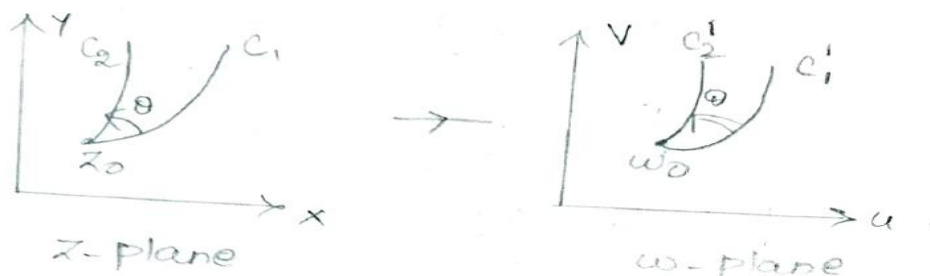




Conformal mapping:

A transformation that preserves angles between every pair of curves through a point both in magnitude and sense, is said to be conformal at that point.



Isogonal mapping:

A transformation under which angles between every pair of curves through a point are preserved in magnitude, but different in direction.



Critical point:

The point z_0 is said to be critical point of the transformation $w = f(z)$ if $f(z)$ is not conformal at z_0 . The critical point is defined by,

$$\frac{dw}{dz} = 0 \text{ and } \frac{dz}{dw} = 0$$



1]. Find the critical point of the transformation

$$w = z + \frac{1}{z}$$

Soln.

Given $w = z + \frac{1}{z}$

$$\frac{dw}{dz} = 1 - \frac{1}{z^2} = \frac{z^2 - 1}{z^2}$$

$$\frac{dz}{dw} = \frac{z^2}{z^2 - 1}$$

Now $\frac{dw}{dz} = 0 \Rightarrow \frac{z^2 - 1}{z^2} = 0$

$$z^2 - 1 = 0$$

$$z^2 = 1$$

$$z = \pm 1$$

and $\frac{dz}{dw} = 0 \Rightarrow \frac{z^2}{z^2 - 1} = 0 \Rightarrow z^2 = 0$
 $z = 0$

The critical points are $0, \pm 1$.

2]. Find the critical point of $w^2 = (z - \alpha)(z - \beta)$

Soln.

Given $w^2 = (z - \alpha)(z - \beta)$

Differentiate w.r. to z,

$$2w \frac{dw}{dz} = (z - \alpha)(1) + (z - \beta)(1)$$
$$= (z - \alpha) + (z - \beta)$$

$$\frac{dw}{dz} = \frac{2z - (\alpha + \beta)}{2w}$$

and $\frac{dz}{dw} = \frac{2w}{2z - (\alpha + \beta)}$

Now $\frac{dw}{dz} = 0 \Rightarrow \frac{2z - (\alpha + \beta)}{2w} = 0$

$$2z = \alpha + \beta$$

$$z = \frac{\alpha + \beta}{2}$$



$$\text{and } \frac{dz}{dw} = 0 \Rightarrow \frac{2w}{2z - (z+\beta)} = 0$$

$$2w = 0$$

$$w = 0$$

$$\sqrt{(z-\alpha)(z-\beta)} = 0$$

$$(z-\alpha)(z-\beta) = 0$$

$$z = \alpha \text{ and } z = \beta$$

\therefore The critical points are $z = \frac{\alpha+\beta}{2}, \alpha, \beta$.

3]. Find the critical point of $w = z^4 - 1$

Soln.

$$\text{Given } w = z^4 - 1$$

$$\text{Now, } \frac{dw}{dz} = 4z^3 \Rightarrow \frac{dw}{dz} = 0 \Rightarrow 4z^3 = 0$$

$$z = 0$$

$$\text{and } \frac{dz}{dw} = \frac{1}{4z^3} = 0, \text{ which is not possible}$$

\therefore The critical point is 0.

4]. Find the critical point of $w = \sin z$

Soln.

$$\text{Given } w = \sin z$$

$$\frac{dw}{dz} = \cos z \Rightarrow \frac{dw}{dz} = 0$$

$$\cos z = 0$$

$$z = \cos^{-1}(0)$$

$$z = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \dots$$

$$z = \pm (2n-1) \frac{\pi}{2} \text{ where } n=1, 2, 3, \dots$$

$$\text{and } \frac{dz}{dw} = 0 \Rightarrow \frac{1}{\cos z} = 0, \text{ which is not possible.}$$

$$\text{How } \frac{1}{\cos z} = 0$$



Fixed point:

A point $z=a$ is said to be fixed point of a mapping $w=f(z)$ if its image under $f(z)$ is itself.

$$\text{i.e., } f(z) = z$$

1] Find the invariant points of $w = \frac{1+z}{1-z}$

Soln.

$$\text{Given } w = \frac{1+z}{1-z}$$

The invariant points are,

$$z = \frac{1+z}{1-z} \quad (\because w = f(z) = z)$$

$$z(1-z) = 1+z$$

$$z - z^2 = 1+z$$

$$z - z^2 - 1 - z = 0$$

$$-z^2 - 1 = 0$$

$$z^2 = -1$$

$$z = \pm i$$

2] Find the fixed points of $w = \frac{2z+6}{z+7}$

Soln.

$$\text{Given } w = \frac{2z+6}{z+7}$$

The fixed points are,

$$z = \frac{2z+6}{z+7}$$

$$z(z+7) = 2z+6$$

$$z^2 + 7z - 2z - 6 = 0$$

$$z^2 + 5z - 6 = 0$$

$$(z+6)(z-1) = 0$$

$$z = 1, -6$$