DYNAMIC ANALYSIS OF SLIDER CRANK MECHANISM

Linear Displacement of PISTON (x)

$$X = r\left[(1-\cos Q) + \left(n - \sqrt{n^2 - Sin^2 Q}\right)\right]$$

Linear Velocity of PISTON (V)

$$V = r.\,\omega\left[\sin Q + \frac{Sin\,2Q}{2n}\right]$$

Linear Acceleration of PISTON (a)

We know that

a = Rate of change of linear Velocity wrt Time

$$= \frac{dV}{dt}$$

$$= \frac{dV}{dQ} \times \frac{dQ}{dt}$$

$$= \omega.\frac{dV}{dQ} \quad \left(\text{Since } \frac{dQ}{dt} = \omega = \text{angular velocity}\right)$$

$$= \omega.\frac{d}{dQ} r.\,\omega\left[\sin Q + \frac{Sin\,2Q}{2n}\right]$$

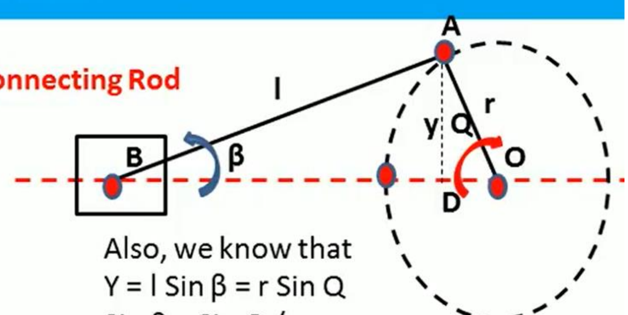
Taking r. ω common and differentiating above eqn-

$$= r.\,\omega^2\left[\cos Q + \frac{Cos\,2Q}{n}\right]$$

Derived

ANGULAR VELOCITY & ACCELERATION OF CONNECTING ROD

Let, $\omega_c = \left(\frac{d\beta}{dt}\right)$ **= angular velocity of Connecting Rod**



We know that

Y = l Sin β = r Sin Q

Or,

$\sin\beta = \frac{Sin\,Q}{n}$ (since n = l/r)

Differentiating both sides w.r.t time

$$\left(\frac{d}{dt}Sin\beta\right) = \frac{d}{dt}\left(\frac{Sin\,Q}{n}\right)$$

$$\left(\frac{d}{d\beta}Sin\beta\right)\left(\frac{d\beta}{dt}\right) = \frac{1}{n}\frac{d}{dQ}\,Sin\,Q\left(\frac{dQ}{dt}\right)$$

$$\cos\beta\left(\frac{d\beta}{dt}\right) = \frac{1}{n}\cos Q\,.\,\omega \quad \left(\text{Since, } \left(\frac{dQ}{dt}\right) = \omega\right)$$

$$\left(\frac{d\beta}{dt}\right) = \frac{\omega.Cos\,Q}{n\,Cos\beta} \qquad ...(1)$$

$\omega_c = \left(\frac{d\beta}{dt}\right)$ = angular velocity of Connecting Rod

Also, we know that

Y = l Sin β = r Sin Q

Sin β = Sin Q / n

$Sin^2\beta + Cos^2\beta = 1$

$\cos\beta = \sqrt{1 - Sin^2\beta}$

$$= \cos\beta = \sqrt{1 - \left(\frac{y}{l}\right)^2}$$

$$= \sqrt{1 - \left(\frac{r\,Sin\,Q}{l}\right)^2}$$

$$\cos\beta = \frac{1}{n}\sqrt{n^2 - Sin^2 Q} \; ... \quad (2)$$

Therefore from equation (1) and (2)

