of freedom.

A little consideration will show that in Fig.7, if a definite displacement (say Θ) is given to the link *AD*, keeping the link *AB* fixed, then the resulting displacements of the remaining two links *BC* and *CD* are also perfectly definite. Thus we see that in a four bar chain, the relative motion is completely constrained. Hence it may be called as a ***constrained kinematic chain***, and it is the basis of all machines.

3. Consider an arrangement of five links, as shown in Fig. 8. In this case,

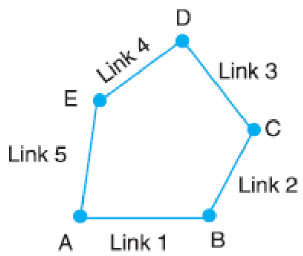
$l = 5$, $p = 5$, and $j = 5$

From equation **(*i*),**

$l = 2p - 4$ or $5 = 2 \times 5 - 4 = 6$

i.e. L.H.S. < R.H.S.

From equation **(*ii*),**



j = 3/2l – 2 or
5 = 3/2X5-2=5.5
i.e. L.H.S. < R.H.S.

Fig. 8. Arrangement of five links

Since the arrangement of five links, as shown in Fig.8 does not satisfy the equations and left hand side is less than right hand side, therefore it is not a kinematic chain. Such a type of chain is called ***unconstrained chain*** *i.e.* the relative motion is not completely constrained. This type of chain is of little practical importance.

4. Consider an arrangement of six links, as shown in Fig.9. This chain is formed by adding two more links in such a way that these two links form a pair with the existing links as well as form themselves a pair. In this case

l = 6, p = 5, and j = 7
From equation ***(i)***,
l = 2 p – 4 or 6 = 2 × 5 – 4 = 6
i.e. L.H.S. = R.H.S.
From equation ***(ii)***,
j = 3/2l – 2 or
7 = 3/2X6-2=7
i.e. L.H.S. = R.H.S.

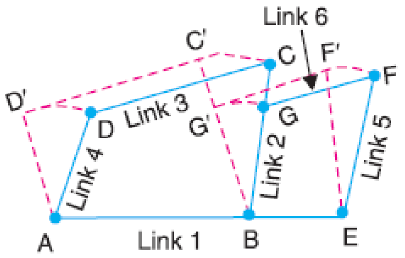


Fig. 9. Arrangement of six links.

Since the arrangement of six links, as shown in Fig.9, satisfies the equations (*i.e.* left hand side is equal to right hand side), therefore it is a kinematic chain.
Note: A chain having more than four links is known as **compound kinematic chain.**

Types of Joints in a Chain

The following types of joints are usually found in a chain:
1. *Binary joint*. When two links are joined at the same connection, the joint is known as binary joint. For example, a chain as shown in Fig.10, has four links and four binary joins at *A*, *B*, *C* and *D*.
In order to determine the nature of chain, *i.e.* whether the chain is a locked chain (or structure) or kinematic chain or unconstrained chain, the following relation between the number of links and the number of binary joints, as given by

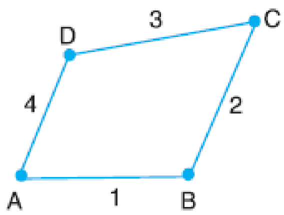


Fig. 10. Kinematic chain with all binary joints.

A.W. Klein, may be used :

j + h/2 = 3/2l – 2 ... ***(i)***
Where *j* = Number of binary joints,
h = Number of higher pairs, and
l = Number of links.
When *h* = 0, the equation ***(i)***, may be written as
j = 3/2l – 2 . . . ***(ii)***
Applying this equation to a chain, as shown in Fig.10, where *l* = 4 and *j* = 4, we have,
4 = 3/2x4-2=4
Since the left hand side is equal to the right hand side, therefore the chain is a kinematic chain or constrained chain.

2. *Ternary joint*. When three links are joined at the same connection, the joint is known as ternary joint. It is equivalent to two binary joints as one of the three links joined carry the pin for the other two links. For example, a chain, as shown in Fig.11, has six links. It has three binary joints at *A*, *B* and *D* and two ternary joints at *C* and *E*. Since one ternary joint is equivalent to two binary joints, therefore equivalent binary joints in a chain, as shown in Fig.11, are
3 + 2 × 2 = 7

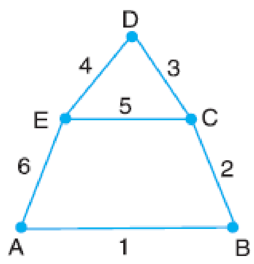


Fig.11. Kinematic chain having binary and ternary joints.

