Mohr's circle for biaxial stresses

Graphical Method for the Stresses on an Oblique Section of a Body

In the previous articles, we have been discussing the analytical method for the determination of normal, shear and resultant stresses across a section. But we shall now discuss a graphical method for this purpose. This is done by drawing a Mohr's Circle of Stresses. The construction of Mohr's Circle of Stresses as well as determination of normal, shear and resultant stresses is very easier than the analytical method. Moreover, there is a little chance of committing any error in this method. In the following pages, we shall draw the Mohr's Circle of Stresses for the following cases :

1. A body subjected to a direct stress in one plane.
2. A body subjected to direct stresses in two mutually perpendicular directions.
3. A body subjected to a simple shear stress.
4. A body subjected to a direct stress in one plane accompanied by a simple shear stress.
5. A body subjected to direct stresses in two mutually perpendicular directions accompanied by a simple shear stress.

Sign Conventions for Graphical Method

Though there are different sign conventions used in different books for graphical method also, yet we shall adopt the following sign conventions, which are widely used and internationally recognised :

1. The angle is taken with reference to the *X-X* axis. All the angles traced in the anticlockwise direction to the *X-X* axis are taken as negative, whereas those in the clockwise direction as positive as shown in Fig. 7.13 (*a*). The value of angle θ, until and unless mentioned is taken as positive and drawn clockwise.
2. The measurements above *X-X* axis and to the right of *Y-Y* axis are taken as positive, whereas those below *X-X* axis and to the left of *Y-Y* axis as negative as shown in Fig 7.13 (*b*) and (*c*).
3. Sometimes there is a slight variation in the results obtained by analytical method and graphical method. The values obtained by graphical method are taken to be correct if they agree upto the first decimal point with values obtained by analytical method, *e.g.*, 8.66 (Analytical) = 8.7 (Graphical), similarly 4.32 (Analytical) = 4.3 (Graphical)

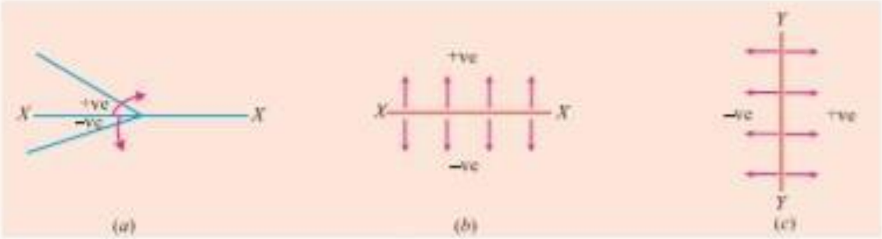


Fig. 7.13

Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to a Direct Stress in One Plane

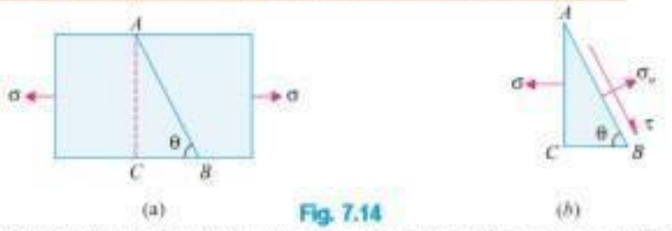


Fig. 7.14

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a direct tensile stress along *X–X* axis as shown in Fig 7.14 (*a*) and (*b*). Now let us consider an oblique section *AB* inclined with *X–X* axis, on which we are required to find out the stresses as shown in the figure.

Let σ = Tensile stress, in *x-x* direction and

θ = Angle which the oblique section *AB* makes with the *x-x* axis in clockwise direction.

First of all, consider the equilibrium of the wedge *ABC*. Now draw the Mohr's* Circle of Stresses as shown in Fig.7.15 and as discussed below :

1. First of all, take some suitable point *O* and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the tensile stress (σ) to some suitable scale and towards right (because σ is tensile). Bisect *OJ* at *C*. Now the point *O* represents the stress system on plane *BC* and the point *J* represents the stress system on plane *AC*.
3. Now with *C* as centre and radius equal to *CO* and or *CJ* draw a circle. It is known as Mohr's Circle for Stresses.

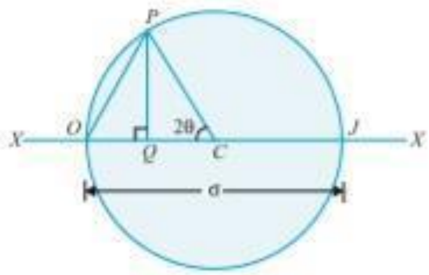


Fig. 7.15

* The diagram was first presented by German Scientist Otto Mohr in 1982.

4. Now through C draw a line CP making an angle of 2θ with CO in the clockwise direction meeting the circle at P. The point P represents the section AB.
5. Through P, draw PQ perpendicular to OX. Join OP.
6. Now OQ, QP and OP will give the normal stress, shear stress and resultant stress respectively to the scale. And the angle POJ is called the angle of obliquity (θ).