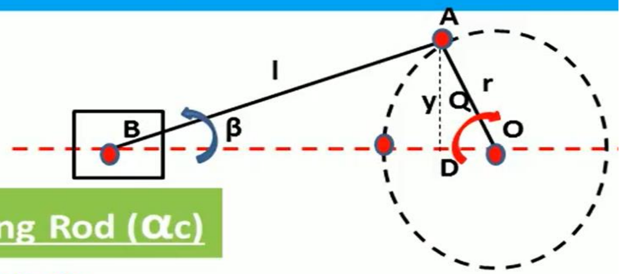
$$\omega_c = \frac{\omega.Cos\,Q}{n\frac{1}{n}\sqrt{n^2 - Sin^2 Q}} = \frac{\omega.Cos\,Q}{\sqrt{n^2 - Sin^2 Q}}$$

Derived

ANGULAR VELOCITY & ACCELERATION OF CONNECTING ROD

Angular Velocity of Connecting Rod

$$\omega_C = \frac{w.Cos\,Q}{n\frac{1}{n}\sqrt{n^2-Sin^2Q}} = \frac{w.Cos\,Q}{\sqrt{n^2-Sin^2Q}}$$



Angular Acceleration of Connecting Rod (αc)

αc = Rate of change of Angular velocity

$$= \frac{d\omega c}{dt} = \frac{d}{dt}\frac{w.Cos\,Q}{\sqrt{n^2-Sin^2Q}}$$

$$= \frac{d}{dQ}\frac{w.Cos\,Q}{\sqrt{n^2-Sin^2Q}}\left(\frac{dQ}{dt}\right)$$

$$= w^2\frac{d}{dQ}\frac{Cos\,Q}{\sqrt{n^2-Sin^2Q}}$$

On Solving

$$\alpha c = -\,w^2 \text{Sin } Q\left[\frac{n^2-1}{(n^2-Sin^2Q)^{3/2}}\right]$$

- Ve sign indicates that angular acceleration tends to decrease the angle β

Forces on the Reciprocating Parts of an Engine, Neglecting the Weight of the Connecting Rod

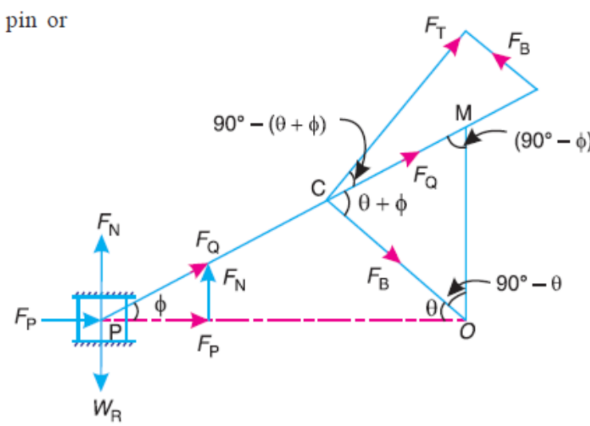
1. ***Piston effort***. It is the net force acting on the piston or crosshead pin, along the line of stroke. It is denoted by F_P

Let m_R = Mass of the reciprocating parts, *e.g.* piston, crosshead pin or gudgeon pin etc., in kg, and

W_R = Weight of the reciprocating parts in newtons = $m_R.g$

We know that acceleration of the reciprocating parts,

$$a_R = a_P = \omega^2.r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$



∴ *Accelerating force or inertia force of the reciprocating parts,

$$F_I = m_R.a_R = m_R.\omega^2.r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

Piston effort, F_P = Net load on the piston ∓ Inertia force

$$= F_L \mp F_I \quad \text{...(Neglecting frictional resistance)}$$

$$= F_L \mp F_I - R_F \quad \text{...(Considering frictional resistance)}$$

where R_F = Frictional resistance.

The –ve sign is used when the piston is accelerated, and +ve sign is used when the piston is retarded.

In a double acting reciprocating steam engine, net load on the piston,

$$F_L = p_1 A_1 - p_2 A_2 = p_1 A_1 - p_2 (A_1 - a)$$

where p_1, A_1 = Pressure and cross-sectional area on the back end side of the piston,

p_2, A_2 = Pressure and cross-sectional area on the crank end side of the piston,

a = Cross-sectional area of the piston rod.