Let us now determine whether this chain is a kinematic chain or not. We know that *l* = 6 and *j* = 7, therefore from equation ***(ii)***,
j = 3/2l-2
7 = 3/2x6-2 = 7
Since left hand side is equal to right hand side, therefore the chain, as shown in Fig. 11, is a kinematic chain or constrained chain.

3. *Quaternary joint.* When four links are joined at the same connection, the joint is called a quaternary joint. It is equivalent to three binary joints. In general, when *l* number of links are joined at the same connection, the joint is equivalent to (*l* – 1) binary joints.
For example consider a chain having eleven links, as shown in Fig. 5.12 (*a*). It has one binary
joint at *D*, four ternary joints at *A*, *B*, *E* and *F*, and two quaternary joints at *C* and *G*. Since one

quaternary joint is equivalent to three binary joints and one ternary joint is equal to two binary joints, therefore total number of binary joints in a chain, as shown in Fig.12 (*a*), are 1 + 4 × 2 + 2 × 3 = 15

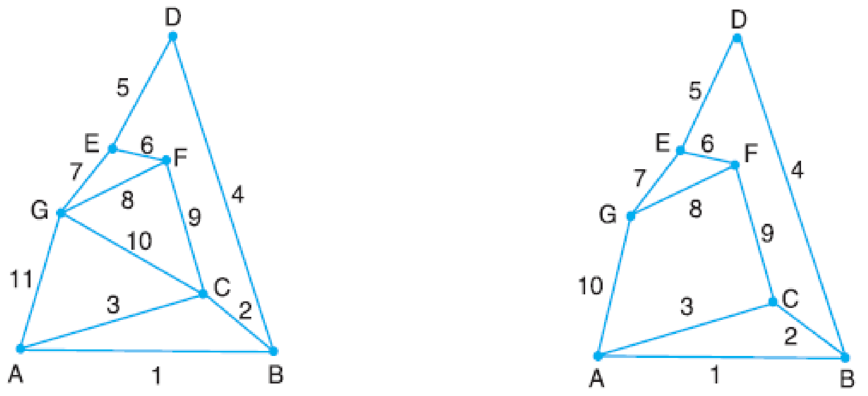


Fig. 12 (*a*) Looked chain having binary, ternary and quaternary joints
(*b*) Kinematic chain having binary and ternary joints

Let us now determine whether the chain, as shown in Fig.12 (*a*), is a kinematic chain or not. We know that l = 11 and j = 15. We know that,
$j = 3/2l-2$ or, 15 = 3/2x11-2 = 14.5, *i.e.*, L.H.S. > R.H.S.

Since the left hand side is greater than right hand side, therefore the chain, as shown in Fig.12 (*a*), is not a kinematic chain. So such a type of chain is called locked chain and forms a rigid frame or structure.

If the link *CG* is removed, as shown in Fig.12 (*b*), it has ten links and has one binary joint at *D* and six ternary joints at *A*, *B*, *C*, *E*, *F* and *G*. Therefore total number of binary joints are 1 + 2 × 6 = 13. We know that
13 = 3/2x10-2 = 13, *i.e.* L.H.S. = R.H.S.
Since left hand side is equal to right hand side, therefore the chain, as shown in Fig.12 (*b*), is a kinematic chain or constrained chain.

Mechanism

When one of the links of a kinematic chain is fixed, the chain is known as ***mechanism***. It may be used for transmitting or transforming motion *e.g.* engine indicators, typewriter etc.
A mechanism with four links is known as ***simple mechanism***, and the mechanism with more than four links is known as ***compound mechanism***. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a ***machine***. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

A little consideration will show that a mechanism may be regarded as a machine in which each part is reduced to the simplest form to transmit the required motion.

Number of Degrees of Freedom for Plane Mechanisms

In the design or analysis of a mechanism, one of the most important concern is the number of degrees of freedom (also called movability) of the mechanism. It is defined as the number of input parameters (usually pair variables) which must be independently controlled in order to bring the mechanism into a useful engineering purpose. It is possible to determine the number of degrees of freedom of a mechanism directly from the number of links and the number and types of joints which it includes.

