SNS COLLEGE OF TECHNOLOGY

16ME207- STRENGTH OF MATERIALS

UNIT II - TORSION AND SPRINGS

Shear stress distribution

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Power transmitted by a shaft

The main purpose of a shaft is to transmit its power to another member. Let a rotating shaft transmitting power from one of its ends to another be considered. Let N be the revolutions per minute (rpm), T be the average torque N · m and ω be the angular speed of the shaft.

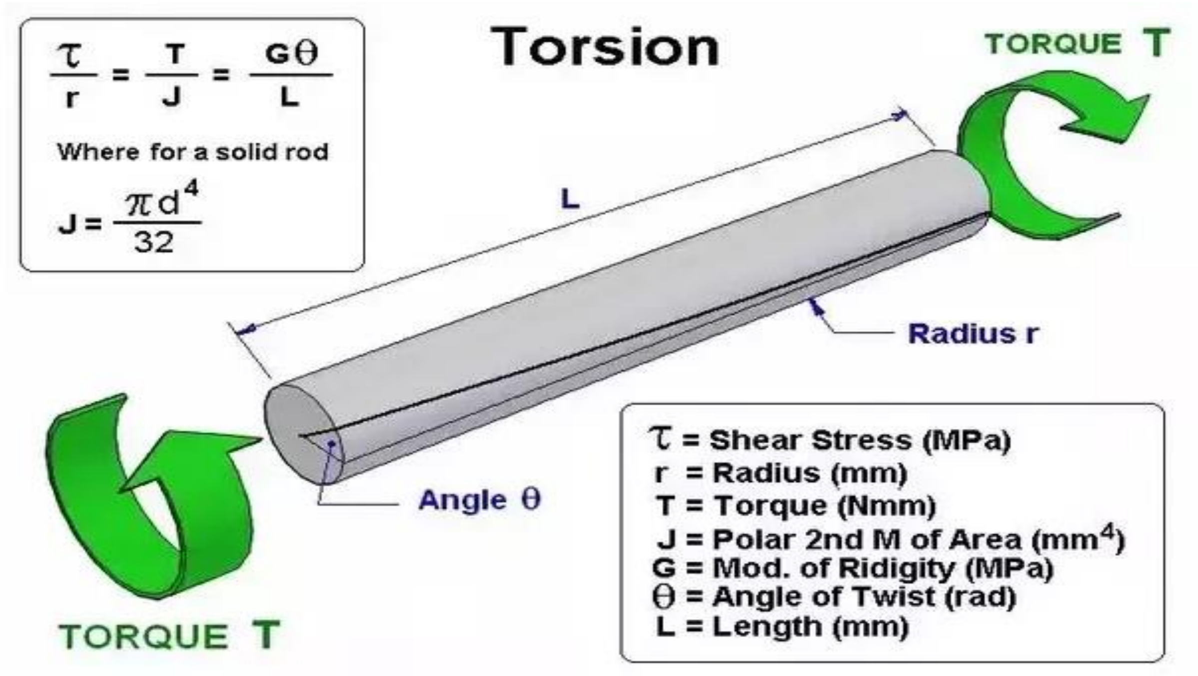
$$\text{Work done per minute} = (\text{Force}) \times (\text{Distance})$$

$$= (\text{Average torque}) \times (\text{Angular displacement})$$

$$= T \times 2\pi N/60$$

Torsional Rigidity

Torsional rigidity is the resistance of the shaft to this shearing stress. The torsional equation is expressed as:



Problem

1. In a tensile test a test piece of 25 mm diameter, 200mm gauge length, stretched 0.0975 mm under a pull of 50 kN. In a torsion test, the same rod twisted 0.025 radian over a length of 200 mm when a torque of 0.4kNm was applied. Evaluate Poisson's ratio and the three elastic moduli for the material.

Solution:-

Young's Modulus

E = Young's Modulus for the test Piece.

