SNS COLLEGE OF TECHNOLOGY

16ME207- STRENGTH OF MATERIALS

UNIT II - TORSION AND SPRINGS

Stepped shaft

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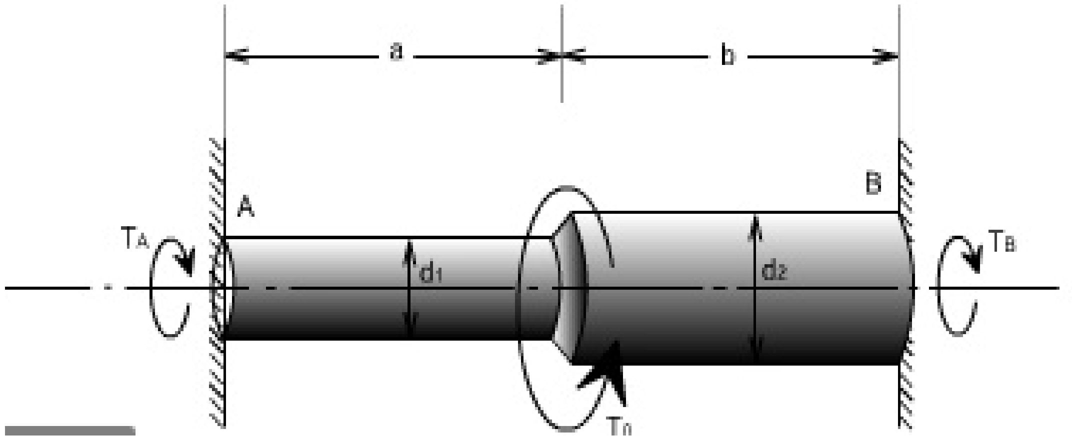
Torsion of Stepped Shafts:

Torque may be applied to a composite shaft at any point with different loading. The composite shaft may be of series or parallel type and the material of the shaft may be of the same or different materials.



Problem

1. A hollow circular shaft 20 mm thick transmit 294 kW at 200 r.p.m. Determine the diameters of the shaft if shear strain due to torsion is not to exceed 8.6 x 10-4. Take: Modulus of rigidity as 80 GN/m2.

Solution

D_H = External diameter of the hollow shaft, m

d_H = Internal diameter of the hollow shaft m, and

t = Thickness of the shaft = (20mm = 0.02m)

$D_H - d_H = 2t = 0.04\,m$ [or $d_H = (D_H - 0.04)\,m$]

Shear strain due to Torsion :-

$e_s = 8.6 \times 10^{-4}$

$$\left.\begin{array}{l} \text{Modulus of rigidity, } C = 80\,GN/m^2 \\ \text{Power Transmitted, } P = 294\,kW \\ \text{Speed, } N = 200\,rpm \end{array}\right\} \text{Given}$$

Diameters of the shaft, D_H and d_H :-

$$P = \frac{2\pi NT}{60 \times 1000}\ kW$$

$$294 = \frac{2\pi \times 200 \times T}{60 \times 1000}\ (\text{or})\ T = \frac{294 \times 60 \times 1000}{2\pi \times 200}$$

$$\boxed{T = 14037\ Nm}$$

$$\frac{T}{I_p} = \frac{\tau}{R}\ (\text{or})\ \tau = \frac{TR}{I_p} = \frac{14037 \times (D_H/2)}{\frac{\pi}{32}(D_H^4 - d_H^4)}$$

$$= \frac{71489.8 D_H}{[D_H^4 - (D_H - 0.04)^4]} \times 10^{-6}\ MN/m^2$$

Problem

$$e_s = \frac{\tau}{C} \quad (\text{or}) \quad \tau = e_s \times C$$

$$= 8.6 \times 10^{-4} \times 80 \times 10^{9} \times 10^{-6}\ MN/m^2$$

$$\tau = 68.6\ MN/m^2$$

$$\boxed{\tau = 68.6\ MN/m^2}$$

$$\frac{71489.8 D_H}{[D_H^4 - (D_H - 0.04)^4]} \times 10^{-6} = 68.8$$

(or)

$$\frac{D_H^4 - (D_H - 0.04)^4}{D_H} = \frac{71489.8}{68.8 \times 10^6} \simeq 1.039 \times 10^{-3}$$

By Trail and Error

$$\boxed{\begin{array}{l} D_H = 0.108\ m\ (\text{or})\ 108\ mm \\ d_H = 108 - (2 \times 20) = 68\ mm \end{array}}$$

2. Two shafts of the same material and same length are subjected to the same torque. If the first shaft is of a solid circular section, and the second shaft is of a hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the two shafts.

Solution :-

D_S = Diameter of the solid shaft.

D_H = External diameter of the Hallow shaft.

d_H = Internal diameter of the Hallow shaft.

τ = Maximum shear stress developed

$d_H = \frac{2}{3} D_H$. (Given).

The torque transmitted by the solid shaft,

$$T_S = \tau . \frac{\pi}{16} D_S^3$$

Torque transmitted by the Hollow shaft,

$$T_H = \tau . \frac{\pi}{16} \left[\frac{D_H^4 - d_H^4}{D_H} \right] = \tau . \frac{\pi}{16} \left[\frac{D_H^4 - (2/3 D_H)^4}{D_H} \right]$$

$$= \tau . \frac{\pi}{16} \left[\frac{D_H^4 - \left(\frac{16}{81} D_H^4 \right)}{D_H} \right] = \tau . \frac{\pi}{16} \times \frac{65}{81} D_H^3$$

Since both the Torques are equal, therefore equating (i) and (ii), we get.

$$T_S = T_H$$

$$\tau . \frac{\pi}{16} D_S^3 = \tau . \frac{\pi}{16} . \frac{65}{81} D_H^3.$$

$$D_H^3 = 1.246 D_S^3 \text{ (or) } D_H = 1.08 D_S$$

Problem

$$\frac{\text{Weight of solid shaft}}{\text{Weight of Hollow Shaft}} = \frac{W_s}{W_H}$$

$$= \frac{A_s \times l_s \times w_s}{A_H \times l_H \times w_H} = \frac{A_s}{A_H}$$

$$\left[\begin{array}{l} \because l_s = l_H \\ w_s = w_H \text{ where, } w \text{ stands for weight density} \end{array}\right]$$

$$= \frac{\frac{\pi}{4} D_s^2}{\frac{\pi}{4}\left[D_H^2 - d_H^2\right]} = \frac{D_s^2}{\left[D_H^2 - \left(\frac{2}{3} D_H\right)^2\right]}$$

$$= \frac{D_s^2}{D_H^2\left(1 - \frac{4}{9}\right)} = \frac{D_s^2}{\frac{5}{9} \times (1.08 D_s^2)} = \frac{1.543}{1}$$

$$\boxed{\frac{W_s}{W_H} = \frac{1.543}{1}}$$