Proof

From the geometry of the Mohr's Circle of Stresses, we find that,

$$OC = CJ = CP = \sigma/2 \qquad \text{... (Radius of the circle)}$$

∴ Normal Stress.

$$\sigma_n = OQ = OC - QC = \left(\frac{\sigma}{2}\right) - \left(\frac{\sigma}{2}\right)\cos 2\theta \qquad \text{...(Same as in Art. 7.7)}$$

and shear stress

$$\tau = QP = CP \sin 2\theta = \frac{\sigma}{2}\sin 2\theta \qquad \text{...(Same as in Art. 7.7)}$$

We also find that maximum shear stress will be equal to the radius of the Mohr's Circle of Stresses *i.e.*, $\frac{\sigma}{2}$. It will happen when 2θ is equal to 90° or 270° *i.e.*, θ is equal to 45° or 135°.

However when $\theta = 45°$ then the shear stress is equal to $\frac{\sigma}{2}$.

And when $\theta = 135°$ then the shear stress is equal to $-\frac{\sigma}{2}$.

Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to Direct Stresses in Two Mutually Perpendicular Directions

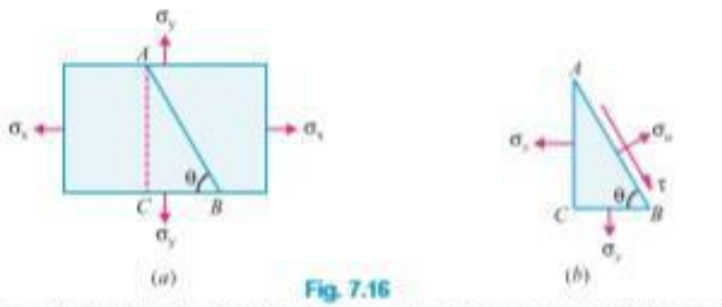


Fig. 7.16

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to direct tensile stresses in two mutually perpendicular directions along x-x and y-y axis as shown in Fig 7.16 (*a*) and (*b*). Now let us consider an oblique section AB inclined with x-x axis on which we are required to find out the stresses as shown in the figure.

Let σ_x = Tensile stress in x-x direction (also termed as major tensile stress),

σ_y = Tensile stress in y-y direction (also termed as minor tensile stress). and

θ = Angle which the oblique section AB makes with x-x axis in clockwise direction.

First of all consider the equilibrium of the wedge ABC. Now draw the Mohr's Circle of Stresses as shown in Fig. 7.17 and as discussed below :

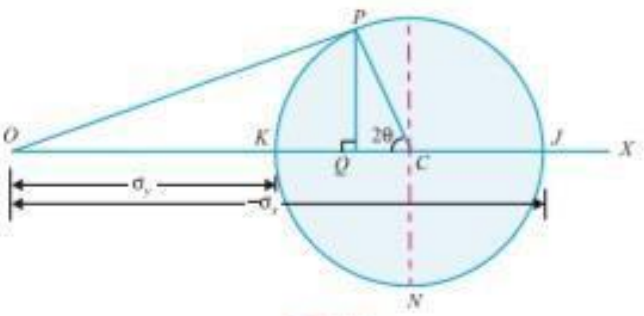


Fig. 7.17

1. First of all, take some suitable point O and draw a horizontal line OX.
2. Cut off OJ and OK equal to the tensile stresses σ_x and σ_y to some suitable scale towards right (because both the stresses are tensile). The point J represents the stress system on plane AC and the point K represents the stress system on plane BC. Bisect JK at C.
3. Now with C as centre and radius equal to CJ or CJ draw a circle. It is known as Mohr's Circle of Stresses.
4. Now through C, draw a line CP making an angle of 2θ with CK in clockwise direction meeting the circle at P. The point P represents the stress systems on the section AB.
5. Through P, draw PQ perpendicular to the line OX. Join OP.
6. Now OQ, QP and OP will give the normal stress, shear stress and resultant stress respectively to the scale. Similarly CM or CN will give the maximum shear stress to the scale. The angle POC is called the angle of obliquity.

Proof

From the geometry of the Mohr's Circle of Stresses, we find that

$$KC = CJ = CP = \frac{\sigma_x - \sigma_y}{2}$$

or
$$OC = OK + KC = \sigma_y + \frac{\sigma_x - \sigma_y}{2} = \frac{2\sigma_y + \sigma_x - \sigma_y}{2} = \frac{\sigma_x + \sigma_y}{2}$$

∴ Normal stress,
$$\sigma_n = OQ = OC - CQ = \frac{\sigma_x - \sigma_y}{2} - CP\cos 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta \qquad \text{...(Same as Art. 7.8)}$$

and shear stress,
$$\tau = QP = CP\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2}\sin 2\theta \qquad \text{...(Same as Art. 7.8)}$$

We also find that the maximum shear stress will be equal to the radius of the Mohr's Circle of Stresses. *i.e.*, $\frac{\sigma_x - \sigma_y}{2}$. It will happen when 2θ is equal to 90° or 270° *i.e.*, when θ is equal to 45° or 135°.