$\delta l = \frac{Wl}{AE}$, Where $A = \frac{\pi}{4} \times \left[\frac{25}{1000}\right]^2 m^2$

$A = 4.91 \times 10^{-4} m^2$.

$$0.0975 \times 10^{-3} = \frac{50 \times 10^3 \times 0.2}{4.91 \times 10^{-4} \times E}$$

$$E = \frac{50 \times 10^3 \times 0.2}{(0.0975 \times 10^{-3}) \times 4.91 \times 10^{-4}} \times 10^{-9} \; GN/m^2.$$

$$\boxed{E = 208 \; GN/m^2}$$

Modulus of Rigidity.

C = Modulus of rigidity for the test piece.

I_p = Polar moment of Inertia of a solid Circular shaft.

$$= \frac{\pi}{32} \times D^4 = \frac{\pi}{32} \times \left[\frac{25}{1000}\right]^4 = 3.835 \times 10^{-8} m^4.$$

Using the following relation, we have.

$$\frac{T}{I_p} = \frac{C\theta}{l}$$

$$C = \frac{Tl}{I_p \theta} = \frac{(0.4 \times 10^3) \times 0.2}{3.835 \times 10^{-8} \times 0.025} \times 10^{-9} \; GN/m^2.$$

Problem

$C = 83.44\ GN/m^2$

Poisson's ratio:-

$1/m$ = Poisson's ratio for the test piece

Using the following relation.

$$C = \frac{mE}{2(m+1)}$$

$$83.44 \times 10^9 = \frac{m \times 208 \times 10^9}{2(m+1)}$$

$$166.88m + 166.88 = 208m$$

$$1/m = 0.246$$

$1/m = 0.246$

Bulk modulus:-

K = Bulk modulus for the test piece

$$K = \frac{mE}{3(m-1)} = \frac{\frac{1}{0.246} \times 208 \times 10^9}{3\left[\frac{1}{0.246} - 1\right]} \times 10^{-9}\ GN/m^2$$

$K = 136.4\ GN/m^2$

2. A solid circular shaft transmits 75 kW power at 200 r.p.m. Calculate the shaft diameter, if the twist in the shaft is not to exceed 1o in 2 meters length of shaft, and shear stress is limited to 50 MN/m2. Take C = 100 GN/m2.

Solution:-

Given:-

$P = 75 kW$; $N = 200 rpm$.

$\theta = 1^\circ = 1 \times \pi/180$ radian; $l = 2 m$.

$\tau = 50 MN/m^2$; $C = 100 GN/m^2$

using the relation:

$$P = \frac{2\pi NT}{60 \times 1000}, \quad 75 = \frac{2\pi \times 200 \times T}{60 \times 1000}$$

$$T = \frac{75 \times 60 \times 1000}{2\pi \times 200} = 3581 \text{ Nm}$$

$$\boxed{T = 3581 \text{ Nm}}$$

First case: Allowable Shear Stress (50 MN/m²), D:

Using the relation, $T = \tau \times \frac{\pi}{16} \times D^3$

$$3581 = 50 \times 10^6 \times \frac{\pi}{16} \times D^3$$

$$D^3 = \frac{3581 \times 16}{50 \times 10^6 \times \pi}$$

$D = 0.0714 m$ (or) $71.4 mm$

$$\boxed{D = 0.0714 m \text{ (or) } 71.4 \text{ mm}}$$

Problem

Second case : Angle of twist (1°), D.

Using the relation,

$$\frac{T}{I_p} = \frac{C\theta}{l}$$

$$\frac{3581}{\frac{\pi}{32} \times D^4} = \frac{100\times10^9 \times 1 \times \pi/180}{2}$$

$$D^4 = \frac{3581 \times 2 \times 180 \times 32}{\pi \times \pi \times 100 \times 10^9}$$

$D = 0.0804m$ (or) $80.4 mm$.

$$\boxed{D = 0.0804m \text{ (or) } 80.4mm}$$

From the above two cases we find that suitable diameter for the shaft is 80.4mm (or) say 80mm (i.e. greater of the two values)