

## Inversion of Mechanism

When one of links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as *inversion of the mechanism*.

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

**Note:** The part of a mechanism which initially moves with respect to the frame or fixed link is called *driver* and that part of the mechanism to which motion is transmitted is called *follower*. Most of the mechanisms are reversible, so that same link can play the role of a driver and follower at different times. For example, in a reciprocating steam engine, the piston is the driver and flywheel is a follower while in a reciprocating air compressor, the flywheel is a driver.

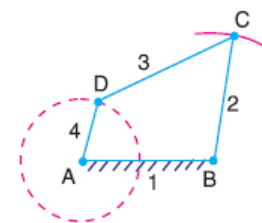
## Types of Kinematic Chains

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:

1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.

## Four Bar Chain or Quadric Cycle Chain

The kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained. The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig.16. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths.



**Fig.16.**Four bar chain.

According to **Grashof's law** for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

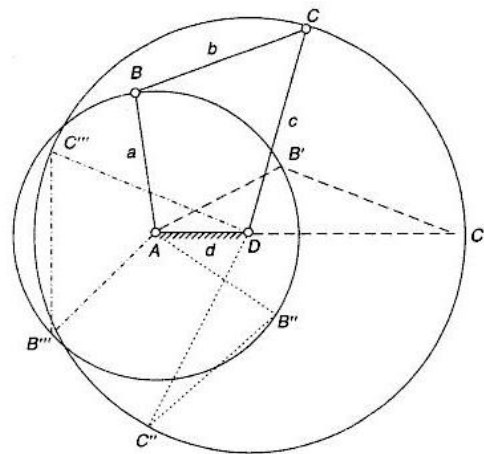
A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such a link is known as **crank** or **driver**. In Fig.16,  $AD$  (link 4) is a crank. The link  $BC$  (link 2) which makes a partial rotation or oscillates is known as **lever** or **rocker** or **follower** and the link  $CD$  (link 3) which connects the crank and lever is called **connecting rod** or **coupler**. The fixed link  $AB$  (link 1) is known as **frame** of the mechanism. When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

### Different class of four bar chain

#### Class-I:

A linkage in which the sum of the lengths of the longest and the shortest links are less than the sum of the other two links, is known as a class-I, four bar linkage.

From the Fig.17 it was clearly shown that the shortest link ( $d$ ) is fixed and the condition that the sum of shortest and longest links is less than the sum of the other two links. The mechanism thus obtained is known as crank-crank or double-crank or drag-crank mechanism or rotary-rotary converter and the figure shows all the three links  $a$ ,  $b$  and  $c$  rotating through one complete revolution.



#### Class-II:

When the sum of the lengths of the largest and the shortest links are more than the sum of the lengths of the other two links, the linkage is known as a class-II, four bar linkage. In such a linkage, fixing of any of the links always results in a rocker-rocker mechanism. In other words, the mechanism and its inversions give the same type of motion (of a double-rocker mechanism).

1. If any of the adjacent link  $d$ , i.e.,  $a$  or  $c$  is fixed,  $d$  can have a full revolution (crank) and the link opposite to it oscillates (rocks). In Fig.18 (a),  $a$  is fixed,  $d$  is the crank and  $b$  oscillates whereas in Fig.18 (b),  $c$  is fixed,  $d$  is the crank and  $b$  oscillates. The mechanism is known as crank-rocker or crank-lever mechanism or rotary-oscillating converter.
2. If the link opposite to the shortest link, i.e., link  $b$  is fixed and the shortest link  $d$  is made a coupler, the other two links  $a$  and  $c$  would oscillate

[Fig.18 (c)]. The mechanism is known as a rocker-rocker or double rocker or double-lever mechanism or oscillating-oscillating converter.

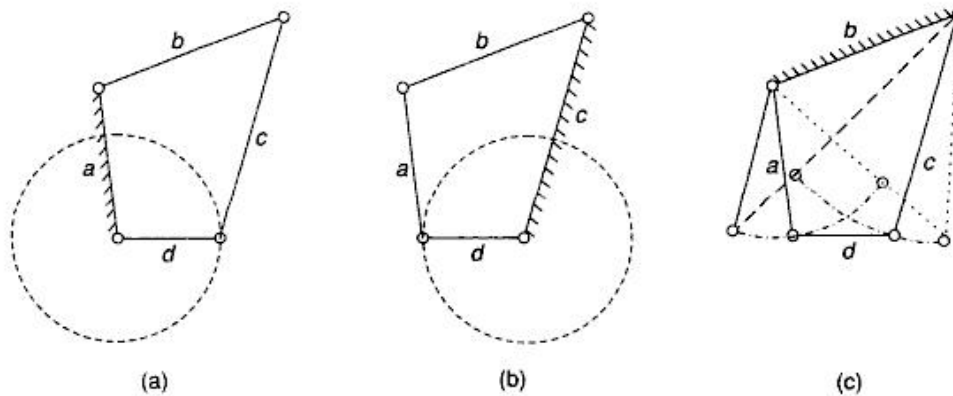


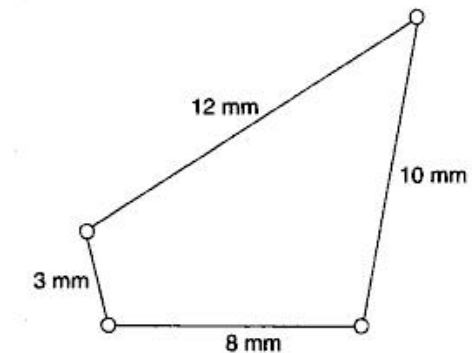
Fig. 18

### Problems:

1. Find all the inversion of the chain given in the figure.

Sol:

- (a) Length of the longest link = 12 mm  
 Length of the shortest link = 3 mm  
 Length of other links = 10 mm and 8 mm  
 Since  $12 + 3 < 10 + 8$ , it belongs to the class-I mechanism and according to Grashoff's law, three distinct inversions are possible.