However when $\theta = 45°$ then the shear stress is equal to $\frac{\sigma_x - \sigma_y}{2}$

And when $\theta = 135°$ then the shear stress will be equal to $\frac{-(\sigma_x - \sigma_y)}{2}$ or $\frac{\sigma_y - \sigma_x}{2}$.

EXAMPLE 7.15. *The stresses at a point of a machine component are 150 MPa and 50 MPa both tensile. Find the intensities of normal, shear and resultant stresses on a plane inclined at an angle of 55° with the axis of major tensile stress.*

Also find the magnitude of the maximum shear stresses in the component.

***SOLUTION.** Given : Tensile stress along horizontal *x-x* axis (σ_x) = 150 MPa ; Tensile stress along vertical y-y axis (σ_y) = 50 MPa and angle made by the plane with the axis of major tensile stress (θ) = 55°.

The given stresses on the planes *AC* and *BC* in the machine component are shown in Fig. 7.18 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.18 (*b*) and as discussed below :

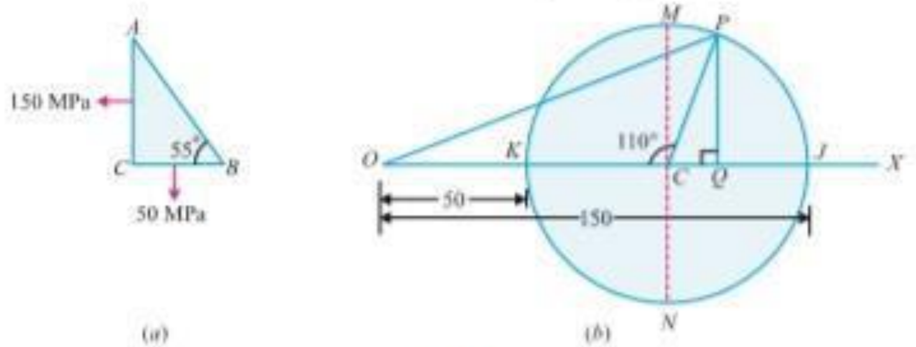


Fig. 7.18

1. First of all, take some suitable point *O* and draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively (*i.e.* 150 MPa and 50 MPa) to some suitable scale towards right. The point *J* represents the stress system on the plane *AC* and the point *K* represents the stress system on the plane *BC*. Bisect *KJ* at *C*.
3. Now with *C* as centre and radius equal to *CJ* or *CK* draw the Mohr's Circle of Stresses.
4. Now through *C* draw two lines *CM* and *CN* at right angles to the line *OX* meeting the circle at *M* and *N*. Also through *C* draw a line *CP* making an angle of 2 × 55° = 110° with *CK* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the plane *AB*.
5. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.

   By measurement, we find that the normal stress (σ_n) = *OQ* = 117.1 MPa ; Shear stress (τ) = *QP* = 47.0 MPa ; Resultant stress (σ_R) = *OP* = 126.2 MPa and maximum shear stress (τ_{max}) = *CM* = ± 50 MPa **Ans.**

16. *The stresses at a point in a component are 100 MPa (tensile) and 50 MPa (compressive). Determine the magnitude of the normal and shear stresses on a plane inclined at an angle of 25° with tensile stress. Also determine the direction of the resultant stress and the magnitude of the maximum intensity of shear stress.*

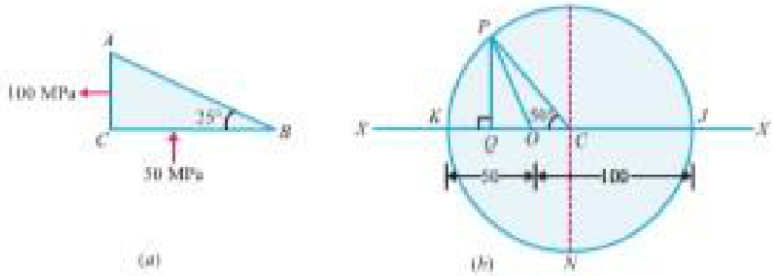


Fig. 7.19

***SOLUTION.** Given : Tensile stress along horizontal *x-x* axis (σ_x) = 100 MPa ; Compressive stress along vertical *y-y* axis (σ_y) = – 50 MPa (Minus sign due to compressive) and angle made by plane with tensile stress (θ) = 25°.

