INTRODUCTION

Dynamic force Analysis- Here effect of inertia forces are also considered along with externally applied forces.
Ex.- Motor running at 90000 rpm. Etc.

Inertia Force:
The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position. It is numerically equal to the accelerating force in magnitude, but ***opposite*** in direction. Mathematically,
Inertia force = – Accelerating force = – $m.a$
where m = Mass of the body, and
a = Linear acceleration of the centre of gravity of the body.

Inertia Torque:
The inertia torque is an imaginary torque, which when applied upon the rigid body, brings it in equilibrium position. It is equal to the accelerating couple in magnitude but ***opposite*** in direction.

D ALEMBERT'S PRINCIPAL

It states that,
if a body is not in Static Equilibrium because of the acceleration it posses, it can be brought to the condition of static equilibrium by introducing on it inertia force which acts through the centre of gravity in the direction opposite to acceleration and is equal to mass times acceleration (ma).
A/C to D Alembert's Principle

$$\sum F + F_i = 0$$

$$\sum T + T_i = 0$$

Also, it states that "Inertia forces and couples and the external forces and torque on the body together gives Static Equilibrium".

- Ve sign indicates that F acts opposite to that of acceleration.

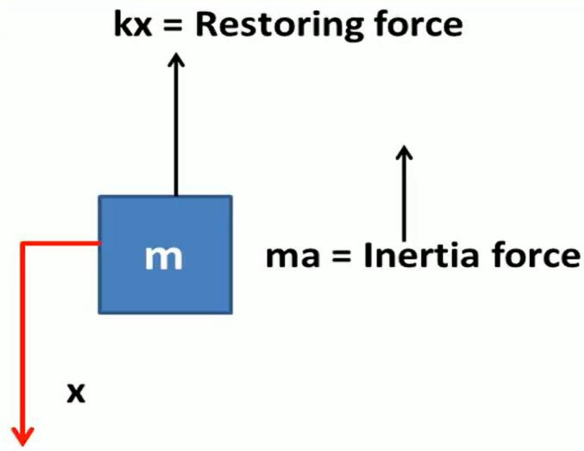
Here,
$\sum F = Vector\ sum\ of\ all\ external\ Forces$
$\sum T = Vector\ sum\ of\ all\ external\ Torgue$

$$F_i = -ma\ (Inertia\ force)$$
$$T_i = -I\alpha\ (Inertia\ Torgue)$$

Here, m = mass of the body
a = Acceleration of centre of mass of the body
I = MOI about an axis passing through the centre of gravity and perpendicular to plane of rotation of body.

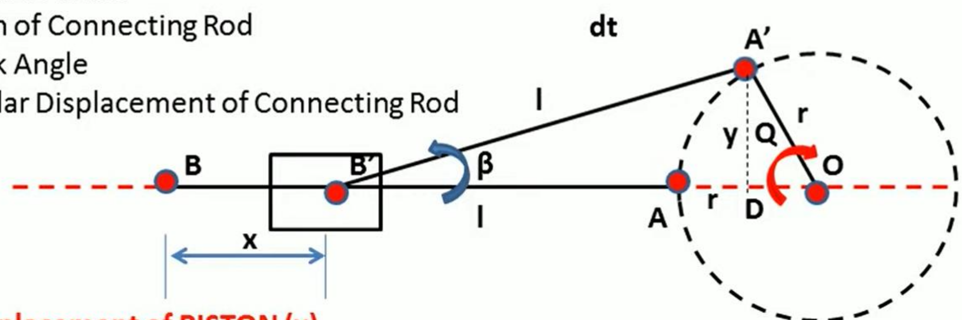
D ALEMBERT'S PRINCIPAL
Continue...

Also we can say that
Inertia force + Restoring Force = zero
Which yields the equation of motion.
As
ma + kx = 0



DYNAMIC ANALYSIS OF SLIDER CRANK MECHANISM

r = Radius of crank
l = Length of Connecting Rod
Q = Crank Angle
β = Angular Displacement of Connecting Rod



Linear Displacement of PISTON (x)

Let, x = Displacement of piston from IDC when the crank has turned through angle Q from IDC.

X = BB' = BO – B'O (From figure)
= (BA + AO) – (B'D + DO)
= (l + r) – (l Cosβ + r Cos Q)

Take l/r = n

Therefore, x = (nr + r) – (nr Cos β + r Cos Q)
= r[(n + 1) – (n Cos β + Cos Q)]

$= r[(n + 1) - (n\frac{1}{n}\sqrt{n^2 - Sin^2 Q} + \text{Cos Q})]$

$= r[(n + 1) - (\sqrt{n^2 - Sin^2 Q} + \text{Cos Q})]$

Also, we know that

Y = l Sin β = r Sin Q

Sin β = Sin Q / n

$Sin^2\beta + Cos^2\beta = 1$

$\text{Cos } \beta = \sqrt{1 - Sin^2\beta}$

$= \text{Cos } \beta = \sqrt{1 - \left(\frac{y}{l}\right)^2}$

$= \sqrt{1 - \left(\frac{r\,Sin\,Q}{l}\right)^2}$

$\text{Cos } \beta = \frac{1}{n}\sqrt{n^2 - Sin^2 Q}$

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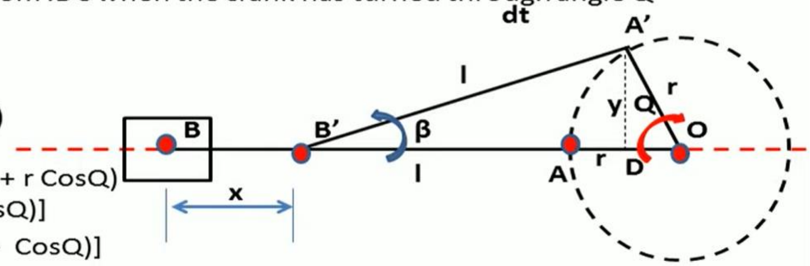

