SNS COLLEGE OF TECHNOLOGY

16ME207- STRENGTH OF MATERIALS

UNIT I - STRESS STRAIN DEFORMATION OF SOLIDS

Stresses in Stepped shafts and varying sections

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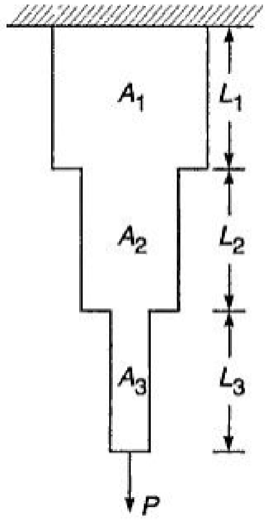
BARS WITH CROSS SECTIONS VARYING IN STEPS

Consider a bar of varying three sections of lengths L1 L2 and L3 having respective areas of cross-sections A1, A2 and A3 subjected to an axial pull P. Let δL1, δL2, δL3 be the changes in length of the respective three sections of the bar, then we have

$$\delta L_1 = \frac{PL_1}{A_1 E}, \quad \delta L_2 = \frac{PL_2}{A_2 E}, \quad \delta L_3 = \frac{PL_3}{A_3 E}$$



Now the total elongation of the bar,

$$\delta L = \delta L_1 + \delta L_2 + \delta L_3 = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E} = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

BARS WITH CONTINUOUSLY VARYING CROSS SECTIONS

Bars with varying Circular cross section

A bar uniformly tapering from a diameter D1 at one end to a diameter D2 at the other end is shown in Figure.

Let P = Axial tensile load on the bar

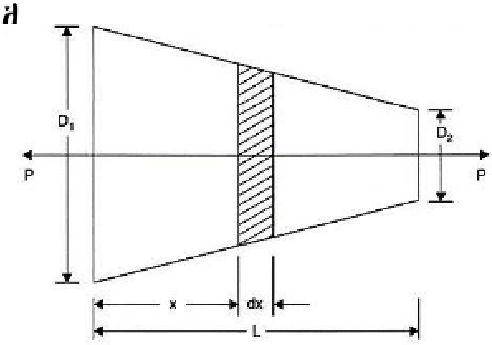
L = Total length of the bar

E = Young's modulus.

Consider a small element of length dx of the bar at a distance x from the left end. Let the diameter of the bar be D at a distance x from the left end

$$D_x = D_1 - \left(\frac{D_1 - D_2}{L}\right) x$$

$$= D_1 - kx \qquad \text{where } k = \frac{D_1 - D_2}{L}$$



BARS WITH CONTINUOUSLY VARYING CROSS SECTIONS

$$A_x = \frac{\pi}{4} D_x^2 = \frac{\pi}{4} (D_1 - k.x)^2.$$

$$\sigma_x = \frac{\text{Load}}{A_x}$$

$$= \frac{P}{\frac{\pi}{4} (D_1 - k.x)^2} = \frac{4P}{\pi (D_1 - k.x)^2}$$

$$e_x = \frac{\text{Stress}}{E} = \frac{\sigma_x}{E}$$

$$= \frac{4P}{\pi (D_1 - k.x)^2} \times \frac{1}{E} = \frac{4P}{\pi E (D_1 - k.x)^2}$$

$$= \text{Strain. } dx = e_x \, . \, dx$$

$$= \frac{4P}{\pi E (D_1 - k \, . \, x)^2} \, . \, dx$$

BARS WITH CONTINUOUSLY VARYING CROSS SECTIONS

∴ Total extension,

$$dL = \int_0^L \frac{4P \cdot dx}{\pi E (D_1 - k.x)^2} = \frac{4P}{\pi E}\int_0^L (D_1 - k.x)^{-2} \cdot dx$$

$$= \frac{4P}{\pi E}\int_0^L \frac{(D_1 - k \cdot x)^{-2} \times (-k)}{(-k)} \cdot dx \quad \text{[Multiplying and dividing by } (-k)\text{]}$$

$$= \frac{4P}{\pi E}\left[\frac{(D_1 - k \cdot x)^{-1}}{(-1) \times (-k)}\right]_0^L = \frac{4P}{\pi E k}\left[\frac{1}{(D_1 - k \cdot x)}\right]_0^L$$

$$= \frac{4P}{\pi E k}\left[\frac{1}{D_1 - k \cdot L} - \frac{1}{D_1 - k \times 0}\right]$$

$$= \frac{4P}{\pi E k}\left[\frac{1}{D_1 - k \cdot L} - \frac{1}{D_1}\right]$$

$$k = \frac{D_1 - D_2}{L}$$

BARS WITH CONTINUOUSLY VARYING CROSS SECTIONS

$$dL = \frac{4P}{\pi E \cdot \left(\frac{D_1 - D_2}{L}\right)} \left[\frac{1}{D_1 - \left(\frac{D_1 - D_2}{L}\right) \cdot L} - \frac{1}{D_1} \right]$$

$$= \frac{4PL}{\pi E \cdot (D_1 - D_2)} \left[\frac{1}{D_1 - D_1 + D_2} - \frac{1}{D_1} \right]$$

$$= \frac{4PL}{\pi E \cdot (D_1 - D_2)} \left[\frac{1}{D_2} - \frac{1}{D_1} \right]$$

$$= \frac{4PL}{\pi E \cdot (D_1 - D_2)} \times \frac{(D_1 - D_2)}{D_1 D_2} = \frac{4PL}{\pi E D_1 D_2}$$

$$\text{Total extension, } dL = \frac{4PL}{\pi E \cdot D^2}$$