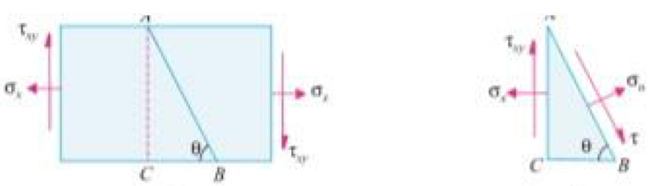
The given stresses on the planes *AC* and *BC* of the component are shown in Fig 7.19 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.19 (*b*) and as discussed below :

1. First of all, take some suitable point *O* and through it draw a horizontal line *XOX*.
2. Cut off *OJ* and *OK* equal to the stresses and respectively (*i.e.*, 100 MPa and – 50 MPa) to some suitable scale such that *J* is towards right (because of tensile stress) and *B* is towards left (because of compressive stress). The point *J* represents the stress system on the plane *AC* and the point *K* represents the stress systems on the plane *BC*. Bisect *KJ* at *C*.
3. Now with *C* as centre and radius equal to *CJ* or *CK* draw the Mohr's Circle of Stresses.
4. Now through *C*, draw two lines *CM* and *CN* at right angles to the line *OX* meeting the circle at *M* and *N*. Also through *C*, draw a line *CP* making an angle of 2 × 25° = 50° with *CK* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the plane *AB*.
5. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.

By measurement, we find that the normal stress (σ_n) = – 23.2 MPa ; Shear stress (τ) = *PQ* = 57.45 MPa; Direction of the resultant stress ∠*POQ* = 68.1° and maximum shear stress (τ_{max}) = *CM* = *CN* = ± 75 MPa **Ans.**

Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to a Direct Stresses in One Plane Accompanied by a Simple Shear Stress

Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to a direct tensile stress along *X-X* axis accompanied by a positive (*i.e.* clockwise) shear stress along *X-X* axis as shown in Fig 7.20 (*a*) and (*b*). Now let us consider an oblique section *AB* … n in the figure



(*a*) (*b*)

Let σ_x = Tensile stress in x-x direction,

τ_{xy} = Positive (*i.e.*, clockwise) shear stress along x-x axis, and

θ = Angle which oblique section AB makes with x-x axis in clock wise direction.

First of all consider the equilibrium of the wedge ABC. **We know that as per the principle of simple shear the face BC of the wedge will also be subjected to an anticlockwise shear stress.** Now draw the Mohr's Circle of Stresses as shown in Fig.7.21 and as discussed below :

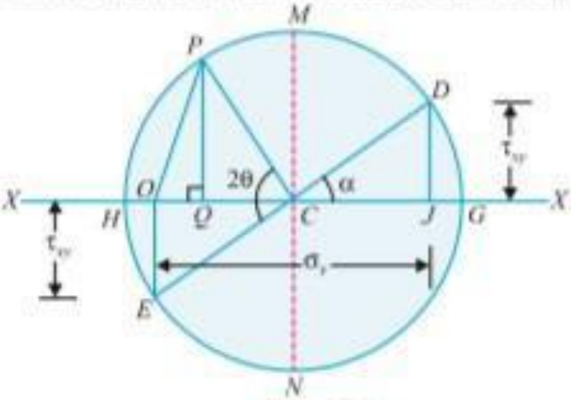


Fig. 7.21

1. First of all, take some suitable point O and through it draw a horizontal line XOX.
2. Cut off OJ equal to the tensile stress σ_x to some suitable scale and towards right (because σ_x is tensile).
3. Now erect a perpendicular at J above the line X-X (because τ_{xy} is positive along x-x axis) and cut off JD equal to the shear stress τ_{xy} to the scale. The point D represents the stress system on plane AC. Similarly, erect a perpendicular below the line x-x (**because τ_{xy} is negative along y-y axis**) and cut off OE equal to the shear stress τ_{xy} to the scale. The point E represents the stress system on plane BC. Join DE and bisect it at C.
4. Now with C as centre and radius equal to CD or CE draw a circle. It is known as Mohr's Circle of Stresses.
5. Now through C, draw a line CP making an angle 2θ with CE in clockwise direction meeting the circle at P. The point P represents the stress system on the section AB.
6. Through P, draw PQ perpendicular to the line OX. Join OP.
7. Now OQ, QP and OP will give the normal, shear and resultant stresses to the scale. And the angle POC is called the angle of obliquity.

Proof

From the geometry of the Mohr's Circle of Stresses, we find that

$$OC = \frac{\sigma_x}{2}$$

and radius of the circle,

$$R = EC = CD = CP = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

Now in the right angled triangle DCJ,

$$\sin\alpha = \frac{DJ}{CD} = \frac{\tau_{xy}}{R} \qquad \text{and} \qquad \cos\alpha = \frac{JC}{CD} = \frac{\sigma_x}{2} \times \frac{1}{R} = \frac{\sigma_x}{2R}$$

and similarly in right angled triangle CPQ,

$$\angle PCQ = (2\theta - \alpha)$$

$$\therefore \quad CQ = CP\cos(2\theta - \alpha) = R[\cos(2\theta - \alpha)]$$
$$= R[\cos\alpha\cos 2\theta + \sin\alpha\sin 2\theta]$$
$$= R\cos\alpha\cos 2\theta + R\sin\alpha\sin 2\theta$$
$$= R \times \frac{\sigma_x}{2R}\cos 2\theta + R \times \frac{\tau_{xy}}{R}\sin 2\theta$$
$$= \frac{\sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