Notes : 1. If 'p' is the net pressure of steam or gas on the piston and D is diameter of the piston, then

Net load on the piston, $F_L = \text{Pressure} \times \text{Area} = p \times \frac{\pi}{4} \times D^2$

2. In case of a vertical engine, the weight of the reciprocating parts assists the piston effort during the downward stroke (*i.e.* when the piston moves from top dead centre to bottom dead centre) and opposes during the upward stroke of the piston (*i.e.* when the piston moves from bottom dead centre to top dead centre).

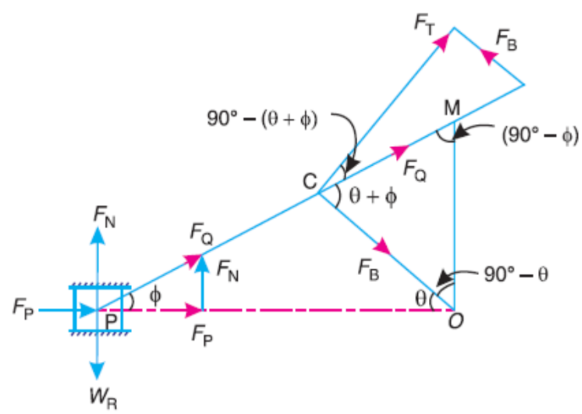
∴ Piston effort, $F_P = F_L \mp F_I \pm W_R - R_F$

2. *Force acting along the connecting rod.* It is denoted by F_Q in Fig. From the geometry of the figure, we find that

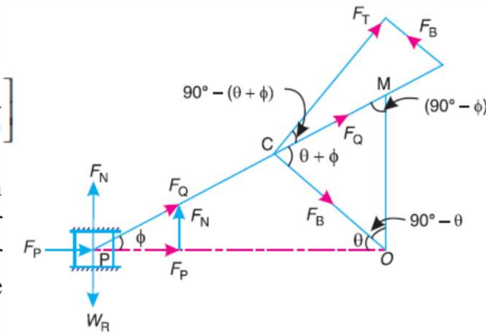
$$F_Q = \frac{F_P}{\cos \phi}$$

We know that $\cos \phi = \sqrt{1 - \frac{\sin^2 \theta}{n^2}}$

$$\therefore \quad F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2 \theta}{n^2}}}$$



3. *Thrust on the sides of the cylinder walls or normal reaction on the guide bars.* It is denoted by F_N in Fig. From the figure, we find that



$$F_N = F_Q \sin\phi = \frac{F_P}{\cos\phi} \times \sin\phi = F_P \tan\phi \qquad \ldots\left[\because F_Q = \frac{F_P}{\cos\phi}\right]$$

4. *Crank-pin effort and thrust on crank shaft bearings.* The force acting on the connecting rod F_Q may be resolved into two components, on perpendicular to the crank and the other along the crank. The component of F_Q perpendicular to the crank is known as ***crank-pin effort*** and it is ***denoted by F_T*** in Fig. The component of F_Q along the crank produces a thrust on the crank shaft bearings and it is denoted by F_B in Fig.

Resolving F_Q perpendicular to the crank,

$$F_T = F_Q \sin(\theta + \phi) = \frac{F_P}{\cos\phi} \times \sin(\theta + \phi)$$

and resolving F_Q along the crank,

$$F_B = F_Q \cos(\theta + \phi) = \frac{F_P}{\cos\phi} \times \cos(\theta + \phi)$$

5. *Crank effort* or ***turning moment or torque on the crank shaft.*** The product of the crankpin effort (F_T) and the crank pin radius (r) is known as ***crank effort*** or ***turning moment*** or ***torque on the crank shaft***. Mathematically,

Crank effort,
$$T = F_T \times r = \frac{F_P \sin(\theta + \phi)}{\cos\phi} \times r$$
$$= \frac{F_P (\sin\theta\cos\phi + \cos\theta\sin\phi)}{\cos\phi} \times r$$
$$= F_P\left(\sin\theta + \cos\theta \times \frac{\sin\phi}{\cos\phi}\right) \times r$$
$$= F_P(\sin\theta + \cos\theta\tan\phi) \times r$$