Shortest link fixed, i.e., when the link with 3 mm length is fixed, the chain will act as double-crank mechanism in which links with lengths of 12 mm and 8 mm will have complete revolutions.

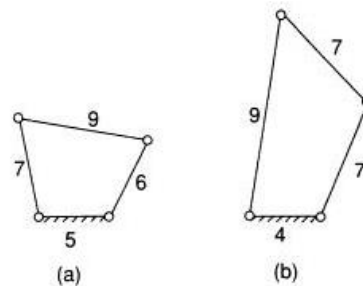
Link opposite to the shortest link fixed, i.e., when the link with 10 mm length is fixed, the chain will act as double-rocker mechanism in which links with lengths of 12 mm and 8 mm will oscillate.

Link adjacent to the shortest link fixed, i.e., when any of the links adjacent to the shortest link, i.e., link with a length of 12 mm or 8 mm is fixed, the chain will act as crank-rocker mechanism in which the shortest link of 3 mm length will revolve and the link with 10 mm length will oscillate.

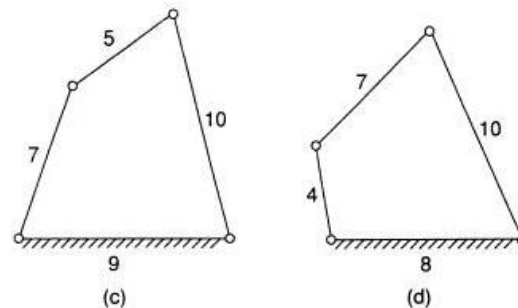
2. The figure shown below are four-link mechanisms. Indicate the type of each mechanism whether crank-rocker or double-crank or double-rocker.

Solution:

- (a) Length of the longest link = 9  
 Length of the shortest link = 5  
 Length of other links = 7 & 6  
 Since  $9+5 > 7+6$ , it belongs to Class-II mechanism. Therefore, it is a double-rocker mechanism.



- (b) Length of the longest link = 9  
 Length of the shortest link = 4  
 Length of other links = 7 & 7  
 Since  $9+4 < 7+7$ , it belongs to Class-I mechanism. In this case as the shortest link is fixed, it is a double-crank mechanism.



- (c) Length of the longest link = 10  
 Length of the shortest link = 5  
 Length of other links = 7 & 9  
 Since  $10+5 < 7+9$ , it belongs to Class-I mechanism. In this case as the link opposite to the shortest link is fixed, it is a double-rocker mechanism.

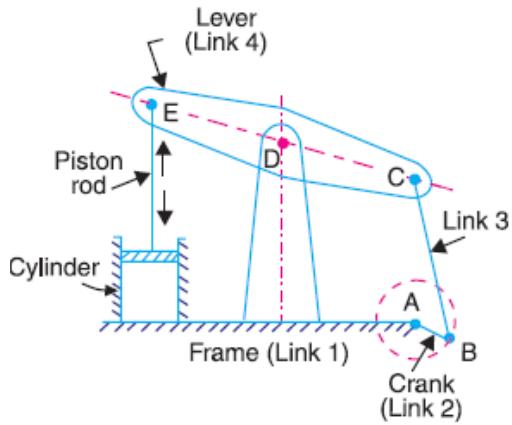
- (d) Length of the longest link = 10  
 Length of the shortest link = 4  
 Length of other links = 8 & 7  
 Since  $10+4 < 8+7$ , it belongs to Class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

### Inversions of Four Bar Chain

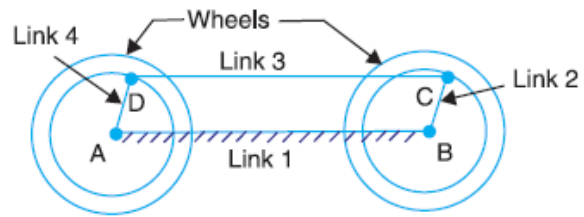
Though there are many inversions of the four bar chain, yet the following are important from the subject point of view:

#### **1. Beam engine (crank and lever mechanism).**

A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig.19. In this mechanism, when the crank rotates about the fixed centre  $A$ , the lever oscillates about a fixed centre  $D$ . The end  $E$  of the lever  $CDE$  is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.