We know that normal stress across the section AB,

$$\sigma_n = OQ = OC - CQ = \frac{\sigma_x}{2} - \left(\frac{\sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta\right)$$
$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad \text{...(Same as in Art. 7.10)}$$

and shear stress,

$$\tau = QP = CP\sin(2\theta - \alpha) = R\sin(2\theta - \alpha)$$
$$= R(\cos\alpha\sin 2\theta - \sin\alpha\cos 2\theta)$$
$$= R\cos\alpha\sin 2\theta - R\sin\alpha\cos 2\theta$$
$$= R \times \frac{\sigma_x}{2R}\sin 2\theta - R \times \frac{\tau_{xy}}{2}\cos 2\theta$$
$$= \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \qquad \text{...(Same as in Art. 7.10)}$$

We also know that maximum stress,

$$\sigma_{max} = OG = OC + CG = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

and minimum stress

$$\sigma_{min} = OH = OC - CH = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

We also find that the maximum shear stress will be equal to the radius of the Mohr's circle of stresses *i.e.*, $\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$. It will happen when $(2\theta - \alpha)$ is equal to 90° or 270°.

However when $(2\theta - \alpha)$ is equal to 90° then the shear stress is equal to $+\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$.

And when $(2\theta - \alpha) = 270°$ then the shear stress is equal to $-\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$.

17. *A plane element in a body is subjected to a tensile stress of 100 MPa accompanied by a clockwise shear stress of 25 MPa. Find (i) the normal and shear stress on a plane inclined at an angle of 20° with the tensile stress ; and (ii) the maximum shear stress on the plane.*

***SOLUTION.** Given : Tensile stress along horizontal x-x axis $(\sigma_x) = 100$ MPa ; Shear stress $(\tau_{xy}) = 25$ MPa and angle made by plane with tensile stress $(\theta) = 20°$.

The given stresses on the element and a complimentary shear stress on the *BC* plane are shown in Fig. 7.22 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig 7.22 (*b*) and as discussed below :

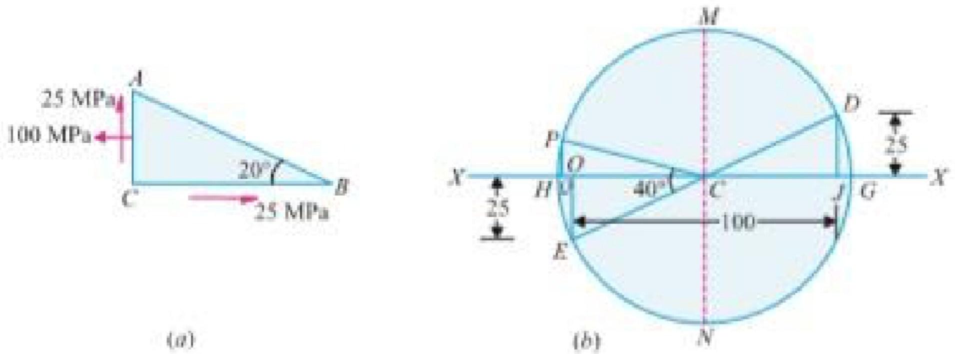
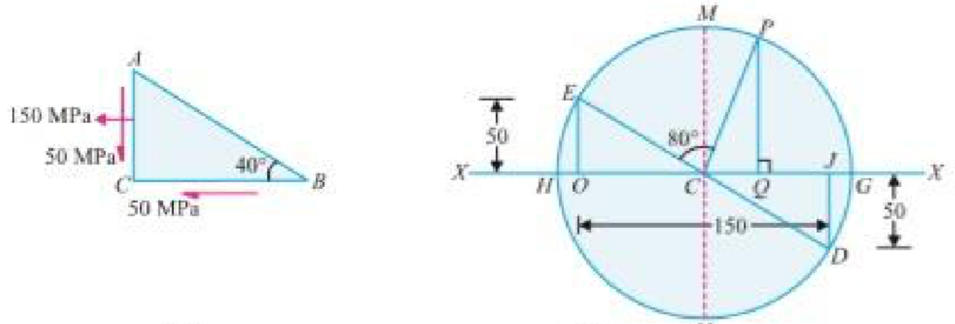


Fig. 7.22

1. First of all, take some suitable point *O*, and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the tensile stress on the plane *AC* (*i.e.*, 100 MPa) to some suitable scale towards right.
3. Now erect a perpendicular at *J* above the line *X-X* and cut off *JD* equal to the positive shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly erect a perpendicular at *O* below the line *X-X* and cut off *OE* equal to the negative shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C*, draw two lines *CM* and *CN* at right angle to the line *OX* meeting the circle at *M* and *N*. Also through *C*, draw a line *CP* making an angle of 2 × 20° = 40° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the normal stress $(\sigma_n) = OQ = 4.4$ MPa (compression) ; Shear stress $(\tau) = QP = 13.0$ MPa and maximum shear stress $(\tau_{max}) = CM = 55.9$ MPa **Ans.**