We know that $l \sin\phi = r \sin\theta$

$$\sin\phi = \frac{r}{l}\sin\theta = \frac{\sin\theta}{n}$$

and
$$\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

∴
$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{n} \times \frac{n}{\sqrt{n^2 - \sin^2\theta}} = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Substituting the value of $\tan\phi$ in equation (*i*), we have crank effort,

$$T = F_P\left(\sin\theta + \frac{\cos\theta\sin\theta}{\sqrt{n^2 - \sin^2\theta}}\right) \times r$$
$$= F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \qquad \ldots(ii)$$

$\ldots(\because 2\cos\theta\sin\theta = \sin 2\theta)$

Problem 1. *The crank-pin circle radius of a horizontal engine is 300 mm. The mass of the reciprocating parts is 250 kg. When the crank has travelled 60° from I.D.C., the difference between the driving and the back pressures is 0.35 N/mm2. The connecting rod length between centres is 1.2 m and the cylinder bore is 0.5 m. If the engine runs at 250 r.p.m. and if the effect of piston rod diameter is neglected, calculate :* **1.** *pressure on slide bars,* **2.** *thrust in the connecting rod,* **3.** *tangential force on the crank-pin, and* **4.** *turning moment on the crank shaft.*

Solution. Given: $r = 300$ mm $= 0.3$ m ; $m_R = 250$ kg; $\theta = 60°$; $p_1 - p_2 = 0.35$ N/mm^2; $l = 1.2$ m ; $D = 0.5$ m $= 500$ mm ; $N = 250$ r.p.m. or $\omega = 2\pi \times 250/60 = 26.2$ rad/s

First of all, let us find out the piston effort (F_P).

We know that net load on the piston,

$$F_L = (p_1 - p_2)\frac{\pi}{4} \times D^2 = 0.35 \times \frac{\pi}{4}(500)^2 = 68730 \text{ N}$$

...($\because$ Force = Pressure × Area)

Ratio of length of connecting rod and crank,

$$n = l/r = 1.2/0.3 = 4$$

and accelerating or inertia force on reciprocating parts,

$$F_I = m_R \cdot \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 250\,(26.2)^2\ 0.3\left(\cos 60° + \frac{\cos 120°}{4}\right) = 19306 \text{ N}$$

$\therefore$ Piston effort, $F_P = F_L - F_I = 68\,730 - 19\,306 = 49\,424$ N $= 49.424$ kN

1. ***Pressure on slide bars***

Let ϕ = Angle of inclination of the connecting rod to the line of stroke.

We know that, $$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 60°}{4} = \frac{0.866}{4} = 0.2165$$

$\therefore$ $\phi = 12.5°$

We know that pressure on the slide bars,

$$F_N = F_P \tan\phi = 49.424 \times \tan 12.5° = 10.96 \text{ kN}$$ **Ans.**

2. ***Thrust in the connecting rod***

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{49.424}{\cos 12.5°} = 50.62 \text{ kN}$$ **Ans.**

3. ***Tangential force on the crank-pin***

We know that tangential force on the crank pin,

$$F_T = F_Q \sin(\theta + \phi) = 50.62 \sin(60° + 12.5°) = 48.28 \text{ kN}$$ **Ans.**

4. ***Turning moment on the crank shaft***

We know that turning moment on the crank shaft,

$$T = F_T \times r = 48.28 \times 0.3 = 14.484 \text{ kN-m}$$ **Ans.**

Problem 2. *The crank and connecting rod of a petrol engine, running at 1800 r.p.m. are 50 mm and 200 mm respectively. The diameter of the piston is 80 mm and the mass of the reciprocating parts is 1 kg. At a point during the power stroke, the pressure on the piston is 0.7 N/mm², when it has moved 10 mm from the inner dead centre. Determine :* ***1.*** *Net load on the gudgeon pin,* ***2.*** *Thrust in the connecting rod,* ***3.*** *Reaction between the piston and cylinder, and* ***4.*** *The engine speed at which the above values become zero.*