**Fig. 19.** Beam engine.

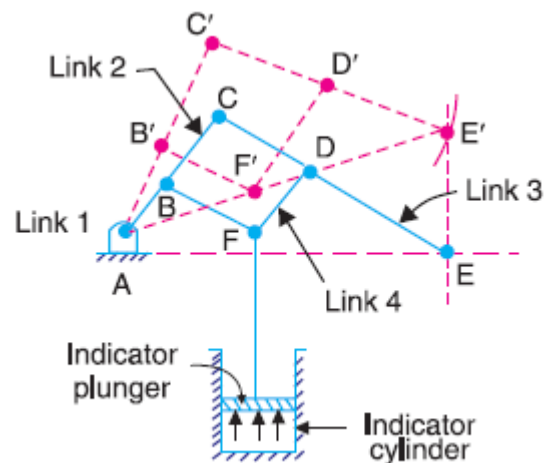


**Fig.20.** Coupling rod of a locomotive.

**2. Coupling rod of a locomotive (Double crank mechanism).** The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in Fig.20. In this mechanism, the links  $AD$  and  $BC$  (having equal length) act as cranks and are connected to the respective wheels. The link  $CD$  acts as a coupling rod and the link  $AB$  is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

**3. Watt's indicator mechanism (Double lever mechanism).**

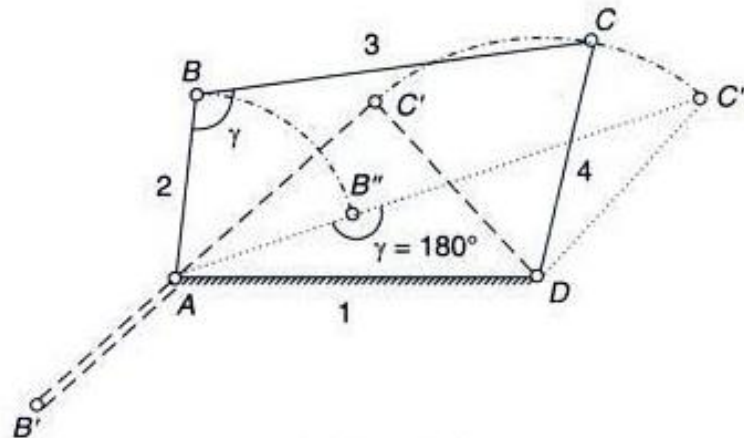
A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in Fig. 21. The four links are: fixed link at  $A$ , link  $AC$ , link  $CE$  and link  $BFD$ . It may be noted that  $BF$  and  $FD$  form one link because these two parts have no relative motion between them. The links  $CE$  and  $BFD$  act as levers. The displacement of the link  $BFD$  is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point  $E$  at the end of the link  $CE$  traces out approximately a straight line.



The initial position of the mechanism is shown in Fig.21 by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.

**Fig. 21.** Watt's indicator mechanism

## MECHANICAL ADVANTAGE



**Fig.22**

The Mechanical Advantage (MA) of a mechanism is the ratio of the output force or torque to the input force or torque at any instant. Thus for the linkage of Fig.22, if friction and inertia forces are ignored and the input torque  $T_2$  is applied to the link 2 to drive the output link 4 with a resisting torque  $T_4$  then,

Power input = Power output

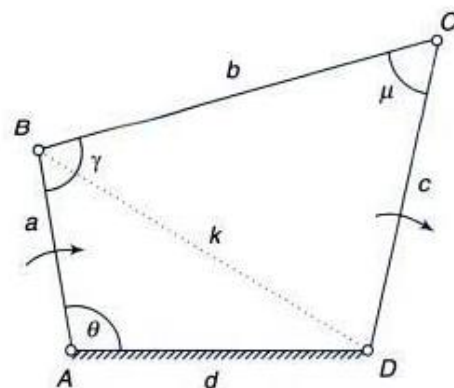
$$T_2 \omega_2 = T_4 \omega_4$$

$$MA = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

Thus, it is the reciprocal of the velocity ratio. In case of crank-rocker mechanisms, the velocity  $\omega_4$  of the output link DC (rocker) becomes zero at the extreme positions ( $AB'C'D$  and  $AB''C''D$ ), i.e., when the input link AB is in line with the coupler BC and the angle  $\gamma$  between them is either zero or  $180^\circ$ , it makes the mechanical advantage to be infinite at such positions. Only a small input torque can overcome a large output torque load. The extreme positions of the linkage are known as toggle positions.

## TRANSMISSION ANGLE

The angle  $\mu$  between the output link and the coupler is known as transmission angle. In Fig.23, if the link AB is the input link, the force applied to the output link DC is transmitted through the coupler BC. For a particular value of force in the coupler rod, the torque transmitted to the output link (about the point D) is maximum when





the transmission angle  $\mu$  is  $90^\circ$ . If links BC and DC become coincident, the transmission angle is zero and the mechanism would lock or jam. If  $\mu$  deviates significantly from  $90^\circ$ , the torque on the output link decreases. Sometimes, it may not be sufficient to overcome the friction in the system and the mechanism may be locked or jammed. Hence  $\mu$  is usually kept more than  $45^\circ$ . The best mechanisms, therefore have a transmission angle that does not deviate much from  $90^\circ$ .