18. *An element in a strained body is subjected to a tensile stress of 150 MPa and a shear stress of 50 MPa tending to rotate the element in an anticlockwise direction. Find (i) the magnitude of the normal and shear stresses on a section inclined at 40° with the tensile stress ; and (ii) the magnitude and direction of maximum shear stress that can exist on the element.*



(a) (b)

*SOLUTION. Given : Tensile stress along horizontal x-x axis (σ_x) = 150 MPa ; Shear stress (τ_{xy}) = – 50 MPa (Minus sign due to anticlockwise) and angle made by section with the tensile stress (θ) = 40°.

The given stresses on the plane *AB* of the element and a complimentary shear stress on the plane *BC* are shown in Fig 7.23 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.23 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the tensile stress on the plane *AC* (*i.e.*, 150 MPa) to some suitable scale towards right.
3. Now erect a perpendicular at *J* below the line *X-X* and cut off *JD* equal to the negative shear stress on the plane *AC* (*i.e.*, 50 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *O* above the line *X-X* and cut off *OE* equal to the positive shear stress on the plane *BC* (*i.e.*, 50 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses meeting the line *X-X* at *G* and *H*.
5. Through *C*, draw two lines *CM* and *CN* at right angles to the line *X-X* meeting the circle at *M* and *N*. Also through *C*, draw a line *CP* making an angle of 2 × 40° = 80° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the Normal stress (σ_n) = *OQ* = 112.2 MPa ; Shear stress (τ) = *QP* = 82.5 MPa and maximum shear stress, that can exist on element (τ_{max}) = ± *CM* = *CN* = 90.14 MPa **Ans.**

19. *An element in a strained body is subjected to a compressive stress of 200 MPa and a clockwise shear stress of 50 MPa on the same plane. Calculate the values of normal and shear stresses on a plane inclined at 35° with the compressive stress. Also calculate the value of maximum shear stress in the element.*

**SOLUTION. Given : Compressive stress along horizontal x-x axis (σ_x) = – 200 MPa (Minus sign due to compressive stress) ; Shear stress (τ_{xy}) = 50 MPa ; and angle made by plane with the compressive stress (θ) = 35°.

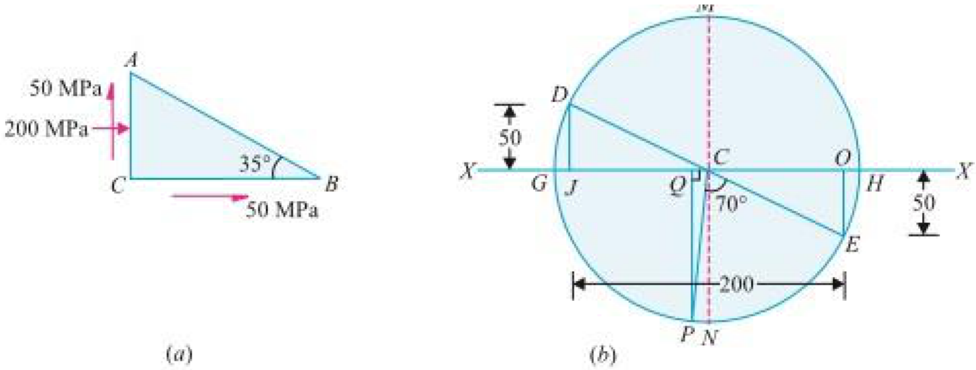


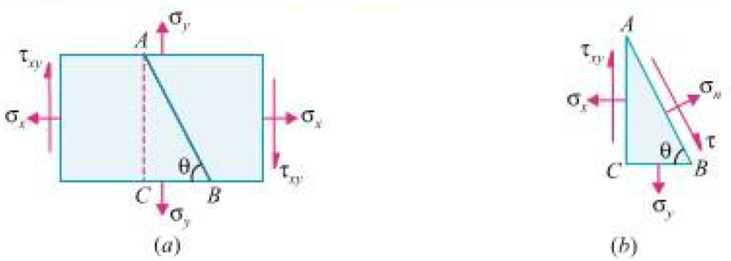
Fig. 7.24

The given stresses on the plane *AC* of the element and a complimentary shear stress on the plane *BC* are shown in Fig. 7.24 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig.7.24 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and through it draw a horizontal line *XOX*.
2. Cut off *OJ* equal to the compressive stress on the plane *AC* (*i.e.*, 200 MPa) to some suitable scale towards left .
3. Now erect a perpendicular at *J* above the line *X-X* and cut off *JD* equal to the positive shear stress on the plane *AC* (*i.e.*, 50 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *O* below the line *X-X* and cut off *OE* equal to the negative shear stress on the plane *BC* (*i.e.*, 50 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses. Meeting the line *X-X* at *G* and *H*.
5. Through *C*, draw two lines *CM* and *CN* at right angles to the line *X-X* meeting the circle at *M* and *N*. Also through *C* draw a line *CP* making an angle of 2 × 35° = 70° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the plane *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the Normal stress (σ_n) = *OQ* = – 112.8 MPa ; Shear stress (τ) = *QP* = – 111.1 MPa and maximum shear stress in the element (t_{max}) = ± *CM* = *CN* = 112.1 MPa **Ans.**

