16ME207- STRENGTH OF MATERIALS

UNIT II - TORSION AND SPRINGS

Compound shafts

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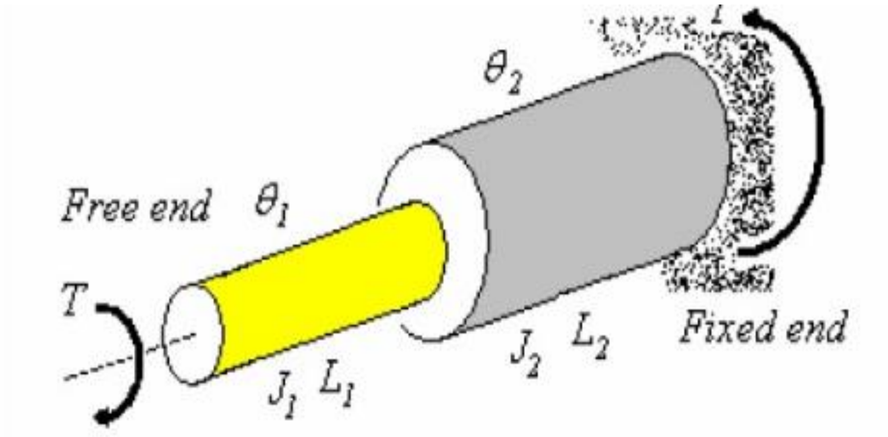
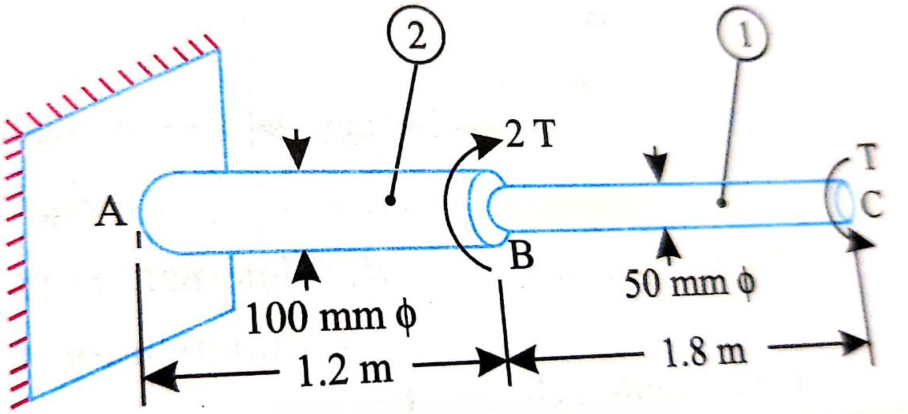
Twist and torsion stiffness - Compound Shafts:

Torsion of shafts in Series. When a shaft is having two different diameters cross section then two equal torques (T) are applied in opposite direction at both ends as shown in the figure. ... Otherwise, one end is fixed and the other end is subjected to a torque T, then also the shafts are said to be in series.



Problem

1. The stepped steel shaft shown in Fig. is subjected to a torque T at the free end and a torque of 2T in the opposite direction at eh junction of two sizes. What is the total angle of twist at the free end, if the maximum shear stress in the shaft is limited to 70 MN/m2 ? Assume the modulus of rigidity to be 84 GN/m2



Problem

Solution :-

Refer the figure.

The Torque 2T at B is equivalent to two torques each of value T. Then BC is subjected to a Torque T at C and an opposite torque T at B, while AB is also subject to equal and opposite torque T at A and B.

for the length BC;

Torque, $T = T$, $l = 1.8\,m$

$$I_p = (\pi/32) \times (0.05)^4$$

$$= 6.136 \times 10^{-7}\,m^4$$

θ_1 = Angle of twist of C relative to B.

$$= \frac{Tl}{CI_p} = \frac{T \times 1.8}{84 \times 10^9 \times 6.136 \times 10^{-7}} \longrightarrow (i)$$

for the length AB;

Torque, $T = T$, $l = 1.2\,m$.

$$I_p = (\pi/32) \times (0.1)^4$$

$$= 9.817 \times 10^{-6}\,m^4.$$

θ_2 = Angle of twist of B relative to A.

$$= \frac{Tl}{CI_p} = \frac{T \times 1.2}{84 \times 10^9 \times 9.817 \times 10^{-6}} \longrightarrow (ii)$$

Problem

θ_1 and θ_2 are in opposite directions.
Hence θ_c, the total angle of twist at C

$$\theta_c = \theta_1 - \theta_2$$

The maximum shear stress occurs in BC, and its value is 70 MN/m² (Given).

Also, $\frac{T}{I_p} = \frac{\tau}{R}$

$$T = \frac{\tau . I_p}{R} = \frac{70\times10^6 \times 6.136\times10^{-7}}{0.025} = 1718.1 \text{ Nm}$$

∴ From equation (i), we have

$$\theta_1 = \frac{1718.1 \times 1.8}{84\times10^9 \times 6.136\times10^{-7}} = 0.06 \text{ radians}$$

and from equation (ii)

$$\theta_2 = \frac{1718.1 \times 1.2}{84\times10^9 \times 9.817\times10^{-6}} = 0.0025 \text{ radians}$$

$$\theta_c = \theta_1 - \theta_2 = 0.06 - 0.0025 = 0.0575$$

$$\theta_c = 0.0575 \text{ radians}$$

$$\theta_c = 0.0575 \times \frac{180}{\pi} = 3.29 \text{ degrees}$$

$$\boxed{\theta_c = 3.29 \text{ degrees}}$$