# Unit V

## Biaxial state of stresses - Thin cylindrical Shell

In engineering field, we daily come across vessels of cylindrical and spherical shapes containing fluids such as tanks, boilers, compressed air receivers etc. Generally, the walls of such vessels are very thin as compared to their diameters. These vessels, when empty, are subjected to atmospheric pressure internally as well as externally. In such a case, the resultant pressure on the walls of the shell is zero. But whenever a vessel is subjected to internal pressure (due to steam, compressed air etc.) its walls are subjected to tensile stresses.

In general, if the thickness of the wall of a shell is less than 1/10th to 1/15th of its diameter, it is known as a thin shell.

## Failure of a Thin Cylindrical Shell due to an Internal Pressure

We have already discussed in the last article that whenever a cylindrical shell is subjected to an internal pressure, its walls are subjected to tensile stresses.

It will be interesting to know that if these stresses exceed the permissible limit, the cylinder is likely to fail in any one of the following two ways as shown in Fig. 31.1 (*a*) and (*b*).

(*a*) Split into two troughs. (*b*) Split into two cylinders.

Fig. 1

1. It may split up into two troughs and
2. It may split up into two cylinders.

## Stresses in a Thin Cylindrical Shell

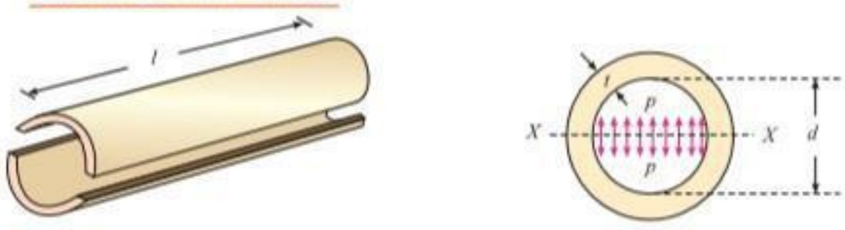
We have already discussed that whenever a cylindrical shell is subjected to an internal pressure, its walls are subjected to tensile stresses. A little consideration will show that the walls of the cylindrical shell will be subjected to the following two types of tensile stresses:

1. Circumferential stress and
2. Longitudinal stress.

In case of thin shells, the stresses are assumed to be uniformly distributed throughout the wall thickness. However, in case of thick shells, the stresses are no longer uniformly distributed and the problem becomes complex. In this chapter, we shall discuss the stress in thin shells only.

NOTE: The above theory also holds good, when the shell is subjected to compressive stress.

## Circumferential Stress




(*a*) (*b*)

Fig. 2

Consider a thin cylindrical shell subjected to an internal pressure as shown in Fig. 31.2(*a*) and (*b*). We know that as a result of the internal pressure, the cylinder has a tendency to split up into two troughs as shown in the figure.

Let $l$ = Length of the shell,

$d$ = Diameter of the shell,

$t$ = Thickness of the shell and

$p$ = Intensity of internal pressure.

Total pressure along the diameter (say X-X axis) of the shell,

$$P = \text{Intensity of internal pressure} \times \text{Area} = p \times d \times l$$

and circumferential stress in the shell,

$$\sigma_c = \frac{\text{Total pressure}}{\text{Resisting section}} = \frac{pdl}{2tl} = \frac{pd}{2t} \qquad \ldots(\because \text{ of two sections})$$

This is a tensile stress across the X-X. It is also known as **hoop stress.**

**Note.** If $\eta$ is the efficiency of the riveted joints of the shell, then stress,

$$\sigma_c = \frac{pd}{2t\eta}$$

## . Longitudinal Stress

Consider the same cylindrical shell, subjected to the same internal pressure as shown in Fig. 31.3 (*a*) and (*b*). We know that as a result of the internal pressure, the cylinder also has a tendency to split into two pieces as shown in the figure.

Let $p$ = Intensity of internal pressure,

$l$ = Length of the shell,

$d$ = Diameter of the shell and

$t$ = Thickness of the shell.

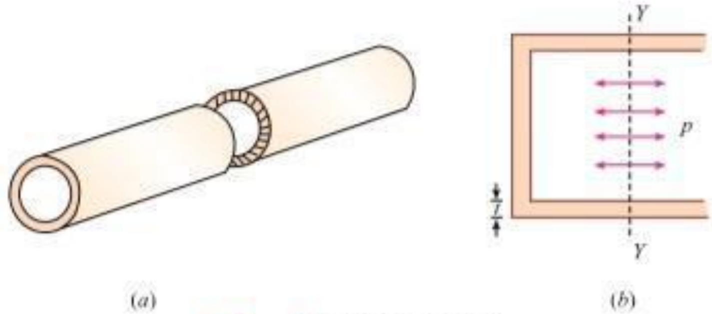



Fig. .3. Longitudinal stress.