Mohr's Circle for Stresses on an Oblique Section of a Body Subjected to Direct Stresses in Two Mutually Perpendicular Directions Accompanied by a Simple Shear Stress



Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to direct tensile stresses in two mutually perpendicular directions along *X-X* and *Y-Y* axes accompanied by a positive (*i.e.*, clockwise) shear stress along *X-X* axis as shown in Fig. 7.25 (*a*) and (*b*). Now let us consider an oblique section *AB* inclined with *X-X* axis on which we are required to find out the stresses as shown in the figure.

Let
σ_x = Tensile stress in *X-X* direction,
σ_y = Tensile stress in *Y-Y* direction,
τ_{xy} = Positive (*i.e.*, clockwise) shear stress along *X-X* axis, and
θ = Angle which the oblique section *AB* makes with *X-X* axis in clockwise direction.

First of all, consider the equilibrium of the wedge *ABC*. We know that as per the principle of simple shear, the face *BC* of the wedge will be subjected to an anticlockwise shear stress equal to τ_{xy} as shown in Fig. 7.25 (*b*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.26 and as discussed below :

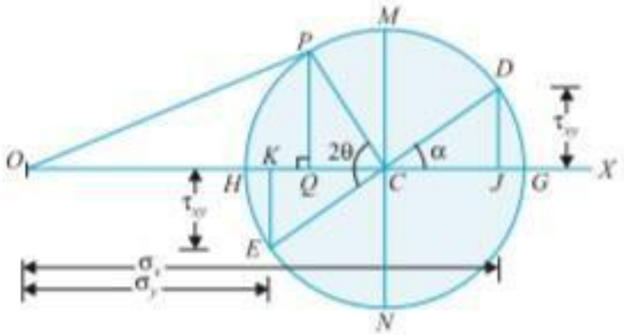
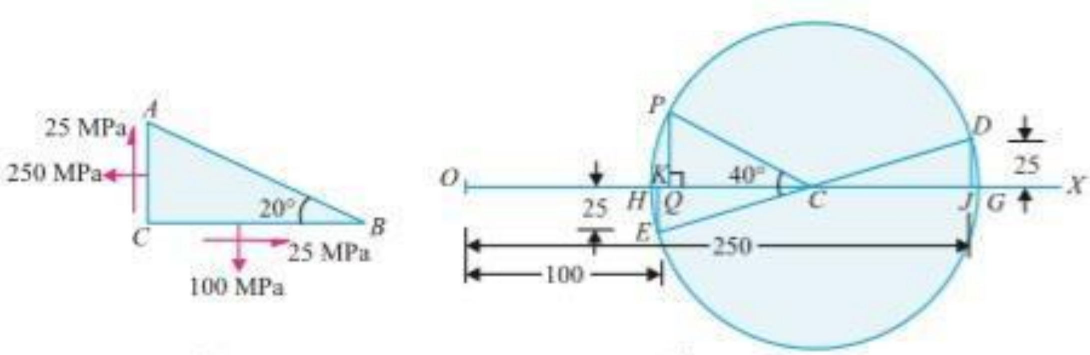


Fig. 7.26

1. First of all, take some suitable point *O* and through it draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively to some suitable scale and towards right (because both the stresses are tensile).
3. Now erect a perpendicular at *J* above the line *X-X* (because τ_{xy} is positive along *X-X* axis) and cut off *JD* equal to the shear stress τ_{xy} to the scale. The point *D* represents the stress system on plane *AC*. Similarly, erect perpendicular below the line *X-X* (because τ_{xy} is negative along *Y-Y* axis) and cut off *KE* equal to the shear stress τ_{xy} to the scale. The point *E* represents the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw a circle. It is known as Mohr's Circle of Stresses.
5. Now through *C*, draw a line *CP* making an angle 2θ with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on section AB.
6. Through *P*, draw *PQ* perpendicular to the line *OX*. Join *OP*.
7. Now *OQ*, *QP* and *OP* will give the normal stress, shear stress and resultant stress respectively to the scale. Similarly *OG* and *OH* will give the maximum and minimum principal shear stresses to the scale. The angle *POC* is called the angle of obliquity.

