Aronhold Kennedy (or Three Centres in Line) Theorem

The Aronhold Kennedy's theorem states that ***if three bodies move relatively to each other*, *they have three instantaneous centres and lie on a straight line.***

Rubbing Velocity at a Pin Joint

The links in a mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as **the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of the pin.**

Consider two links *OA* and *OB* connected by a pin joint at *O* as shown in Fig.3.

Let ω_1 = Angular velocity of the link *OA* or the angular velocity of the point *A* with respect to *O*.

ω_2 = Angular velocity of the link *OB* or the angular velocity of the point *B* with respect to *O*, and

r = Radius of the pin.

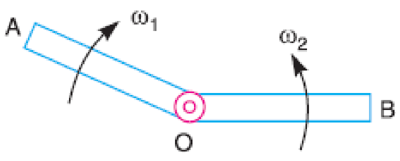


Fig. 3. Links connected by pin joints

According to the definition,

Rubbing velocity at the pin joint *O*

$= (\omega_1 - \omega_2)\, r$, if the links move in the same direction

$= (\omega_1 + \omega_2)\, r$, if the links move in the opposite direction

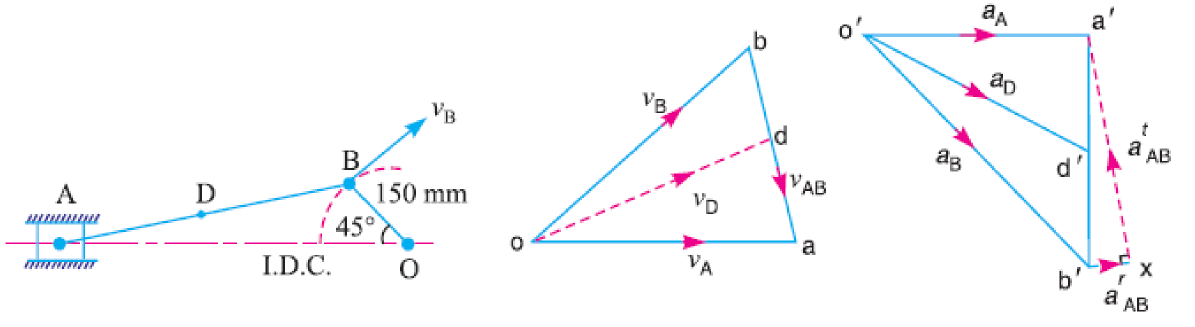
Velocity and Acceleration of a Point on a Link by Relative Velocity Method

Problem 1. *The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine:* ***1.*** *Linear velocity and acceleration of the midpoint of the connecting rod, and* ***2.*** *angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.*

Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; *OB* = 150 mm = 0.15 m ; *BA* = 600 mm = 0.6 m

We know that linear velocity of *B* with respect to *O* or velocity of *B*,

$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713$ m/s



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

1. Linear velocity of the midpoint of the connecting rod

First of all draw the space diagram, to some suitable scale; as shown in Fig. Now the velocity diagram, as shown in Fig., is drawn as discussed below:

1. Draw vector *ob* perpendicular to *BO*, to some suitable scale, to represent the velocity of *B* with respect to *O* or simply velocity of *B i.e.* v_{BO} or v_B, such that vector *ob* = $v_{BO} = v_B = 4.713$ m/s

2. From point *b*, draw vector *ba* perpendicular to *BA* to represent the velocity of *A* with respect to *B i.e.* v_{AB} , and from point *o* draw vector *oa* parallel to the motion of *A* (which is along *AO*) to represent the velocity of *A i.e.* v_A. The vectors *ba* and *oa* intersect a

By measurement, we find that velocity of *A* with respect to *B*,

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$

3. In order to find the velocity of the midpoint *D* of the connecting rod *AB*, divide the vector *ba* at *d* in the same ratio as *D* divides *AB*, in the space diagram. In other words, *bd / ba = BD/BA*

Note: Since *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

4. Join *od*. Now the vector *od* represents the velocity of the midpoint *D* of the connecting rod *i.e.* v_D.

By measurement, we find that

v_D = vector *od* = 4.1 m/s **Ans.**

Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of *B* with respect to *O* or the acceleration of *B*,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

and the radial component of the acceleraiton of *A* with respect to *B*,

$$a^r_{AB} = \frac{v^2_{AB}}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. is drawn as discussed below:

1. Draw vector *o′ b′* parallel to *BO*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *O* or simply acceleration of *B i.e.* a^r_{BO} or a_B, such that

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

Note: Since the crank *OB* rotates at a constant speed, therefore there will be no tangential component of the acceleration of *B* with respect to *O*.