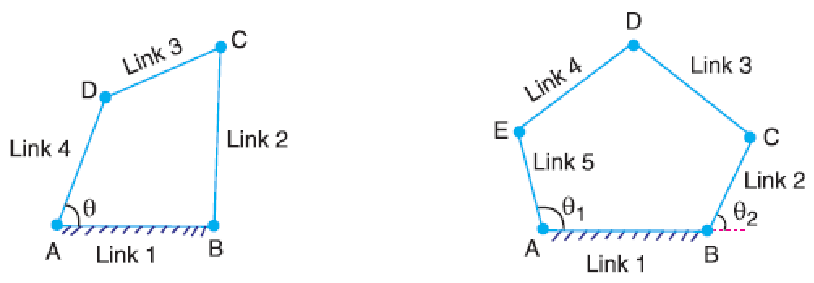


Fig. 13 (*a*) Four bar chain. (*b*) Five bar chain.

Consider a four bar chain, as shown in Fig.13 (*a*). A little consideration will show that only one variable such as ϴ is needed to define the relative positions of all the links. In other words, we say that the number of degrees of freedom of a four bar chain is one. Now, let us consider a five bar chain, as shown in Fig.13 (*b*). In this case two variables such as ϴ1 and ϴ2 are needed to define completely the relative positions of all the links. Thus, we say that the number of degrees of freedom is two.

Now let us consider a plane mechanism with *l* number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be (l – 1) and thus the total number of degrees of freedom will be 3 (l – 1) before they are connected to any other link. In general, a mechanism with *l* number of links connected by *j* number of binary joints or lower pairs (*i.e.* single degree of freedom pairs) and *h* number of higher pairs (*i.e.* two degree of freedom pairs), then the number of degrees of freedom of a mechanism is given by

$n = 3(l - 1) - 2j - h$ ... **(*i*)**

This equation is called Kutzbach criterion for the movability of a mechanism having plane motion.

If there are no two degree of freedom pairs (*i.e.* higher pairs), then $h = 0$. Substituting $h = 0$ in equation (*i*), we have

$n = 3(l - 1) - 2j$ ... **(*ii*)**

Application of Kutzbach Criterion to Plane Mechanisms

We have discussed in the previous article that Kutzbach criterion for determining the number of degrees of freedom or movability (n) of a plane mechanism is $n = 3 (l - 1) - 2 j - h$

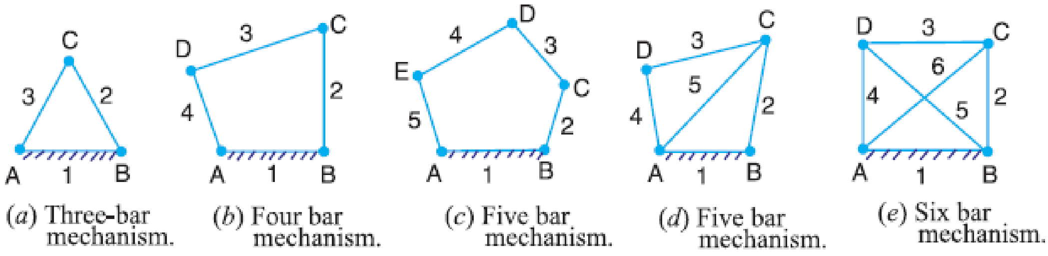


Fig. 14. Plane mechanisms.

The number of degrees of freedom or movability (n) for some simple mechanisms having no higher pair (*i.e.* $h = 0$), as shown in Fig.14, are determined as follows :

1. The mechanism, as shown in Fig.14 (*a*), has three links and three binary joints, *i.e.* $l = 3$ and $j = 3$.

$\therefore n = 3 (3 - 1) - 2 \times 3 = 0$

2. The mechanism, as shown in Fig.14 (*b*), has four links and four binary joints, *i.e.* $l = 4$ and $j = 4$.

$\therefore n = 3 (4 - 1) - 2 \times 4 = 1$

3. The mechanism, as shown in Fig.14 (*c*), has five links and five binary joints, *i.e.* $l = 5$, and $j = 5$.

$\therefore n = 3 (5 - 1) - 2 \times 5 = 2$

4. The mechanism, as shown in Fig.14 (*d*), has five links and six equivalent binary joints (because there are two binary joints at *B* and *D*, and two ternary joints at *A* and *C*), *i.e.* $l = 5$ and $j = 6$.

$\therefore n = 3 (5 - 1) - 2 \times 6 = 0$

5. The mechanism, as shown in Fig.14 (*e*), has six links and eight equivalent binary joints (because there are four ternary joints at *A*, *B*, *C* and *D*), *i.e.* $l = 6$ and $j = 8$.

