Gear Trains

Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels:

1. Simple gear train, **2.** Compound gear train, **3.** Reverted gear train, and **4.** Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Epicyclic Gear Train

A simple epicyclic gear train is shown in Fig.1, where a gear *A* and the arm *C* have a common axis at *O*1 about which they can rotate. The gear *B* meshes with gear *A* and has its axis on the arm at *O*2, about which the gear *B* can rotate. If the arm is fixed, the gear train is simple and gear *A* can drive gear *B* or ***vice- versa***, but if gear *A* is fixed and the arm is rotated about the axis of gear *A* (*i.e.* *O*1), then the gear *B* is forced to rotate ***upon*** and ***around*** gear *A*. Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi.*** means upon and ***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound.***

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

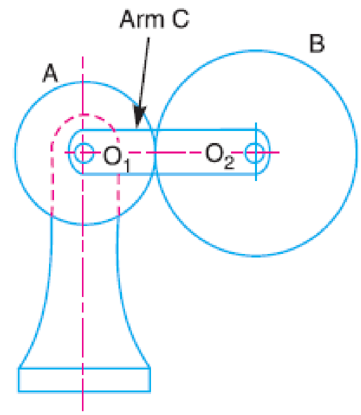


Fig.1. Epicyclic gear train.

Velocity Ratio of Epicyclic Gear Train

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular method, and **2.** Algebraic method.

These methods are discussed, in detail, as follows:

1. *Tabular method*. Consider an epicyclic gear train as shown in Fig. 1.

Let T_A = Number of teeth on gear *A*, and T_B = Number of teeth on gear *B*.

We know that $N_B / N_A = T_A / T_B$. Since NA = 1 revolution, therefore $N_B = T_A / T_B$.

Step No.	*Conditions of motion*	*Revolutions of elements*		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through $+ x$ revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

2. *Algebraic method.* In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train *viz.* some element is fixed and the other has specified motion. These two conditions are sufficient to solve all the equations ; and hence to determine the motion of any element in the epicyclic gear train.

Let the arm C be fixed in an epicyclic gear train as shown in Fig. 13.6. Therefore speed of the gear A relative to the arm C

$$= N_A - N_C$$

and speed of the gear B relative to the arm C,

$$= N_B - N_C$$

Since the gears A and B are meshing directly, therefore they will revolve in *opposite* directions.

$$\therefore \quad \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

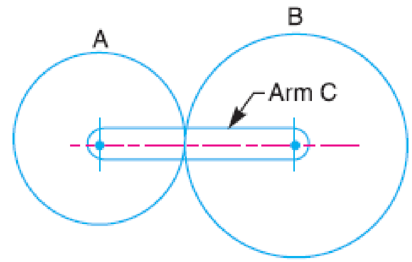
Since the arm C is fixed, therefore its speed, $N_C = 0$.

$$\therefore \quad \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If the gear A is fixed, then $N_A = 0$.

$$\frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B} \quad \text{or} \quad \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Problem 1. *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B?*



Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

1. *Tabular method*

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +\,150 \text{ r.p.m.}$$

Also the gear A is fixed, therefore

$$x + y = 0 \quad \text{or} \quad x = -y = -\,150 \text{ r.p.m.}$$

$$\therefore \text{ Speed of gear } B, \quad N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +\,270 \text{ r.p.m.}$$

$$= 270 \text{ r.p.m. (anticlockwise)} \quad \textbf{Ans.}$$

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -\,300 \quad \text{or} \quad x = -\,300 - y = -\,300 - 150 = -\,450 \text{ r.p.m.}$$

$\therefore$ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +\,510 \text{ r.p.m.}$$

$$= 510 \text{ r.p.m. (anticlockwise)} \quad \textbf{Ans.}$$

2. *Algebraic method*

Let N_A = Speed of gear A.

N_B = Speed of gear B, and

N_C = Speed of arm C.