2. The acceleration of *A* with respect to *B* has the following two components:
(*a*) The radial component of the acceleration of *A* with respect to *B i.e.* a^r_{AB}, and
(*b*) The tangential component of the acceleration of *A* with respect to *B i.e.* a^t_{AB}.
These two components are mutually perpendicular.
Therefore from point *b'*, draw vector *b' x* parallel to *AB* to represent a^r_{AB} = 9.3 m/s^2 and from point *x* draw vector *xa'* perpendicular to vector *b' x* whose magnitude is yet unknown.
3. Now from *o'*, draw vector *o' a'* parallel to the path of motion of *A* (which is along *AO*) to represent the acceleration of *A i.e.* a_A. The vectors *xa'* and *o' a'* intersect at *a'*. Join *a' b'*.
4. In order to find the acceleration of the midpoint *D* of the connecting rod *AB*, divide the vector *a' b'* at *d'* in the same ratio as *D* divides *AB*. In other words
b'd' / *b'a'* = *BD* / *BA*
5. Join *o' d'*. The vector *o' d'* represents the acceleration of midpoint *D* of the connecting rod *i.e.* a_D.
By measurement, we find that
a_D = vector *o' d'* = 117 m/s^2 **Ans.**

2. *Angular velocity of the connecting rod*

We know that angular velocity of the connecting rod *AB*,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B\text{)} \quad \textbf{Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a^t_{AB} = 103 \text{ m/s}^2$$

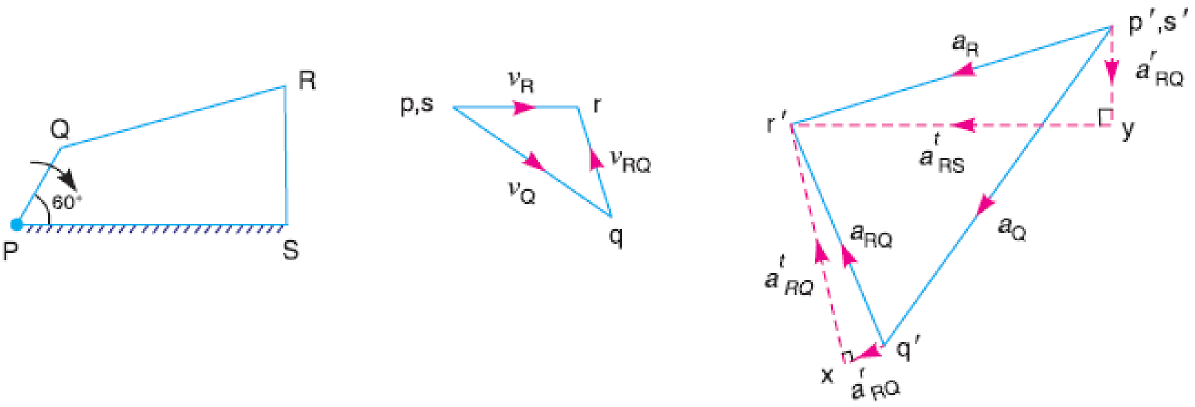
We know that angular acceleration of the connecting rod *AB*,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B\text{)} \quad \textbf{Ans.}$$

Problem 2. *PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm; QR = 175 mm; RS = 112.5 mm; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.*

Solution. Given: ω_{QP} = 10 rad/s; *PQ* = 62.5 mm = 0.0625 m; *QR* = 175 mm = 0.175 m; *RS* = 112.5 mm = 0.1125 m; *PS* = 200 mm = 0.2 m

We know that velocity of *Q* with respect to *P* or velocity of *Q*,
$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625$ m/s



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Angular velocity of links QR and RS

vector $pq = v_{QP} = v_Q = 0.625$ m/s

v_{RQ} = vector *qr* = 0.333 m/s, and $v_{RS} = v_R$ = vector *sr* = 0.426 m/s

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise)}$$ **Ans.**

$$\omega_{RS} = \frac{v_{RS}}{SR} = \frac{0.426}{0.1125} = 3.78 \text{ rad/s (Clockwise)}$$ **Ans.**

$$a^r_{QP} = a_{QP} = a_Q = \frac{v_{QP}^2}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Angular acceleration of links QR and RS

$$a^r_{QP} = a_{QP} = a_Q = \frac{v^2_{QP}}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

$$a^r_{RQ} = \frac{v^2_{RQ}}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

$$a^r_{RS} = a_{RS} = a_R = \frac{v^2_{RS}}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

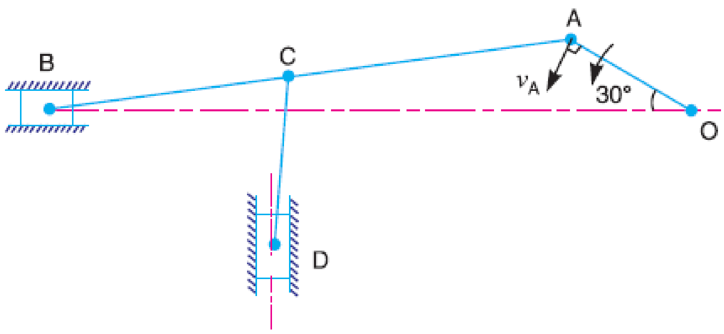
a^t_{RQ} = vector xr' = 4.1 m/s^2 and a^t_{RS} = vector yr' = 5.3 m/s^2

$$\alpha_{QR} = \frac{a^t_{RQ}}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