20. *A point is subjected to a tensile stress of 250 MPa in the horizontal direction and another tensile stress of 100 MPa in the vertical direction. The point is also subjected to a simple shear stress of 25 MPa, such that when it is associated with the major tensile stress, it tends to rotate the element in the clockwise direction. What is the magnitude of the normal and shear stresses inclined on a section at an angle of 20° with the major tensile stress ?*

***SOLUTION.** Given : Tensile stress in horizontal direction (σ_x) = 250 MPa ; Tensile stress in vertical direction (σ_y) = 100 MPa ; Shear stress (τ) = 25 MPa and angle made by section with major tensile stress (θ) = 20°.



(*a*) (*b*)

The given stresses on the face *AC* of the point alongwith a tensile stress on the plane *BC* and a complimentary shear stress on the plane *BC* are shown in Fig 7.27 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.27 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and through it draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively (*i.e.*, 250 MPa and 100 MPa) to some suitable scale towards right.
3. Now erect a perpendicular at *J* above the line *OX* and cut off *JD* equal to the positive shear stress on the plane *AC* (*i.e.*, 25 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *K* below the *OX* and cut off *KE* equal to the negative shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C* draw a line *CP* making an angle of 2 × 20° = 440° with *CE* in clockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section to *AB*.
6. Through *P*, draw *PQ* perpendicular to the line *OX*.

   By measurement, we find that the normal stress, (σ_x) = *OQ* = 101.5 MPa and shear stress τ = *QP* = 29.0 MPa **Ans.**

21. *A point in a strained material is subjected to the stresses as shown in Fig. 7.29. Find graphically, or otherwise, the normal and shear stresses on the section AB.*

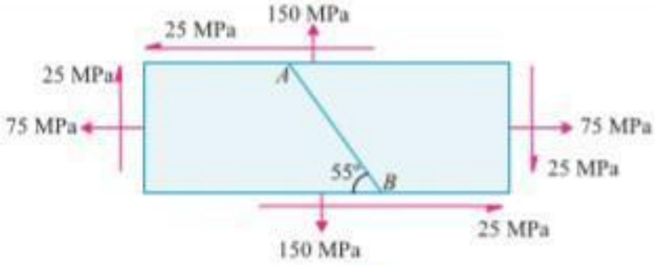


Fig. 7.29

*Solution. Given : Tensile stress along horizontal *x-x* axis (σ_x) = 75 MPa ; Tensile stress along vertical y-y axis (σ_y) = 150 MPa ; Shear stress (τ_{xy}) = 25 MPa and angle made by section with horizontal tensile stress in clockwise direction (θ) = 55°.

The given stresses on the planes *AC* and *BC* are shown in Fig.7.30 (*a*). Now draw the Mohr's Circle of Stresses as shown in Fig. 7.30 (*b*) and as discussed below :

1. First of all, take some suitable point *O*, and draw a horizontal line *OX*.
2. Cut off *OJ* and *OK* equal to the tensile stresses σ_x and σ_y respectively (*i.e.*,75 MPa and 150 MPa) to some suitable scale towards right.

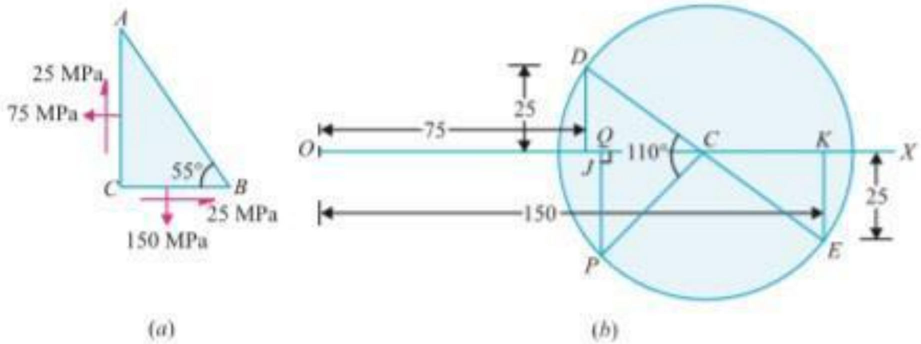


Fig. 7.30

3. Now erect a perpendicular at J above the line *OX* and cut off *JD* equal to the positive shear stress on the plane *AC* (*i.e.*, 25 MPa) to the scale. The point *D* represents the stress system on the plane *AC*. Similarly, erect a perpendicular at *K* below the line *OX* and cut off *KE* equal to the negative shear stress on the plane *BC* (*i.e.*, 25 MPa) to the scale. The point *E* represents the stress system on the plane *BC*. Join *DE* and bisect it at *C*.
4. Now with *C* as centre and radius equal to *CD* or *CE* draw the Mohr's Circle of Stresses.
5. Now through *C* draw a line *CP* making an angle of 2 × 55° = 110° with *CD* in an anticlockwise direction meeting the circle at *P*. The point *P* represents the stress system on the section *AB*.

   By measurement, we find that the normal stress (σ_n) = *OQ* = 76.1 MPa and shear stress (τ) = *PQ* = – 26.7 MPa. **Ans.**