X = BB' = BO – B'O (From figure)

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$$X = r [(1- Cos Q) + (n - \sqrt{n^2 - Sin^2Q})]$$

Derived

If l >> r than n^2 *will be very large*

Therefore, $\sqrt{n^2 - Sin^2Q} = \sqrt{n^2} = n$

Hence, $x = r (1 – Cos Q)$

Therefore, from above equation we can say that,

If length of Connecting rod is large the piston executes **SIMPLE HARMONIC MOTION**

DYNAMIC ANALYSIS OF SLIDER CRANK MECHANISM

Linear Displacement of PISTON (x)

$$X = r [(1- Cos Q) + (n - \sqrt{n^2 - Sin^2Q})]$$

Linear Velocity of PISTON (V)

We know that

V = Rate of change of linear Displacement wrt Time

$$= \frac{dX}{dt}$$

$$= \frac{dX}{dQ} X \frac{dQ}{dt} = w. \frac{dX}{dQ} \quad \text{(Since } \frac{dQ}{dt} = w = \text{angular velocity)}$$

$$= w. \frac{d}{dQ}\{r [(1- Cos Q) + (n - \sqrt{n^2 - Sin^2Q})]\}$$

Taking r common and differentiating above eqn-

$$= r.w [0 – (- Sin Q) + 0 - \frac{1}{2}(n^2 – Sin^2Q)^{-1/2} X (-2Sin Q . Cos Q)$$

$$V = r.w[sin Q + \frac{Sin 2Q}{2\sqrt{n^2 – Sin^2Q}}]$$

Derived

Again, if n^2 *is larger as compared to* Sin^2Q

$$V = r.w\left[sin Q + \frac{Sin 2Q}{2n} \right]$$

Derived

DYNAMIC ANALYSIS OF SLIDER CRANK MECHANISM

Linear Displacement of PISTON (x)

$$X = r\,[\,(1 - \cos Q) + (n - \sqrt{n^2 - Sin^2 Q}\,)]$$

Linear Velocity of PISTON (V)

$$V = r.\,\omega\left[\,\sin Q + \frac{Sin\,2Q}{2n}\right]$$

Linear Acceleration of PISTON (a)

We know that

a = Rate of change of linear Velocity wrt Time

$$= \frac{dV}{dt}$$

$$= \frac{dV}{dQ}\ X\ \frac{dQ}{dt}$$

$$= \omega.\,\frac{dV}{dQ} \quad \left(\text{Since } \frac{dQ}{dt} = \omega = \text{angular velocity}\right)$$

$$= \omega.\,\frac{d}{dQ} r.\,\omega\left[\,\sin Q + \frac{Sin\,2Q}{2n}\right]$$

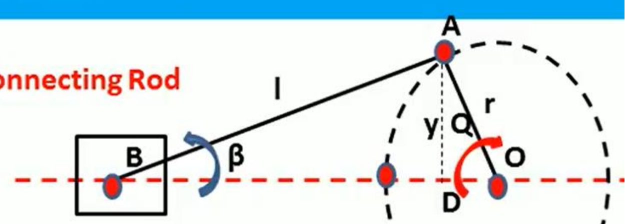
Taking r. ω common and differentiating above eqn-

$$= r.\,\omega^2\left[\cos Q + \frac{Cos\,2Q}{n}\right]$$

Derived

ANGULAR VELOCITY & ACCELERATION OF CONNECTING ROD

Let, $\omega c = \left(\frac{d\beta}{dt}\right)$ = angular velocity of Connecting Rod



We know that

Y = l Sin β = r Sin Q

Or,

$$\text{Sin } \beta = \frac{Sin\,Q}{n} \quad (\text{since } n = l/r)$$

Differentiating both sides w.r.t time

$$\left(\frac{d}{dt}Sin\beta\right) = \frac{d}{dt}\left(\frac{Sin\,Q}{n}\right)$$

$$\left(\frac{d}{d\beta}Sin\beta\right)\left(\frac{d\beta}{dt}\right) = \frac{1}{n}\frac{d}{dQ}\ Sin\ Q\left(\frac{dQ}{dt}\right)$$

$$\text{Cos } \beta\left(\frac{d\beta}{dt}\right) = \frac{1}{n}\text{Cos } Q\,.\,\omega \quad \left(\text{Since, } \left(\frac{dQ}{dt}\right) = \omega\right)$$

$$\left(\frac{d\beta}{dt}\right) = \frac{\omega.Cos\,Q}{n\,Cos\beta} \qquad \ldots(1)$$

$\omega c = \left(\frac{d\beta}{dt}\right)$ = angular velocity of Connecting Rod

Also, we know that

Y = l Sin β = r Sin Q

Sin β = Sin Q / n

$Sin^2\beta + Cos^2\beta = 1$

$$\text{Cos } \beta = \sqrt{1 - Sin^2\beta}$$

$$= \text{Cos } \beta = \sqrt{1 - \left(\frac{y}{l}\right)^2}$$

$$= \sqrt{1 - \left(\frac{r\,Sin\,Q}{l}\right)^2}$$

$$\text{Cos } \beta = \frac{1}{n}\sqrt{n^2 - Sin^2 Q} \ \ldots \quad (2)$$

Therefore from equation (1) and (2)

$$\omega c = \frac{\omega.Cos\,Q}{n\frac{1}{n}\sqrt{n^2 - Sin^2 Q}} = \frac{\omega.Cos\,Q}{\sqrt{n^2 - Sin^2 Q}}$$

Derived