Applying cosine law to triangles ABD and BCD (Fig.23)

$$a^2 + d^2 - 2ad \cos \theta = k^2 \quad (i)$$

$$\text{and } b^2 + c^2 - 2bc \cos \mu = k^2 \quad (ii)$$

From (i) and (ii),

$$a^2 + d^2 - 2ad \cos \theta = b^2 + c^2 - 2bc \cos \mu$$

$$\text{or } a^2 + d^2 - b^2 - c^2 - 2ad \cos \theta + 2bc \cos \mu = 0$$

### Problems:

1. Find the maximum and minimum transmission angles for the mechanisms shown in figure below.

Solution:

- (a) Length of the longest link = 3  
 Length of the shortest link = 1  
 Length of other links = 3 & 2  
 Since  $3+1 < 3+2$ , it belongs to the class I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

The maximum transmission angle is when  $\theta$  is  $180^\circ$  (Fig.a)

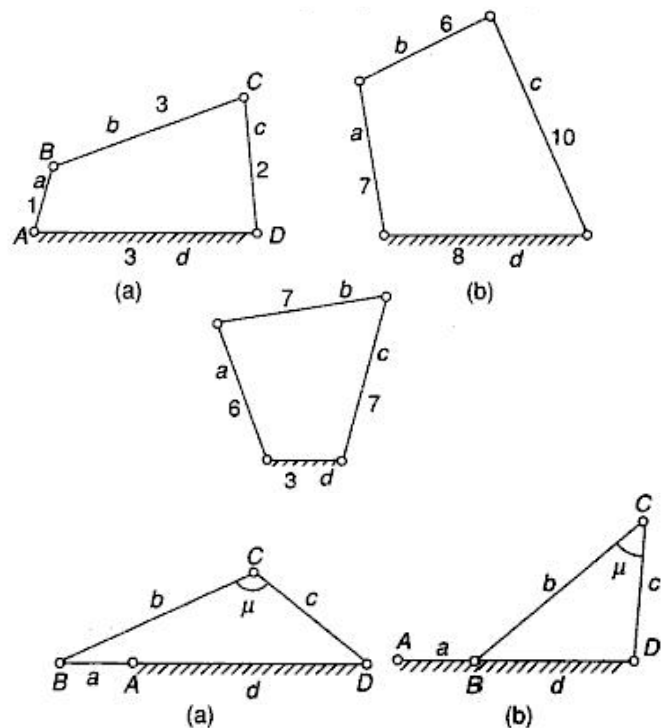
$$\begin{aligned} \text{Thus } (a+d)^2 &= b^2 + c^2 - 2bc \cos \mu \\ (1+3)^2 &= 3^2 + 2^2 - 2 \times 3 \times 2 \cos \mu \\ 16 &= 9 + 4 - 12 \cos \mu \\ \cos \mu &= -\frac{3}{12} = -0.25 \end{aligned}$$

$$\mu = 104.5^\circ$$

The minimum transmission angle is when  $\theta$  is  $90^\circ$  (Fig.b)

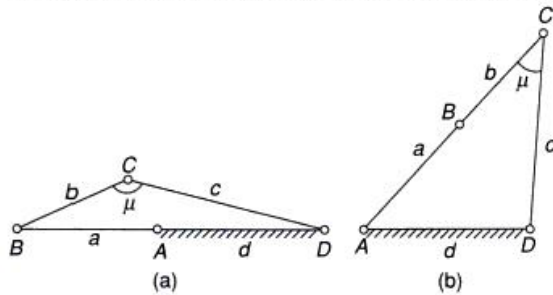
$$\begin{aligned} \text{Thus } (d-a)^2 &= b^2 + c^2 - 2bc \cos \mu \\ (3-1)^2 &= 3^2 + 2^2 - 2 \times 3 \times 2 \cos \mu \\ 4 &= 9 + 4 - 12 \cos \mu \\ \cos \mu &= \frac{3}{4} = 0.75 \end{aligned}$$

$$\mu = 41.4^\circ$$



- (b) In this mechanism,  
 Length of the longest link = 10  
 Length of the shortest link = 6  
 Length of other links = 8 and 7

Since  $10 + 6 > 8 + 7$ , it belongs to the class-II mechanism and thus is a double-rocker mechanism.



Maximum transmission angle is when  $\theta$  is  $180^\circ$  [Fig. 1.48(a)],

$$\begin{aligned} \text{Thus, } (a + d)^2 &= b^2 + c^2 - 2bc \cos \mu \\ (7 + 8)^2 &= 6^2 + 10^2 - 2 \times 6 \times 10 \cos \mu \\ 225 &= 36 + 100 - 120 \cos \mu \\ \cos \mu &= -\frac{89}{120} = -0.742 \end{aligned}$$

$$\mu = 137.9^\circ$$

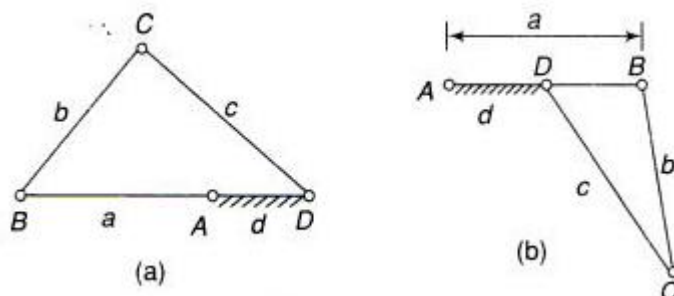
Minimum transmission angle is when the angle at  $B$  is  $180^\circ$  [Fig. 1.48(b)],

$$\begin{aligned} \text{Thus, } d^2 &= (a + b)^2 + c^2 - 2(a + b)c \cos \mu \\ 8^2 &= (7 + 6)^2 + 10^2 - 2(7 + 6) \times 10 \times \cos \mu \\ 64 &= 169 + 100 - 260 \cos \mu \\ \cos \mu &= \frac{205}{260} = 0.788 \end{aligned}$$

$$\mu = 38^\circ$$

- (c) In this mechanism,  
 Length of the longest link = 7  
 Length of the shortest link = 3  
 Length of other links = 6 and 6

Since  $7 + 3 < 6 + 6$ , it belongs to the class-I mechanism. In this case as the shortest link is fixed, it is a double-crank or drag-link mechanism.