$\therefore n = 3 (6 - 1) - 2 \times 8 = - 1$

It may be noted that

(*a*) When $n = 0$, then the mechanism forms a structure and no relative motion between the links is possible, as shown in Fig. 14 (*a*) and (*d*).

(*b*) When $n = 1$, then the mechanism can be driven by a single input motion, as shown in Fig.14 (*b*).

(*c*) When $n = 2$, then two separate input motions are necessary to produce constrained motion for the mechanism, as shown in Fig.14 (*c*).

(*d*) When $n = - 1$ or less, then there are redundant constraints in the chain and it forms a statically indeterminate structure, as shown in Fig. 14 (*e*).

The application of Kutzbach's criterion applied to mechanisms with a higher pair or two degree of freedom joints is shown in Fig. 15.

Fig. 15. Mechanism with a higher pair.

In Fig. 15 (*a*), there are three links, two binary joints and one higher pair, *i.e.* $l = 3$, $j = 2$ and $h = 1$.

$\therefore n = 3\,(3 - 1) - 2 \times 2 - 1 = 1$

In Fig. 15 (*b*), there are four links, three binary joints and one higher pair, *i.e.* $l = 4$, $j = 3$ and $h = 1$

$\therefore n = 3\,(4 - 1) - 2 \times 3 - 1 = 2$

Here it has been assumed that the slipping is possible between the links (*i.e.* between the wheel and the fixed link). However if the friction at the contact is high enough to prevent slipping, the joint will be counted as one degree of freedom pair, because only one relative motion will be possible between the links.

Grubler's Criterion for Plane Mechanisms

The Grubler's criterion applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting $n = 1$ and $h = 0$ in Kutzbach equation, we have

$$1 = 3\,(l - 1) - 2\,j \text{ or } 3l - 2j - 4 = 0$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.

A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints cannot have odd number of links. The simplest possible machanisms of this type are a four bar mechanism and a slider-crank mechanism in which $l = 4$ and $j = 4$.

Problems:

Example 1.1 *For the kinematic linkages shown in Fig. 1.18, calculate the following:*

- *the number of binary links (N_b)*
- *the number of ternary links (N_t)*
- *the number of other (quaternary, etc.) links (N_o)*
- *the number of total links (N)*
- *the number of loops (L)*
- *the number of joints or pairs (P_1)*
- *the number of degrees of freedom (F)*

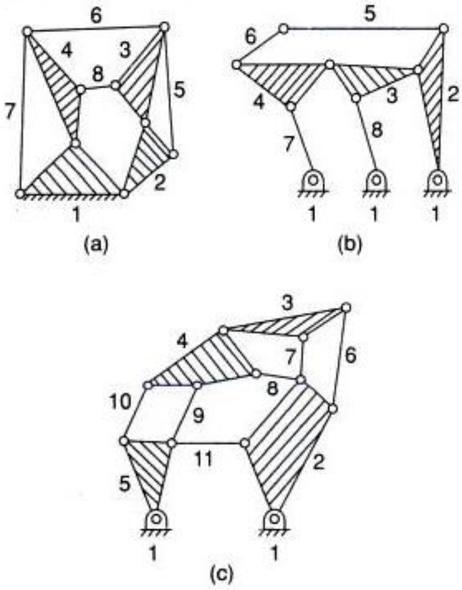


Fig. 1.18

Solution

(a) $N_b = 4; N_t = 4; N_o = 0; N = 8; L = 4$

$P_1 = 11$ by counting

or $P_1 = (N + L - 1) = 11$

$F = 3(N - 1) - 2P_1$

$= 3(8 - 1) - 2 \times 11 = -1$

or $F = N - (2L + 1)$

$= 8 - (2 \times 4 + 1) = -1$

The linkage has negative degree of freedom and thus is a superstructure.

(b) $N_b = 4; N_t = 4; N_o = 0; N = 8; L = 3$

$P_1 = 10$ (by counting)

or $P_1 = (N + L - 1) = 10$

$F = N - (2L + 1) = 8 - (2 \times 3 + 1) = 1$

or $F = 3(N - 1) - 2P_1$

$= 3(8 - 1) - 2 \times 10 = 1$

i.e., the linkage has a constrained motion when one of the seven moving links is driven by an external source.

(c) $N_b = 7; N_t = 2; N_o = 2; N = 11$

$L = 5; P_1 = 15$

$F = N - (2L + 1) = 11 - (2 \times 5 + 1) = 0$

Therefore, the linkage is a structure.