

INTRODUCTION

Dynamic force Analysis- Here effect of inertia forces are also considered along with externally applied forces.

Ex.- Motor running at 90000 rpm. Etc.

Inertia Force:

The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position. It is numerically equal to the accelerating force in magnitude, but **opposite** in direction. Mathematically,

Inertia force = $-$ Accelerating force = $- m.a$

where m = Mass of the body, and

a = Linear acceleration of the centre of gravity of the body.



Inertia Torque:

The inertia torque is an imaginary torque, which when applied upon the rigid body, brings it in equilibrium position. It is equal to the accelerating couple in magnitude but **opposite** in direction.

D ALEMBERT'S PRINCIPAL

It states that,

if a body is not in Static Equilibrium because of the acceleration it posses, it can be brought to the condition of static equilibrium by introducing on it inertia force which acts through the centre of gravity in the direction opposite to acceleration and is equal to mass times acceleration (ma).

A/C to D Alembert's Principle

$$\sum F + F_i = 0$$

$$\sum T + T_i = 0$$

Also, it states that
"Inertia forces and couples and the external forces and torque on the body together gives Static Equilibrium".

Here,

$\sum F$ = Vector sum of all external Forces

$\sum T$ = Vector sum of all external Torque

$$F_i = -ma \text{ (Inertia force)}$$

$$T_i = -I\alpha \text{ (Inertia Torque)}$$

- Ve sign indicates that F acts opposite to that of acceleration.

Here, m = mass of the body

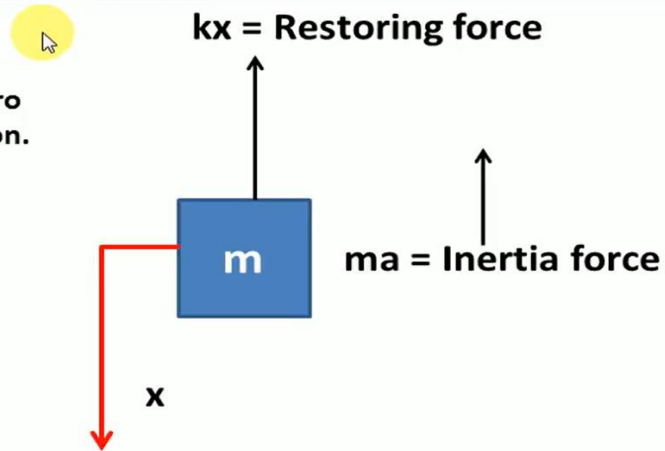
a = Acceleration of centre of mass of the body

I = MOI about an axis passing through the centre of gravity and perpendicular to plane of rotation of body.

D ALEMBERT'S PRINCIPAL

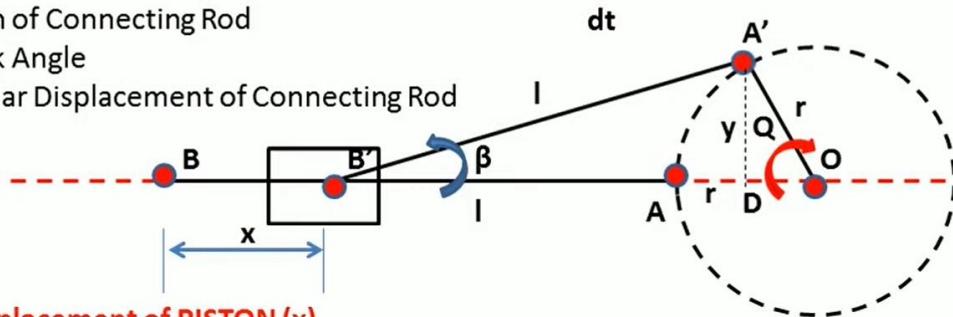
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Also we can say that
Inertia force + Restoring Force = zero
 Which yields the equation of motion.
 As
 $ma + kx = 0$



DYNAMIC ANALYSIS OF SLIDER CRANK MECHANISM

- r = Radius of crank
- l = Length of Connecting Rod
- Q = Crank Angle
- β = Angular Displacement of Connecting Rod



Linear Displacement of PISTON (x)

Let, x = Displacement of piston from IDC when the crank has turned through angle Q from IDC.

$$\begin{aligned} X = BB' &= BO - B'O \text{ (From figure)} \\ &= (BA + AO) - (B'D + DO) \\ &= (l + r) - (l \cos \beta + r \cos Q) \end{aligned}$$

Take $l/r = n$

$$\begin{aligned} \text{Therefore, } x &= (nr + r) - (nr \cos \beta + r \cos Q) \\ &= r[(n + 1) - (n \cos \beta + \cos Q)] \\ &= r[(n + 1) - (n \frac{1}{n} \sqrt{n^2 - \sin^2 Q} + \cos Q)] \\ &= r[(n + 1) - (\sqrt{n^2 - \sin^2 Q} + \cos Q)] \end{aligned}$$

Also, we know that

$$\begin{aligned} Y &= l \sin \beta = r \sin Q \\ \sin \beta &= \sin Q / n \\ \sin^2 \beta + \cos^2 \beta &= 1 \end{aligned}$$

$$= \sqrt{1 - \left(\frac{r \sin Q}{l}\right)^2}$$

$$\cos \beta = \sqrt{1 - \sin^2 \beta}$$

$$\cos \beta = \frac{1}{n} \sqrt{n^2 - \sin^2 Q}$$

$$= \cos \beta = \sqrt{1 - \left(\frac{y}{l}\right)^2}$$