Maximum transmission angle is when  $\theta$  is  $180^\circ$   
 [Fig. 1.49(a)],

$$\begin{aligned} \text{Thus } (a + d)^2 &= b^2 + c^2 - 2bc \cos \mu \\ (6 + 3)^2 &= 6^2 + 7^2 - 2 \times 6 \times 7 \cos \mu \\ 81 &= 36 + 49 - 84 \cos \mu \\ \cos \mu &= \frac{4}{84} = 0.476 \end{aligned}$$

$$\mu = 87.27^\circ$$

Minimum transmission angle is when  $\theta$  is  $0^\circ$   
 [Fig. 1.49(b)],

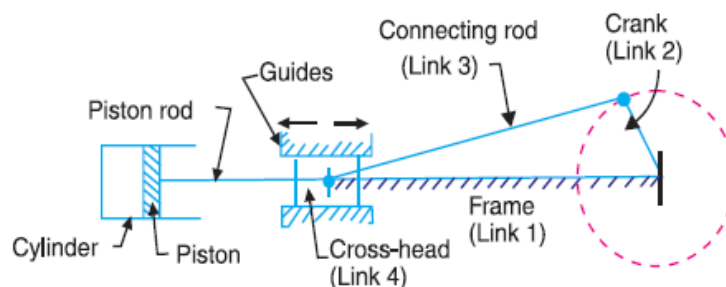
$$\begin{aligned} \text{Thus } (a - d)^2 &= b^2 + c^2 - 2bc \cos \mu \\ (6 - 3)^2 &= 6^2 + 7^2 - 2 \times 6 \times 7 \cos \mu \\ 9 &= 36 + 49 - 84 \cos \mu \\ \cos \mu &= \frac{76}{84} = 0.9048 \end{aligned}$$

$$\mu = 25.2^\circ$$

### Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa.

In a single slider crank chain, as shown in Fig. 24, the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.



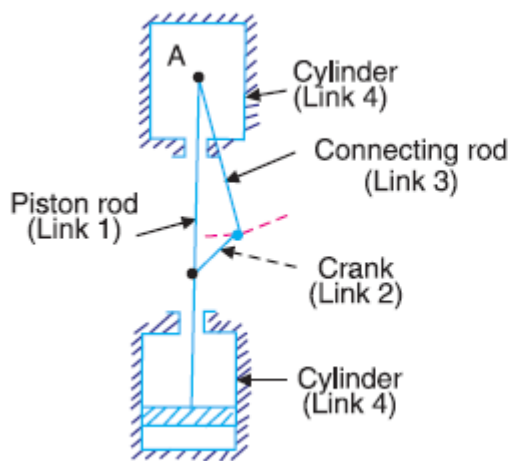
**Fig. 24.** Single slider crank chain.

The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

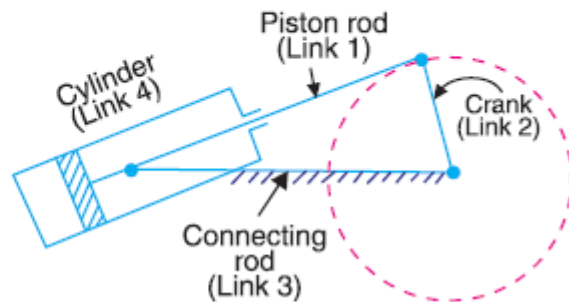
## Inversions of Single Slider Crank Chain

There are four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

**1. Pendulum pump or Bull engine.** In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (*i.e.* sliding pair), as shown in Fig. 25. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig. 25.



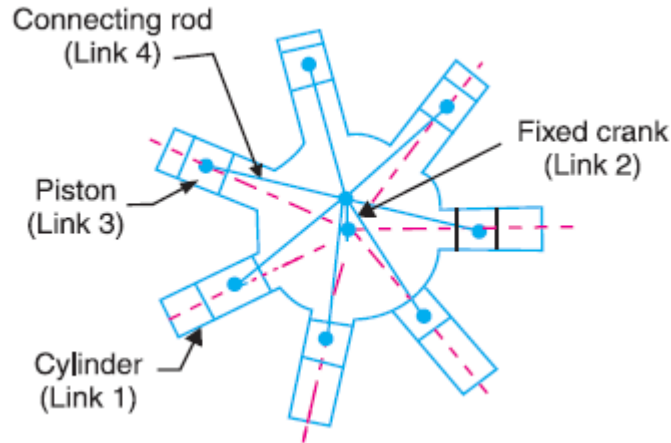
**Fig. 25.** Pendulum pump.