Total pressure along its length (say Y-Y axis) of the shell

$$P = \text{Intensity of internal pressure} \times \text{Area}$$

$$= p \times \frac{\pi}{4}(d)^2$$

and longitudinal stress in the shell,

$$\sigma_l = \frac{\text{Total pressure}}{\text{Resisting section}} = \frac{p \times \frac{\pi}{4}(d)^2}{\pi d t} = \frac{pd}{4t}$$

This is also a tensile stress across the section Y-Y. It may be noted that the longitudinal stress is half of the circumferential or hoop stress.

**Note.** If $\eta$ is the efficiency of the riveted joints of the shell, then the stress,

$$\sigma_l = \frac{pd}{4t\eta}$$

**1.** *A stream boiler of 800 mm diameter is made up of 10 mm thick plates. If the boiler is subjected to an internal pressure of 2.5 MPa, find the circumferential and longitudinal stresses induced in the boiler plates.*

**Solution.** Given : Diameter of boiler ($d$) = 800 mm ; Thickness of plates ($t$) = 10 mm and internal pressure ($p$) = 2.5 MPa = 2.5 N/mm$^2$.

***Circumferential stress induced in the boiler plates***

We know that circumferential stress induced in the boiler plates,

$$\sigma_c = \frac{pd}{2t} = \frac{2.5 \times 800}{2 \times 10} = 100 \text{ N/mm}^2 = \mathbf{100 \text{ MPa}} \qquad \textbf{Ans.}$$

***Longitudinal stress induced in the boiler plates***

We also know that longitudinal stress induced in the boiler plates,

$$\sigma_l = \frac{pd}{4t} = \frac{2.5 \times 800}{4 \times 10} = 50 \text{ N/mm}^2 = \mathbf{50 \text{ MPa}} \qquad \textbf{Ans.}$$

**2.** *A cylindrical shell of 1.3 m diameter is made up of 18 mm thick plates. Find the circumferential and longitudinal stress in the plates, if the boiler is subjected to an internal pressure of 2.4 MPa. Take efficiency of the joints as 70%.*

**Solution.** Given: Diameter of shell ($d$) = 1.3 m = $1.3 \times 10^3$ mm ; Thickness of plates ($t$) = 18 mm; Internal pressure ($p$) = 2.4 MPa = 2.4 N/mm$^2$ and efficiency ($\eta$) = 70% = 0.7.

***Circumferential stress***

We know that circumferential stress,

$$\sigma_c = \frac{pd}{2t\eta} = \frac{2.4 \times (1.3 \times 10^3)}{2 \times 18 \times 0.7} = 124 \text{ N/mm}^2 = \mathbf{124 \text{ MPa}} \qquad \textbf{Ans.}$$

***Longitudinal stress***

We also know that longitudinal stress,

$$\sigma_l = \frac{pd}{4t\eta} = \frac{2.4 \times (1.3 \times 10^3)}{4 \times 18 \times 0.7} = 62 \text{ N/mm}^2 = \mathbf{62 \text{ MPa}} \qquad \textbf{Ans.}$$

**3.** *A gas cylinder of internal diameter 40 mm is 5 mm thick. If the tensile stress in the material is not to exceed 30 MPa, find the maximum pressure which can be allowed in the cylinder.*

**Solution.** Given: Diameter of cylinder ($d$) = 40 mm ; Thickness of plates ($t$) = 5 mm and tensile stress ($\sigma_c$) = 30 MPa = 30 N/mm$^2$.

Let $p$ = Maximum pressure which can be allowed in the cylinder.

We know that circumferential stress ($\sigma_c$),

$$30 = \frac{pd}{2t} = \frac{p \times 40}{2 \times 5} = 4p$$

$$\therefore \quad p = \frac{30}{4} = 7.5 \text{ N/mm}^2 = \mathbf{7.5\ MPa} \quad \textbf{Ans.}$$

**Note: 1.** Since the circumferential stress ($\sigma_c$) is double the longitudinal stress ($\sigma_l$), therefore in order to find the maximum pressure the given stress should be taken as circumferential stress.

