

SNS COLLEGE OF TECHNOLOGY

SIS

(An Autonomous Institution)
Coimbatore-641035.

UNIT-III COMPLEX DIFFERENTIATION

HARMONIC FUNCTION

taplace equation:

3 d + 3 d = 0 % called Laplace equation.

Harmong coquation:

Any function with a variables having the good order partial desiratives which satisfies laplace eqn. B called a barmonec eqn.

conjugate Hasmonic punction:

If u and v core has monic functions such that utiv is analytec, then earn is called the conjugate has monic of the other.

Hore u is conjugate has monic of v and v is conjugate has monic of u.

I. prove that $u = e^{x} \cos y$ & bosimon?c. Soln.

Creen
$$u = e^{x} \cos y$$

$$\frac{\partial u}{\partial x} = e^{x} \cos y \quad \left| \frac{\partial u}{\partial y} = -e^{x} \sin y \right|$$

$$\frac{\partial^{2} u}{\partial x^{2}} = e^{x} \cos y \quad \left| \frac{\partial^{2} u}{\partial y^{2}} = -e^{x} \cos y \right|$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{x} \cos y - e^{x} \cos y$$

Hence u satisfies laplace equation.

The function us barmonic.

EJ. PHOVE that $u = \frac{1}{2} \log (x^2 + y^2)$ is has months.



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Solh.

Leven
$$u = \frac{1}{2} \log (x^2 + y^2)$$
 $\frac{\partial u}{\partial x} = \frac{1}{2} \frac{1}{x^2 + y^2} (2x)$
 $\frac{\partial u}{\partial x} = \frac{1}{2} \frac{1}{x^2 + y^2} (2x)$
 $\frac{\partial^2 u}{\partial x^2} = \frac{(x^2 + y^2)(1) - x(2x)}{(x^2 + y^2)^2}$
 $= \frac{x^2 + y^2 - 2x^2}{(x^2 + y^2)^2}$
 $\frac{\partial u}{\partial y^2} = \frac{1}{(x^2 + y^2)^2} (2x^2 + y^2)^2$
 $= \frac{x^2 + y^2}{(x^2 + y^2)^2} + \frac{x^2 - y^2}{(x^2 + y^2)^2}$
 $= \frac{x^2 + y^2}{(x^2 + y^2)^2} + \frac{x^2 - y^2}{(x^2 + y^2)^2}$

Thence $u = \frac{x^2 + y^2 + x^2 - y^2}{(x^2 + y^2)^2}$

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