Assuming the arm C to be fixed, speed of gear A relative to arm C

$$= N_A - N_C$$

and speed of gear B relative to arm $C = N_B - N_C$

Since the gears A and B revolve in *opposite* directions, therefore

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \quad \text{...}(i)$$

Speed of gear B when gear A is fixed

When gear A is fixed, the arm rotates at 150 r.p.m. in the anticlockwise direction, *i.e.*

$$N_A = 0, \quad \text{and} \quad N_C = +150 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{0 - 150} = -\frac{36}{45} = -0.8 \quad \text{...[From equation } (i)]$$

or $$N_B = -150 \times -0.8 + 150 = 120 + 150 = 270 \text{ r.p.m.} \quad \textbf{Ans.}$$

Speed of gear B when gear A makes 300 r.p.m. clockwise

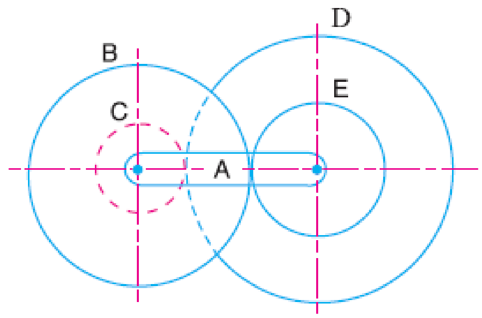
Since the gear A makes 300 r.p.m. clockwise, therefore

$$N_A = -300 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{-300 - 150} = -\frac{36}{45} = -0.8$$

or $$N_B = -450 \times -0.8 + 150 = 360 + 150 = 510 \text{ r.p.m.} \quad \textbf{Ans.}$$

Problem 2. *In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.*



Solution. Given: $T_B = 75$; $T_C = 30$; $T_D = 90$; $N_A = 100$ r.p.m. (clockwise)

The reverted epicyclic gear train is shown in Fig. First of all, let us find the number of teeth on gear E (T_E). Let dB , d_C , d_D and d_E be the pitch circle diameters of gears B, C, D and E respectively. From the geometry of the figure,

$d_B + d_E = d_C + d_D$

Since the number of teeth on each gear, for the same module, are proportional to their pitch circle diameters, therefore

$T_B + T_E = T_C + T_D$

$\therefore T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$

The table of motions is drawn as follows:

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D-E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D-E* rotated through + *x* revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Since the gear *B* is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_E}{T_B} = 0 \quad \text{or} \quad y - x \times \frac{45}{75} = 0$$

$$\therefore \qquad y - 0.6 = 0 \qquad \text{...(}i\text{)}$$

Also the arm *A* makes 100 r.p.m. clockwise, therefore

$$y = -100 \qquad \text{...(}ii\text{)}$$

Substituting $y = -100$ in equation (*i*), we get

$$-100 - 0.6\,x = 0 \quad \text{or} \quad x = -100 / 0.6 = -166.67$$

From the fourth row of the table, speed of gear *C*,

$$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = +\,400 \text{ r.p.m.}$$

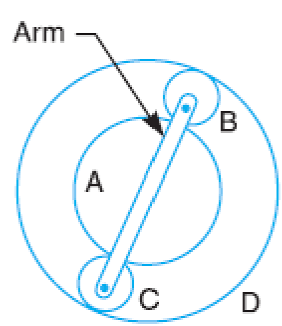
$$= 400 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

Problem 3. *An epicyclic train of gears is arranged as shown in Fig. How many revolutions does the arm, to which the pinions B and C are attached, make?*

1. *When A makes one revolution clockwise and D makes half a revolution anticlockwise, and*

2. *When A makes one revolution clockwise and D is stationary?*

The number of teeth on the gears A and D are 40 and 90 respectively.