Solution. Given : $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.52$ rad/s ; $r = 50$ mm $= 0.05$ m; $l = 200$ mm ; $D = 80$ mm ; $m_R = 1$ kg ; $p = 0.7$ N/mm² ; $x = 10$ mm

1. Net load on the gudgeon pin

We know that load on the piston,

$$F_L = \frac{\pi}{4} D^2 \times p = \frac{\pi}{4} \times (80)^2 \times 0.7 = 3520 \text{ N}$$

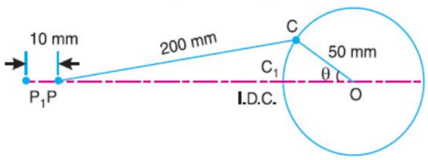


Fig. 15.10

When the piston has moved 10 mm from the inner dead centre, *i.e.* when $P_1P = 10$ mm, the crank rotates from OC_1 to OC through an angle θ as shown in Fig. 15.10.

By measurement, we find that *$\theta = 33°$.

We know that ratio of lengths of connecting rod and crank,

$$n = l/r = 200/50 = 4$$

* The angle θ may also be obtained as follows:

We know that $x = r\left[(1 - \cos\theta) + \frac{\sin^2\theta}{2n}\right] = r\left[(1 - \cos\theta) + \frac{1 - \cos^2\theta}{2n}\right]$

$$10 = 50\left[(1 - \cos\theta) + \frac{1 - \cos^2\theta}{2 \times 4}\right] = \frac{50}{8}\left[(8 - 8\cos\theta + 1 - \cos^2\theta\right]$$

$$= 50 - 50\cos\theta + 6.25 - 6.25\cos^2\theta$$

or $6.25\cos^2\theta + 50\cos\theta - 56.25 = 0$

Solving this quadratic equation, we get $\theta = 33.14°$

and inertia force on the reciprocating parts,

$$F_I = m_R . a_R = m_R . \omega^2 . r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= 1 \times (188.52)^2 \times 0.05\left(\cos 33° + \frac{\cos 66°}{4}\right) = 1671 \text{ N}$$

We know that net load on the gudgeon pin,

$$F_P = F_L - F_I = 3520 - 1671 = 1849 \text{ N}$$ **Ans.**

2. Thrust in the connecting rod

Let ϕ = Angle of inclination of the connecting rod to the line of stroke.

We know that, $\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 33°}{4} = \frac{0.5446}{4} = 0.1361$

∴ $\phi = 7.82°$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{1849}{\cos 7.82°} = 1866.3 \text{ N}$$ **Ans.**

3. *Reaction between the piston and cylinder*

We know that reaction between the piston and cylinder,

$$F_N = F_P \tan\phi = 1849 \tan 7.82° = 254 \text{ N} \quad \textbf{Ans.}$$

4. *Engine speed at which the above values will become zero*

A little consideration will show that the above values will become zero, if the inertia force on the reciprocating parts (F_I) is equal to the load on the piston (F_L). Let ω_1 be the speed in rad/s, at which $F_I = F_L$.

$$\therefore \quad m_R\,(\omega_1)^2\,r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = \frac{\pi}{4}D^2 \times p$$

$$1\,(\omega_1)^2 \times 0.05\left(\cos 33° + \frac{\cos 66°}{4}\right) = \frac{\pi}{4} \times (80)^2 \times 0.7 \quad \text{or} \quad 0.0\,47\,(\omega_1)^2 = 3520$$

$$\therefore \quad (\omega_1)^2 = 3520 / 0.047 = 74\,894 \text{ or } \omega_1 = 273.6 \text{ rad/s}$$

$\therefore$ Corresponding speed in r.p.m.,

$$N_1 = 273.6 \times 60 / 2\pi = 2612 \text{ r.p.m.} \quad \textbf{Ans.}$$