**Fig. 26.** Oscillating cylinder engine.

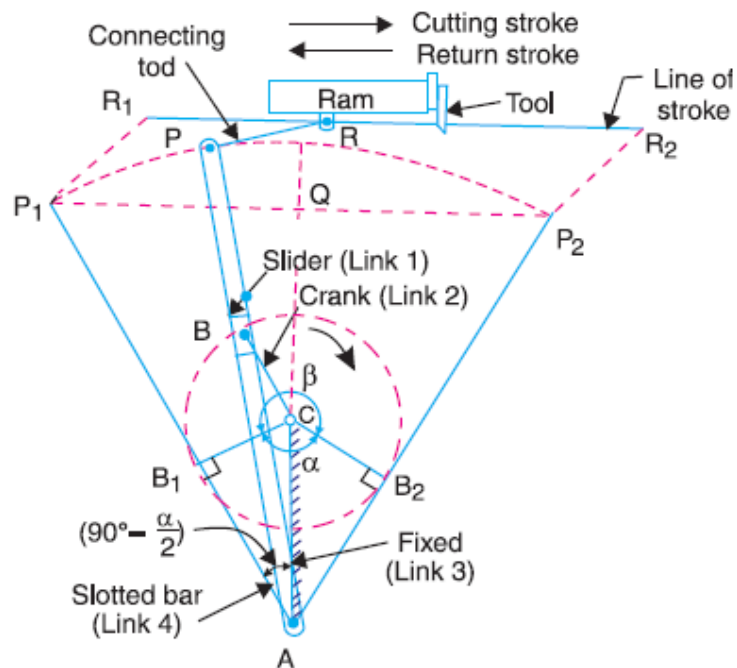
**2. Oscillating cylinder engine.** The arrangement of oscillating cylinder engine mechanism, as shown in Fig.26, is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

**3. Rotary internal combustion engine or Gnome engine.** Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre D, as shown in Fig.27, while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.



**Fig. 27.** Rotary internal combustion engine.

**4. Crank and slotted lever quick return motion mechanism.** This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines. In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig.28. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke  $R_1R_2$ . The line of stroke of the ram (i.e.  $R_1R_2$ ) is perpendicular to AC produced.



**Fig. 28.** Crank and slotted lever quick return motion mechanism.

In the extreme positions,  $AP_1$  and  $AP_2$  are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position  $CB_1$  to  $CB_2$  (or through an angle  $\beta$ ) in the clockwise direction. The return stroke occurs when the crank rotates from the position  $CB_2$  to  $CB_1$  (or through angle  $\alpha$ ) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}$$

Since the tool travels a distance of  $R_1 R_2$  during cutting and return stroke, therefore travel of the tool or length of stroke

$$\begin{aligned} &= R_1 R_2 = P_1 P_2 = 2P_1 Q = 2AP_1 \sin \angle P_1 A Q \\ &= 2AP_1 \sin \left( 90^\circ - \frac{\alpha}{2} \right) = 2AP \cos \frac{\alpha}{2} \quad \dots (\because AP_1 = AP) \\ &= 2AP \times \frac{CB_1}{AC} \quad \dots \left( \because \cos \frac{\alpha}{2} = \frac{CB_1}{AC} \right) \\ &= 2AP \times \frac{CB}{AC} \quad \dots (\because CB_1 = CB) \end{aligned}$$

**5. Whitworth quick return motion mechanism.** This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link  $CD$  (link 2) forming the turning pair is fixed, as shown in Fig.29. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank  $CA$  (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at  $A$  slides along the slotted bar  $PA$  (link 1) which oscillates at a pivoted point  $D$ . The connecting rod  $PR$  carries the ram at  $R$  to which a cutting tool is fixed. The motion of the tool is constrained along the line  $RD$  produced, *i.e.* along a line passing through  $D$  and perpendicular to  $CD$ .

When the driving crank  $CA$  moves from the position  $CA_1$  to  $CA_2$  (or the link  $DP$  from the position  $DP_1$  to  $DP_2$ ) through an angle  $\alpha$  in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance  $2 PD$ .

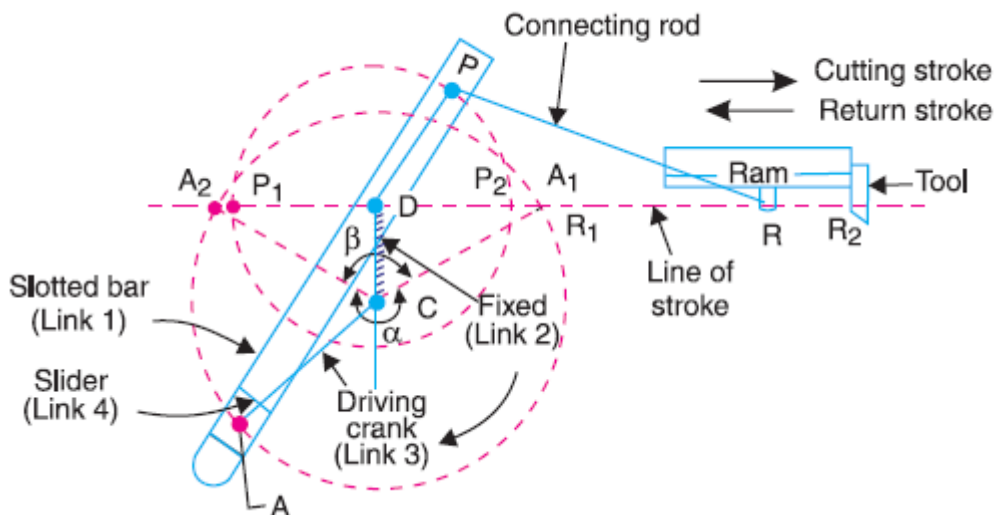
Now when the driving crank moves from the position  $CA_2$  to  $CA_1$  (or the link  $DP$  from  $DP_2$  to  $DP_1$ ) through an angle  $\beta$  in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (*i.e.* during forward or cutting stroke) will be equal to the time taken by the driving crank to move from  $CA_2$  to  $CA_1$ . Similarly, the time taken during the right to left movement of the ram (or during the idle or return

stroke) will be equal to the time taken by the driving crank to move from  $CA_2$  to  $CA_1$ .