**2.** If however, we take the given tensile stress of 30 N/mm$^2$ as the longitudinal stress, then

$$30 = \frac{pd}{4t} = \frac{p \times 40}{4 \times 5} = 2p$$

$$\therefore \quad p = \frac{30}{4} = 15 \text{ N/mm}^2 = 15 \text{ MPa}$$

Now we shall provide a pressure of 7.5 MPa *i.e.* (Lesser of the two values) obtained by using the tensile stress as circumferential stress and longitudinal stress.

## Design of Thin Cylindrical Shells

Designing of thin cylindrical shell involves calculating the thickness ($t$) of a cylindrical shell for the given length ($l$), diameter ($d$), intensity of maximum internal pressure ($p$) and circumferential stress ($\sigma_c$). The required thickness of the shell is calculated from the relation.

$$t = \frac{pd}{2\sigma_c} \qquad \text{... (See Article 31.4)}$$

If the thickness so obtained, is not a round figure, then next higher value is provided.

**Note:** The thickness obtained from the longitudinal stress will be half of the thickness obtained from circumferential stress. Thus, it should not be accepted.

**4.** *A thin cylindrical shell of 400 mm diameter is to be designed for an internal pressure of 2.4 MPa. Find the suitable thickness of the shell, if the allowable circumferential stress is 50 MPa.*

**Solution.** Given: Diameter of shell ($d$) = 400 mm ; Internal pressure ($p$) = 2.4 MPa = 2.4 N/mm$^2$ and circumferential stress ($\sigma_c$) = 50 MPa = 50 N/mm$^2$.

We know that thickness of the shell,

$$t = \frac{pd}{2\sigma_c} = \frac{2.4 \times 400}{2 \times 50} = \mathbf{9.6\ mm} \quad \textbf{say} \quad \mathbf{10\ mm} \qquad \textbf{Ans.}$$

**5.** *A cylindrical shell of 500 mm diameter is required to withstand an internal pressure of 4 MPa. Find the minimum thickness of the shell, if maximum tensile strength in the plate material is 400 MPa and efficiency of the joints is 65%. Take factor of safety as 5.*

**Solution.** Given: Diameter of shell ($d$) = 500 mm ; Internal pressure ($p$) = 4 MPa = 4 N/mm$^2$; Tensile strength = 400 MPa = 400 N/mm$^2$ ; Efficiency ($\eta$) = 65% = 0.65 and factor of safety = 5.

We know that allowable tensile stress (*i.e.*, circumferential stress),

$$\sigma_c = \frac{\text{Tensile strength}}{\text{Factor of safety}} = \frac{400}{5} = 80 \text{ N/mm}^2$$

and minimum thickness of shell,

$$t = \frac{pd}{2\sigma_c\,\eta} = \frac{4 \times 500}{2 \times 80 \times 0.65} = \mathbf{19.2\ mm} \quad \textbf{say} \quad \mathbf{20\ mm} \qquad \textbf{Ans.}$$

## Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure

We have already discussed in the chapter on Elastic Constants that lateral strain is always accompanied by a linear strain. It is thus obvious that in a thin cylindrical shell subjected to an internal pressure, its walls will also be subjected to lateral strain. The effect of the lateral strains is to cause some change in the dimensions (*i.e.*, length and diameter) of the shell. Now consider a thin cylindrical shell subjected to an internal pressure.

Let $l$ = Length of the shell,

$d$ = Diameter of the shell,

$t$ = Thickness of the shell and

$p$ = Intensity of the internal pressure.

We know that the circumferential stress,

$$\sigma_c = \frac{pd}{2t}$$

and longitudinal stress, $$\sigma_l = \frac{pd}{4t}$$

Now let $\delta d$ = Change in diameter of the shell,

$\delta l$ = Change in the length of the shell and

$\frac{1}{m}$ = Poisson's ratio.

Now changes in diameter and length may be found out from the above equations, as usual (*i.e.*, by multiplying the strain and the corresponding linear dimension).

$$\therefore \quad \delta d = \varepsilon_1 \cdot d = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right) \times d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right)$$

and $$\delta l = \varepsilon_2 \cdot l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) \times l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

**6.** *A cylindrical thin drum 800 mm in diameter and 4 m long is made of 10 mm thick plates. If the drum is subjected to an internal pressure of 2.5 MPa, determine its changes in diameter and length. Take E as 200 GPa and Poisson's ratio as 0.25.*

**Solution.** Given: Diameter of drum ($d$) = 800 mm ; Length of drum ($l$) = 4 m = $4 \times 10^3$ mm ; Thickness of plates ($t$) = 10 mm ; Internal pressure ($p$) = 2.5 MPa = 2.5 N/mm$^2$ ; Modulus of elasticity ($E$) = 200 GPa = $200 \times 10^3$ N/mm$^2$ and poisson's ratio $\left(\frac{1}{m}\right) = 0.25$.