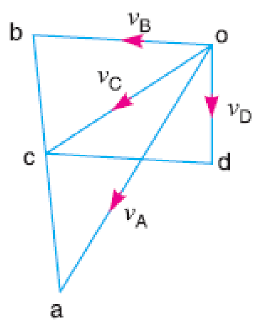
$$\alpha_{RS} = \frac{a^t_{RS}}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise)}$$ **Ans.**

Problem 3. *In the mechanism, as shown in Fig., the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm. For the given configuration, determine:* **1.** *velocities of sliding at B and D,* **2.** *Angular velocity of CD,* **3.** *linear acceleration of D, and* **4.** *angular acceleration of CD.*

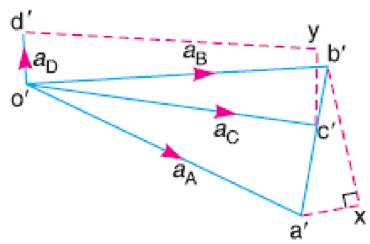
Solution. Given : N_{AO} = 20 r.p.m. or $\omega_{AO} = 2\pi \times 20/60 = 2.1$ rad/s ; OA = 300 mm = 0.3 m ; AB = 1200 mm = 1.2 m ; BC = CD = 450 mm = 0.45 m



(*a*) Space diagram.



(*b*) Velocity diagram.



(*c*) Acceleration diagram.

v_B = vector ob = 0.4 m/s **Ans.**

v_D = vector od = 0.24 m/s **Ans.**

$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s (Anticlockwise). } \textbf{Ans.}$$

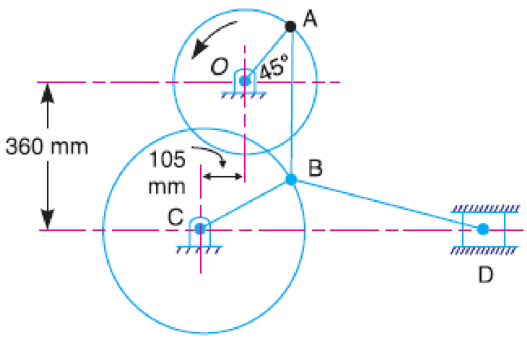
$$a_D = \text{vector } o'd' = 0.16 \text{ m/s}^2 \quad \textbf{Ans.}$$

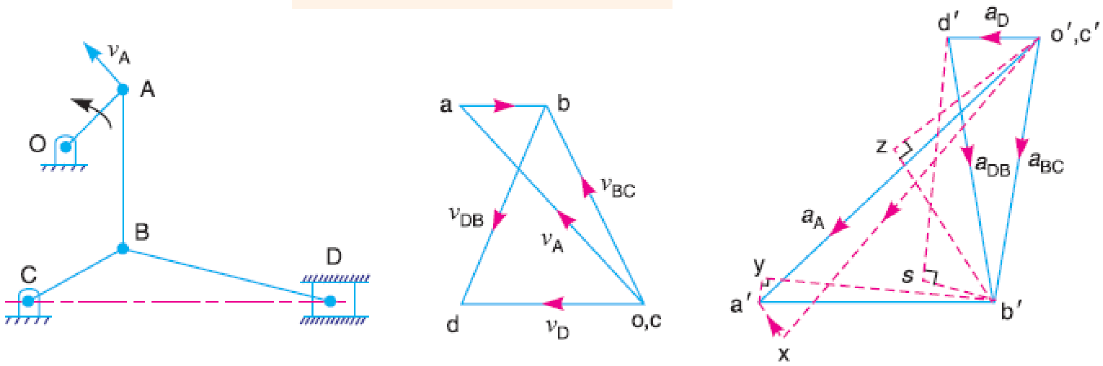
$$\alpha_{CD} = \frac{a^t_{DC}}{CD} = \frac{1.28}{0.45} = 2.84 \text{ rad/s}^2 \text{ (Clockwise) } \textbf{Ans.}$$

Problem 4. *In the toggle mechanism shown in Fig., the slider D is constrained to move on a horizontal path. The crank OA is rotating in the counter-clockwise direction at a speed of 180 r.p.m. increasing at the rate of 50 rad/s^2. The dimensions of the various links are as follows:*

OA = 180 mm; CB = 240 mm; AB = 360 mm; and BD = 540 mm.

For the given configuration, find **1.** *Velocity of slider D and angular velocity of BD, and* **2.** *Acceleration of slider D and angular acceleration of BD.*





(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.