Since the crank link  $CA$  rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \quad \text{or} \quad \frac{360^\circ - \beta}{\beta}$$



**Fig. 29.** Whitworth quick return motion mechanism.

**Problem 1.** In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

**Solution.** Given :  $AC = 240$  mm ;  $CB_1 = 120$  mm ;  $AP_1 = 450$  mm

**Inclination of the slotted bar with the vertical**

Let  $\angle CAB_1 =$  Inclination of the slotted bar with the vertical.

The extreme positions of the crank are shown in Fig. 5.29. We know that

$$\begin{aligned} \sin \angle CAB_1 &= \sin \left( 90^\circ - \frac{\alpha}{2} \right) \\ &= \frac{B_1C}{AC} = \frac{120}{240} = 0.5 \\ \therefore \angle CAB_1 &= 90^\circ - \frac{\alpha}{2} \\ &= \sin^{-1} 0.5 = 30^\circ \text{ Ans.} \end{aligned}$$

**Time ratio of cutting stroke to the return stroke**

We know that

$$90^\circ - \alpha/2 = 30^\circ$$

$$\therefore \alpha/2 = 90^\circ - 30^\circ = 60^\circ$$

or

$$\alpha = 2 \times 60^\circ = 120^\circ$$

$$\therefore \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360^\circ - \alpha}{\alpha} = \frac{360^\circ - 120^\circ}{120^\circ} = 2 \text{ Ans.}$$

**Length of the stroke**

We know that length of the stroke,

$$\begin{aligned} R_1R_2 &= P_1P_2 = 2 P_1Q = 2 AP_1 \sin (90^\circ - \alpha/2) \\ &= 2 \times 450 \sin (90^\circ - 60^\circ) = 900 \times 0.5 = 450 \text{ mm Ans.} \end{aligned}$$

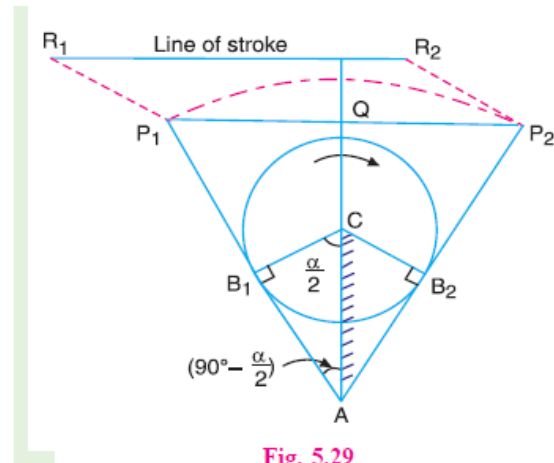


Fig. 5.29



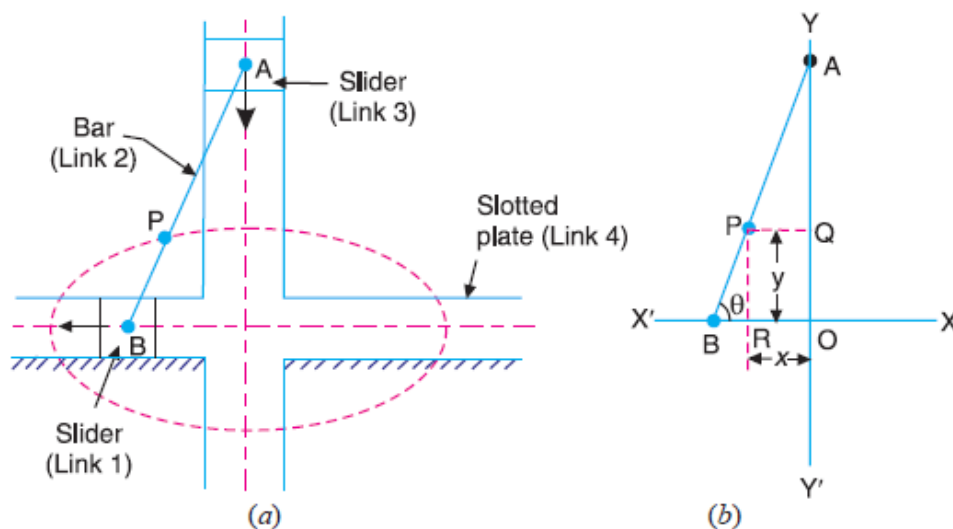
## Double Slider Crank Chain

A kinematic chain which consists of two turning pairs and two sliding pairs is known as *double slider crank chain*, as shown in Fig.30. From the Fig. we can see that the link 2 and link 1 form one turning pair and link 2 and link 3 form the second turning pair. The link 3 and link 4 form one sliding pair and link 1 and link 4 form the second sliding pair.