***Change in diameter***

We know that change in diameter,

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{1}{2m}\right) = \frac{2.5 \times (800)^2}{2 \times 10 \times (200 \times 10^3)}\left(1 - \frac{0.25}{2}\right) \text{mm}$$

$$= \mathbf{0.35\ mm} \quad \textbf{Ans.}$$

***Change in length***

We also know that change in length,

$$\delta l = \frac{pdl}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) = \frac{2.5 \times 800 \times (4 \times 10^3)}{2 \times 10 \times (200 \times 10^3)}\left(\frac{1}{2} - 0.25\right) \text{mm}$$

$$= \mathbf{0.5\ mm} \quad \textbf{Ans.}$$

## Change in Volume of a Thin Cylindrical Shell due to an Internal Pressure

We have already discussed in the last article, that there is always an increase in the length and diameter of a thin cylindrical shell due to an internal pressure. A little consideration will show that increase in the length and diameter of the shell will also increase its volume. Now consider a thin cylindrical shell subjected to an internal pressure.

Let $l$ = Original length,

$d$ = Original diameter,

$\delta l$ = Change in length due to pressure and

$\delta d$ = Change in diameter due to pressure.

We know that original volume,

$$V = \frac{\pi}{4} \times d^2 \times l = \left[\frac{\pi}{4}(d + \delta d)^2 \times (l \times \delta l)\right] - \frac{\pi}{4} \times d^2 \times l$$

$$= \frac{\pi}{4}(d^2 \cdot \delta l + 2dl \cdot \delta d) \qquad \text{...(Neglecting small quantities)}$$

$$\therefore \quad \frac{\delta V}{V} = \frac{\frac{\pi}{4}(d^2 \cdot \delta l + 2dl \cdot \delta d)}{\frac{\pi}{4} \times d^2 \times l} = \frac{\delta l}{l} + \frac{2\delta d}{d} = \varepsilon_l + 2\varepsilon_c$$

or $\delta V = V(\varepsilon_l + 2\varepsilon_c)$

where $\varepsilon_c$ = Circumferential strain and

$\varepsilon_l$ = Longitudinal strain.

*7. A cylindrical vessel 2 m long and 500 mm in diameter with 10 mm thick plates is subjected to an internal pressure of 3 MPa. Calculate the change in volume of the vessel. Take E = 200 GPa and Poisson's ratio = 0.3 for the vessel material.*

**Solution.** Given: Length of vessel $(l) = 2 \text{ m} = 2 \times 10^3$ mm ; Diameter of vessel $(d) = 500$ mm ; Thickness of plates $(t) = 10$ mm ; Internal pressure $(p) = 3 \text{ MPa} = 3 \text{ N/mm}^2$ ; Modulus of elasticity $(E) = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$ and poisson's ratio $\left(\frac{1}{m}\right) = 0.3$.

We know that circumferential strain,

$$\varepsilon_c = \frac{pd}{2tE}\left(1 - \frac{1}{2m}\right) = \frac{3 \times 500}{2 \times 10 \times (200 \times 10^3)}\left(1 - \frac{0.3}{2}\right) = 0.32 \times 10^{-3} \quad \text{...}(i)$$

and logitudinal strain,

$$\varepsilon_l = \frac{pd}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right) = \frac{3 \times 500}{2 \times 10 \times (200 \times 10^3)}\left(\frac{1}{2} - 0.3\right) = 0.075 \times 10^{-3} \quad \text{...}(ii)$$

We also know that original volume of the vessel,

$$V = \frac{\pi}{4}(d)^2 \times l = \frac{\pi}{4}(500)^2 \times (2 \times 10^3) = 392.7 \times 10^6 \text{ mm}^3$$

$\therefore$ Change in volume,

$$\delta V = V(\varepsilon_c + 2\varepsilon_l) = 392.7 \times 10^6 [0.32 \times 10^{-3} + (2 \times 0.075 \times 10^{-3})] \text{ mm}^3$$

$$= \mathbf{185 \times 10^3 \text{ mm}^3} \qquad \textbf{Ans.}$$