16ME207- STRENGTH OF MATERIALS

UNIT- IV DEFLECTION OF BEAMS AND BUCKLING OF COLUMNS

Equivalent length of a column - Problems

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Equivalent length of a column

TABLE : 1

Case	End Conditions.	Equivalent length, l_e.	Buckling load, Euler.
1.	Both ends hinged (or) pin jointed (or) rounded (or) free.	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	One end fixed, Other end free	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	One end fixed, Other end pin jointed.	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	Both ends fixed (or) encastered.	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

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Equivalent length of a column - Problems

A slender pin ended aluminum column 1.8 m long and of circular cross-section is to have an outside diameter of 50 mm. Calculate the necessary internal diameter to prevent failure by buckling if the actual load applied is 13.6 kN and the critical load applied is twice the actual load. Take E for aluminum as 70 GN/m2.

Solution :-

Outside diameter of the Column,

$D = 50\text{mm} = 0.05\text{m}$

Inside diameter of the Column, d.

Area of the Column,

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(0.05^2 - d^2)$$

Moment of Inertia of the Column,

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(0.05^4 - d^4)$$

Also, Critical load = 2 × Safe load (Given).

= 2 × 13.6 = 27.2 kN.

End Conditions :- Pin ended.

$l_e = l = 1.8\text{m}.$

Using the relation, $P_{Euler} = \frac{\pi^2 EI}{l_e^2}$, We get.

$$27.2\times10^3 = \frac{\pi^2 \times 70\times10^9 \times \frac{\pi}{64}(0.05^4 - d^4)}{1.8^2}$$

$$(0.05^4 - d^4) = \frac{27.2\times10^3 \times 1.8^2 \times 64}{\pi^2 \times 70\times10^9 \times \pi} = 2.6\times10^{-6}$$

$$d^4 = 6.25\times10^{-6} - 2.6\times10^{-6} = 3.65\times10^{-6}.$$

$d = 0.0437\text{m} = 43.7\text{mm}.$

Inside diameter, d = 43.7mm

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Equivalent length of a column - Problems

A bar of length 4m when used as a simply supported beam and subjected to a u.d.l. of 30 kN/m over the whole span, deflects 15 mm at the centre. Determine the crippling loads when it is used as a column with following end conditions:a) Both ends pin-jointed;b) One end fixed and other end hinged;c) Both ends fixed.

Solution:-

Given Data:-

Length of the bar $l = 4m$

uniformly distributed load, $W = 30 kN/m$.

Deflection, $\delta = 15 mm = 0.015 m$.

We know that, $\delta = \frac{5Wl^4}{384EI}$ (X)

$$0.015 = \frac{5 \times (30 \times 10^3) \times 4^4}{384 EI}$$

$$EI = \frac{5 \times (30 \times 10^3) \times 4^4}{0.015 \times 384} = 6.66 \times 10^6 Nm^2$$

i) $P_{Euler} = \frac{\pi^2 EI}{l_e^2}$ ($l_e = l = 4m$).

$$= \frac{\pi^2 \times 6.66 \times 10^6}{4^2} = 4108 kN$$

ii) $P_{Euler} = \frac{\pi^2 EI}{l_e^2}$ $\left[l_e = \frac{l}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.83m\right]$

$$= \frac{\pi^2 EI}{2.83^2} = \frac{\pi^2 \times 6.66 \times 10^6}{2.83^2} = 8207 kN.$$

ii) $P_{Euler} = \frac{\pi^2 EI}{l_e^2}$ $\left[l_e = \frac{l}{2} = \frac{4}{2} = 2m\right]$

$$= \frac{\pi^2 EI}{2^2} = \frac{\pi^2 \times 6.66 \times 10^6}{2^2} = 16432 kN.$$