## Inversions of Double Slider Crank Chain

The following three inversions of a double slider crank chain are important from the subject point of view:

**1. Elliptical trammels.** It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Fig.30. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link  $AB$  (link 2) is a bar which forms turning pair with links 1 and 3. When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as  $P$  traces out an ellipse on the surface of link 4, as shown in Fig. 30 (a). A little consideration will show that  $AP$  and  $BP$  are the semi-major axis and semi-minor axis of the ellipse respectively. This can be proved as follows:



**Fig. 30.** Elliptical trammels.

Let us take  $OX$  and  $OY$  as horizontal and vertical axes and let the link  $BA$  be inclined at an angle  $\theta$  with the horizontal, as shown in Fig.30 (b). Now the coordinates of the point  $P$  on the link  $BA$  will be

$$x = PQ = AP \cos \theta; \text{ and } y = PR = BP \sin \theta$$

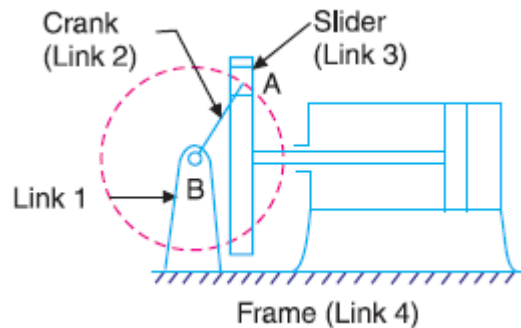
or 
$$\frac{x}{AP} = \cos \theta; \text{ and } \frac{y}{BP} = \sin \theta$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2 \theta + \sin^2 \theta = 1$$

This is the equation of an ellipse. Hence the path traced by point  $P$  is an ellipse whose semi major axis is  $AP$  and semi-minor axis is  $BP$ .

**2. Scotch yoke mechanism.** This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig.31, link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about  $B$  as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.

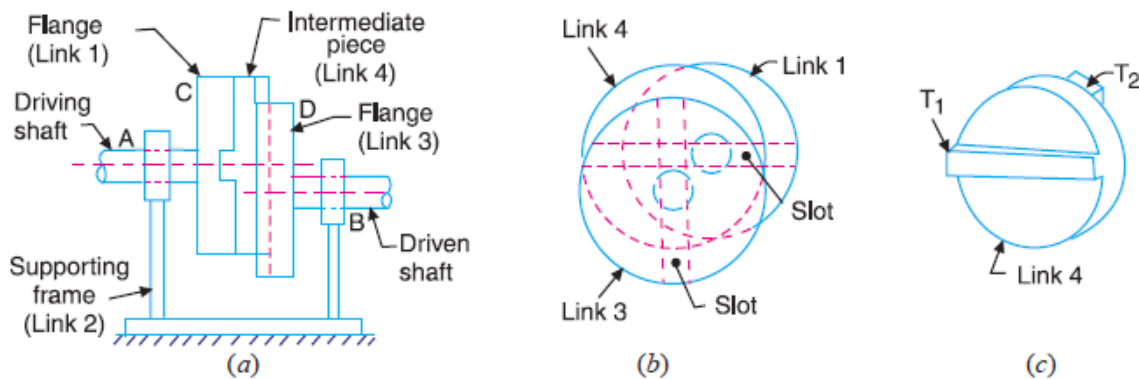


**Fig.31.** Scotch yoke mechanism.

**3. Oldham's coupling.** An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Fig.32 (a). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces, as shown in Fig.32 (b). The intermediate piece (link 4) which is a circular disc, have two tongues (*i.e.* diametrical projections)  $T_1$  and  $T_2$  on each face at right angles to each other, as shown in Fig.32 (c). The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.

When the driving shaft *A* is rotated, the flange *C* (link 1) causes the intermediate piece (link 4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange *D* (link 3) at the same angle and thus the shaft *B* rotates. Hence links 1, 3 and 4 have the same angular velocity at every instant. A little consideration will show, that there is a sliding motion between the link 4 and each of the other links 1 and 3.



**Fig.32.** Oldham's coupling.

If the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

Let  $\omega$  = Angular velocity of each shaft in rad/s, and  
 $r$  = Distance between the axes of the shafts in metres.

$\therefore$  Maximum sliding speed of each tongue (in m/s),

$$v = \omega \cdot r$$

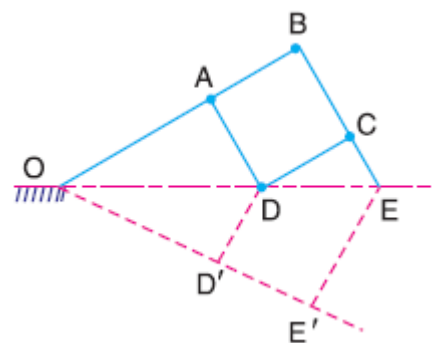
### Common Mechanisms - Straight line mechanism, Dwell mechanism

#### Pantograph:

A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point. It consists of a jointed parallelogram *ABCD* as shown in Fig.33. It is made up of bars connected by turning pairs. The bars *BA* and *BC* are extended to *O* and *E* respectively, such that  $OA/OB = AD/BE$

Thus, for all relative positions of the bars, the triangles *OAD* and *OBE* are similar and the points *O*, *D* and *E* are in one straight line. It may be proved that point *E* traces out the same path as described by point *D*.

From similar triangles *OAD* and *OBE*, we find that,  $OD/OE = AD/BE$



**Fig. 33.** Pantograph.