Solution. Given : $T_A = 40$; $T_D = 90$

First of all, let us find the number of teeth on gears B and C (*i.e.* T_B and T_C). Let d_A, d_B, d_C and d_D be the pitch circle diameters of gears A, B, C and D respectively. Therefore from the geometry of the figure,

$$d_A + d_B + d_C = d_D \quad \text{or} \quad d_A + 2\,d_B = d_D \qquad ...(\because d_B = d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \quad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(\because T_B = T_C)$$

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm	*Gear A*	*Compound gear B-C*	*Gear D*
1.	Arm fixed, gear A rotates through -1 revolution (*i.e.* 1 rev. clockwise)	0	-1	$+\dfrac{T_A}{T_B}$	$+\dfrac{T_A}{T_B}\times\dfrac{T_B}{T_D} = +\dfrac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x\times\dfrac{T_A}{T_B}$	$+x\times\dfrac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\dfrac{T_A}{T_B} - y$	$x\times\dfrac{T_A}{T_D} - y$

1. ***Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise***

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1 \quad \text{or} \quad x + y = 1 \qquad ...(i)$$

Also, the gear D makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D} - y = \frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90} - y = \frac{1}{2}$$

$$\therefore \quad 40\,x - 90\,y = 45 \quad \text{or} \quad x - 2.25\,y = 1.125 \qquad ...(ii)$$

From equations (*i*) and (*ii*), $x = 1.04$ and $y = -0.04$

$$\therefore \quad \text{Speed of arm} = -y = -(-0.04) = +0.04$$

$$= 0.04 \text{ revolution anticlockwise} \quad \textbf{Ans.}$$

2. ***Speed of arm when A makes 1 revolution clockwise and D is stationary***

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1 \quad \text{or} \quad x + y = 1 \qquad ...(iii)$$

Also the gear D is stationary, therefore

$$x\times\frac{T_A}{T_D} - y = 0 \quad \text{or} \quad x\times\frac{40}{90} - y = 0$$

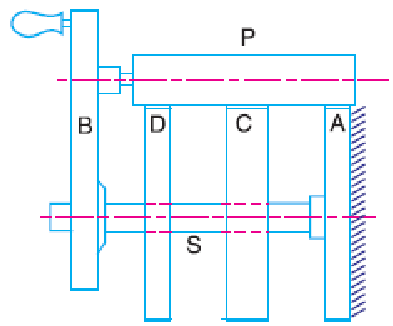
$$\therefore \quad 40\,x - 90\,y = 0 \quad \text{or} \quad x - 2.25\,y = 0 \qquad ...(iv)$$

From equations (*iii*) and (*iv*),

$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$\therefore$ Speed of arm $= -y = -0.308 = 0.308$ revolution clockwise **Ans.**

Problem 4. *The Fig. shows an epicyclic gear train known as Ferguson's paradox. Gear A is fixed to the frame and is, therefore, stationary. The arm B and gears C and D are free to rotate on the shaft S. Gears A, C and D have 100, 101 and 99 teeth respectively. The planet gear has 20 teeth. The pitch circle diameters of all are the same so that the planet gear P meshes with all of them. Determine the revolutions of gears C and D for one revolution of the arm B.*



Solution. Given: $T_A = 100$; $T_C = 101$; $T_D = 99$; $T_P = 20$

Step No.	Conditions of motion	Revolutions of elements			
		Arm B	Gear A	Gear C	Gear D
1.	Arm B fixed, gear A rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$+\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_C}{T_D}=+\frac{T_A}{T_D}$
2.	Arm B fixed, gear A rotated through $+x$ revolutions	0	$+x$	$+x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_D}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_D}$

The arm B makes one revolution, therefore

$$y = 1$$

Since the gear A is fixed, therefore from the fourth row of the table,

$$x + y = 0 \quad \text{or} \quad x = -y = -1$$

Let N_C and N_D = Revolutions of gears C and D respectively.

From the fourth row of the table, the revolutions of gear C,

$$N_C = y + x \times \frac{T_A}{T_C} = 1 - 1 \times \frac{100}{101} = +\frac{1}{101} \quad \textbf{Ans.}$$

and the revolutions of gear D,

$$N_D = y + x \times \frac{T_A}{T_D} = 1 - \frac{100}{99} = -\frac{1}{99} \quad \textbf{Ans.}$$

From above we see that for one revolution of the arm B, the gear C rotates through 1/101 revolutions in the same direction and the gear D rotates through 1/99 revolutions in the opposite direction.